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THIRD ALL-UNION SYMPOSIUM ON WAVE DIFFRACTION.(U)
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FOREIGN TECHNOLOGY DIVISION

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THIRD ALL-UNION SYMPOSIUM ON WAVE DIFFRACTION

FOREIGN TECHNOLOGY DIVISION

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THIRD ALL-UNION SYMPOSIUM ON WAVE DIFFRACTION

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U. S. BOARD ON GEOGRAPHIC NAMES TRANSLITERATION SYSTEM

Block	Italic	Transliteration	Block	Italic	Transliteration
А а	*А а*	A, a	Р р	*Р р*	R, r
Б б	*Б б*	B, b	С с	*С с*	S, s
В в	*В в*	V, v	Т т	*Т т*	T, t
Г г	*Г г*	G, g	У у	*У у*	U, u
Д д	*Д д*	D, d	Ф ф	*Ф ф*	F, f
Е е	*Е е*	Ye, ye; E, e*	Х х	*Х х*	Kh, kh
Ж ж	*Ж ж*	Zh, zh	Ц ц	*Ц ц*	Ts, ts
З з	*З з*	Z, z	Ч ч	*Ч ч*	Ch, ch
И и	*И и*	I, i	Ш ш	*Ш ш*	Sh, sh
Й й	*Й й*	Y, y	Щ щ	*Щ щ*	Shch, shch
К к	*К к*	K, k	Ъ ъ	*Ъ ъ*	"
Л л	*Л л*	L, l	Ы ы	*Ы ы*	Y, y
М м	*М м*	M, m	Ь ь	*Ь ь*	'
Н н	*Н н*	N, n	Э э	*Э э*	E, e
О о	*О о*	O, o	Ю ю	*Ю ю*	Yu, yu
П п	*П п*	P, p	Я я	*Я я*	Ya, ya

*ye initially, after vowels, and after ъ, ь; e elsewhere.
When written as ё in Russian, transliterate as yё or ё.

RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

Russian	English	Russian	English	Russian	English
sin	sin	sh	sinh	arc sh	$\sinh^{-1}$
cos	cos	ch	cosh	arc ch	$\cosh^{-1}$
tg	tan	th	tanh	arc th	$\tanh^{-1}$
ctg	cot	cth	coth	arc cth	$\coth^{-1}$
sec	sec	sch	sech	arc sch	sech^{-1}
cosec	csc	csch	csch	arc csch	csch^{-1}

Russian	English
rot	curl
lg	log

GRAPHICS DISCLAIMER

All figures, graphics, tables, equations, etc. merged into this translation were extracted from the best quality copy available.

THIRD ALL-UNION SYMPOSIUM ON WAVE DIFFRACTION.

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From the Organizing Committee.

The third All-Union symposium on the wave diffraction is called according to the plan/layout of the actions of the Academy of Sciences of the USSR (Sections of the diffraction of electromagnetic, acoustic and other waves of Council for the acoustics of the AS USSR) for the coordination of scientific research in the region of the theory of diffraction.

Symposium was conducted in Tbilisi from 24 to 30 September, 1964 by the Section of diffraction and by State Committee for radio electronics together with the Academy of Sciences of Georgian SSR, with the participation of Acoustic institute, Tbilisi State university and Georgian polytechnic institute.

The organization committee of symposium included: V. D. Kupradze (chairman), G. D. Malyuzhinets (representative of GKRE, deputy chairman), D. Z. Avazashvili, V. A. Borovikov, L. A. Vaynshteyn, S. S. Voyt, G. V. Glenkin, N. A. Kuz'min, G. I. Makarov, V. A. Marchenko, M. A. Miller, G. I. Petrashen' and Yu. A. Ukhanov (the scientific secretary of organization committee and the sections of

wave diffraction).

Into the present collection entered all in proper time sent by the authors abstracts of the reports, accepted for listening by the symposium. The part of these reports in the program of symposium is united into the reviews. Abstracts in the collector/collection are united into three sections:

I. Development of the methods of solving the diffraction problems, connected with the linear partial differential equations.

II. Specific problems of the theory of diffraction.

III. Use/application of theory of diffraction and adjacent questions (in particular, some wave problems, connected with the statistics and the nonlinearity, and also with the use of geometric-optical methods).

Of these sections the first, as the largest/coarsest in turn, is decomposed into fifteen subsections.

Pages 3-10. (No typing).

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PREFACE.

The use of laws governing the diffraction, i.e., the behavior of waves in the conditions, complicated by natural or technical facts, is basis all of the expanding practical uses/applications of wave processes in different branches of science and engineering, in particular in radio engineering, acoustics, seismic surveying, hydrodynamics, etc.

The contemporary technical developments, connected with radiation, propagation, reflection and reception/procedure of waves, cannot be efficient without conducting of the diffraction calculations, which lead to ever more complicated mathematical problems also of the needing development new mathematical methods.

As a result of these necessities of development of wave technology in the latter/last 10-15 years the development of the theory of diffraction both for the USSR and abroad was very intensified, about which testify thousands of scientific works, published on questions of diffraction.

Rapidly is changed very character of this development. Ever greater role is assigned to mathematical methods and relatively smaller - physical intuition. Occurs a break with more than one-hundred year's tradition, which related the theory of diffraction in essence to the field of physical sciences and establishing when the separate results, obtained strictly mathematically, appeared to be museum rarities not affecting the matter. Now, when yearly are published the solutions or the research of the solutions of such problems which for many years seemed insurmountably difficult for the mathematical examination, the theory of diffraction on the eyes is converted from the branch of physics into the section of applied mathematics, connected with physics with its applications/appendices section, mathematical content of which is the development of the efficient analytical and computational methods of solving different types of problems for the steady-state (elliptic) and unsteady wave equations.

This rearrangement of the developing theory of diffraction is complicated by the effect of the tradition mentioned above, and also by the fact that at present is not yet solved the problem of VUZ preparation and distribution of specialists in the theory of wave diffraction. Under these conditions the especially important coordinating role play those called of times in two years according to the plan of the Academy of Sciences the symposia on the wave

diffraction (1960, 1962, 1964), which preceded two conferences of the united sections of the diffraction of acoustic, electromagnetic, elastic and hydrodynamic waves at the All-Union acoustic conferences (1957, 1958).

Experiment of conducting conferences and symposia on the wave diffraction 1957-1962 shows their stimulating effect on the development of the theory of diffraction. Participation in the symposia of a considerable quantity of representatives of the wave branches of technology contributes to the introduction of the more advanced methods of diffraction calculations and thereby it raises the efficiency of resolving the complex problems of wave technology.

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At the same time is discovered, on one hand, the mathematical qualification of the physicists and engineers increasing from the symposium to the symposium, is occupied by diffraction calculations for the technical developments, and, on the other hand, the increasing interest in the diffraction problems on the part of the mathematicians and their fruitful participation in the development, these tasks. To the enlistment of the mathematicians contributes the fact that, in contrast to the specialized conferences on one or the other branch of wave technology, on the symposia on the wave

diffraction the sufficient accent is placed on the systematic, in particular the mathematical, side of the discussed works.

The present collection of abstracts shows that the third symposium on the wave diffraction is further space, which draws together the theory of diffraction with mathematics and which demonstrates the power of mathematical methods (among which are for the first time proposed or substantially developed) for the solution of new diffraction problems.

In spite of the squishiness of presentation in some abstracts, collector/collection is of interest not only for participants in the symposium, but also for other readers, who desire to be acquainted with the contemporary development of the theory of diffraction in statu nascendi, or to obtain representation about the diversity of mathematical methods utilized in this theory.

Section of the diffraction of electromagnetic, acoustic and other waves. Council on Acoustics of the AS USSR.

I. DEVELOPMENT OF METHODS OF SOLVING DIFFRACTION PROBLEMS CONNECTED WITH LINEAR PARTIAL DIFFERENTIAL EQUATIONS.

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1. PHYSICAL SETTINGS AND THE MATHEMATICAL FORMULATIONS OF PROBLEMS. QUESTIONS OF UNIQUENESS AND EXISTENCE.

SOME QUESTIONS OF THE RADIOWAVE PROPAGATION OF THE SDV RANGE.

E. M. Gyunninen, G. I. Makarov, V. V. Novikov.

In recent years considerable attention is given to questions of the propagation of the electromagnetic waves of low frequency in the waveguide channel the Earth - ionosphere. In the present report is given the survey/coverage of the basic tasks, which appear during the study of this problem, and the results, obtained in recent years by different authors. At the same time we will give some results of studies and numerical calculations of electromagnetic field, carried out in the department of radiophysics of LGU.

It is interesting, in the first place, to note two different mathematical approaches to the construction of the solution of the problem in question. The equations of Maxwell in the case of the isotropic ionosphere and point vertical source (electrical or

magnetic) are reduced with the harmonic dependence on the time to the two-dimensional equation of Helmholtz for the potentials, which by the method of separation of variable/alternating is reduced to two ordinary differential equations for the radial and azimuth functions. For further construction of the solution it is possible to use either its own azimuth functions or its own radial functions. In the first case we come to the decision in the form of the series/row of zonal harmonics (series/row of Debye), in the second case - to the series/row of normal waves. Series/row on the normal waves can be obtained also from the series/row of Debye with the help of the conversion of Watson.

The ionosphere is inhomogeneous medium; however, to construct the general analytical solution of task with the heterogeneous ionosphere is impossible. Therefore in the majority of works is used during the construction of decision the model of the ionosphere in the form of homogeneous medium with the sharp lower spherical boundary. In some works is used the more composite model of the ionosphere in the form of the medium, which consists of the spherical concentric layers with constant properties.

During the calculation of the field of the electromagnetic waves, which extend in the waveguide channel, it is possible to go three variously. The first path is based on the use of a series/row

of Debye (Joler). However, the direct addition of the series/row of Debye is hindered/hampered by his poor convergence; therefore is used the method of an improvement in the convergence, which consists in the fact that from each term of series/row they subtract its asymptotic representation at the high frequencies. The series/row of Debye proves to be possible to use for the calculations of field at the low frequencies (to 50 kHz), with exception of the regions of the high fading of field.

The alternate path consists in the use of a series/row on the normal waves (P. Ye. Krasnushkin).

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In this case it is necessary to find eigenvalues of differential equation for the radial functions, which are zero complicated transcendental equation. The solution of this transcendental equation presents basic and actually only difficulty during the calculation of field. The analysis of convergence of series on the normal waves shows that it is expedient to use at large distances from the source (order 1000 km and further).

Finally, the third possible path, convenient with the small distances from the source, lies in the fact that to represent the

solution as expansion whose each member is represented by series/row or integral, which makes sense repeatedly of the reflected from the ionosphere and earth's surface of wave. Depending on the mutual location of source and observation point one or another term of expansion can make sense of "straight/direct" or "diffraction" ray/beam.

In the case of "straight/direct" rays/beams the solution is represented in the form of reflecting formulas (corresponding integrals are computed by the steepest descent method). However, in the case of "diffraction" rays/beams the numerical possible integration of integrals for the ducts/contours, close to the ducts/contours of the fastest descent. Calculation on the deductions in this latter/last case is hindered/hampered in connection with an increase in the multiplicity of poles with an increase in the number of reflections. This path possesses the advantage that it makes it possible simply to take into account the heterogeneity of the ionosphere on the height/altitude, and also the effect of the magnetic field of the Earth.

In the report are given the results of research and numerical calculations of the eigenvalues of waveguide task and structure of electromagnetic field. Are given also the results of the calculations of reflection coefficients from the heterogeneous isotropic and

anisotropic ionosphere at different angles of incidence over a wide range of the frequencies which are used during the determination of field with the use of resolution in terms of the "rays/beams". In the latter case is conducted the evaluation of the effect of the magnetic field of the Earth.

DIFFRACTION OF LONG WAVES IN THE OCEAN.

P. S. Lineykin.

We examine the effect of solid walls on the propagation of the long waves in the uniform ocean of constant depth on the revolving Earth. Research is produced on the basis of the model of the so-called β- plane where a change in Coriolis's parameters along the meridian is approximated by linear function. Is found out the distribution of speeds and the increase of the level of the floating surface of ocean.

PROBLEMS OF THE DIFFRACTION AND PROPAGATION OF ELECTROMAGNETIC WAVES IN A PLASMA.

N. A. Kuz'min.

The flow of works on the theory of waves in the plasma is extremely great, and to do the comprehensive survey/coverage in one report is very difficult; therefore we previously reject the complete illumination of entire material on this question, and we will be bounded to the task of the familiarization of the specialists in the theory of wave diffraction only with the basic results of the theory of electromagnetic waves in the plasma and with some questions in examination of which we took personal part. As a whole the material of report will make it possible to obtain representation about the character of differential equations, about the types of boundary conditions, by which they must satisfy waves in the plasma, and also about the degree of approximation of theory during the study of the effects, which are of interest for radio engineering. Sometimes will be reported the concrete/specific/actual results of theory.

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Sufficiently complete material according to the theory of electromagnetic waves in the plasma and on general/common/total questions of the electrodynamics of plasma with the detailed bibliography is in the monographs of V. L. Ginsburg, V. P. Silin and A. A. Rukhadze, V. D. Shafranova, also, in the collectors/collections "Questions of the theory of plasma" edited by M. A. Leontovich, published in the USSR, and in the monographs of L. Spitzer, T. Kh. Stiks, V. Alis, Ridbek, D. Denisse and D. Delcroix, etc., published outside the boundary.

During the study of the effects of propagation and diffraction of electromagnetic waves in the plasma for purposes of radio engineering in the majority of the cases it suffices to proceed from the results of the phenomenological theory according to which can be introduced the tensor of the dielectric constant of medium (plasma) from the equation of motion of electron without taking into account their thermal agitation. This approach makes it possible in many instances to obtain the results, which are coordinated with the experimental data, and at the same time making it possible to analyze in the final formulas the interesting physical corollaries of theory. A strict formulation of the problem with the self-consistent equations of Maxwell and the kinetic theory in many practical important cases proves to be unsolvable in the explicit analytical form. In connection with this it is important for those cases when

there are solutions for the self-consistent system of equations, to come to light/detect/expose the limits of the applicability of the phenomenological theory of "average/mean electron". All these questions will be presented in the report, and also adjoining them: the account of spatial dispersion, the effect of the temperature in the "cold" plasma, absorption by Landau, high-frequency energy losses in the plasma as a result of the internal collisions of the particles of different type, etc.

In the report will be given some results according to the theory of the propagation of waves in the plasma, placed into the waveguide with the conducting walls. For the waveguide tasks with gyrotropic plasma in the magnetostatic field, directed along the axis of waveguide, it succeeds the boundary-value problem about eigenvalues of leading to the variational problem of stationary functional for special potentials. High-frequency energy losses in the plasma-type waveguide, as it will be shown, are caused by a number of factors: first, by the internal collisions of particles in the waveguide volume of space, in the second place, by the effects of the collision of particles near the wall with the wall of waveguide (theoretical result of B. T. Kormilitsyn) and, thirdly, the heat losses in the walls of waveguide.

In a number of cases of task the diffractions of electromagnetic

waves on the surfaces with the laminar gyrotropic plasma can be strictly solved by Wiener-Hopf-Foch's method, if for the boundary conditions of the task of using generalized impedance boundary conditions for the tensor media. In the report will be given the boundary conditions indicated for the anisotropic laminar media.

From the diffraction tasks will be briefly reported the results on the wave dissipation during the simplest plasma formation/educations (sphere, cylinder).

THE TENSOR OF THE EFFICIENT DIELECTRIC CONSTANT OF HETEROGENEOUS MAGNETICALLY ACTIVE PLASMA.

Yu. A. Ryzhov, V. V. Tamoykin.

It is known that for describing the middle field in the randomly inhomogeneous medium it is convenient to use an efficient dielectric constant. In the series/row of works this value was calculated for the isotropic weakly inhomogeneous medium. Is of interest obtaining formulas for the efficient dielectric constant of heterogeneous magnetically active plasma.

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Calculation is conducted on the assumption that the fluctuations of electronic concentration are assigned. Is examined the uniform probability field of the fluctuations of electron density.

The tensor of the efficient dielectric constant of magnetically active inhomogeneous plasma is determined with the help of the relationship/ratio

$$\langle D_i \rangle = \hat{\varepsilon}_{ij}^{\text{эф}} \langle E_j \rangle = \varepsilon_{ij}^{0} \langle E_j \rangle + \langle \Delta\varepsilon_{ij} e_j \rangle . \qquad (1)$$

where $\langle D_i\rangle$, $\langle E_i\rangle$ - average/mean values of electrical induction and field in the inhomogeneous medium, ε^0_{ij} - average/mean value of the tensor of dielectric const.nt of magnetically active plasma, $\Delta\varepsilon_{ij}$ - the fluctuation tensor of dielectric constant, e_i - fluctuation field in the plasma. Fluctuation field e_j is described with the help of the equation

$$\Delta e_i - \frac{\partial^2 e_j}{\partial x_i \partial x_j} + k_0^2 \varepsilon^0_{ij} e_j = -k_0^2 \Delta\varepsilon_{ij} \langle E_j\rangle, \qquad (2)$$

which is correct in the slightly inhomogeneous plasma.

It is easy to obtain following connection/communication of vectors <D> and <E>:

$$\langle D_i\rangle = \varepsilon^0_{ij}\langle E_j\rangle + \frac{k_0^2}{2(2\pi)^3} e_{k\alpha\beta} e_{imn} \int \langle \Delta\varepsilon_{ij}(\mathbf{r})\, \Delta\varepsilon_{kp}(\mathbf{r}_1)\rangle D_{\alpha m} D_{\beta n} \times$$
$$\times \langle E_p(\mathbf{r}_1)\rangle I_0(\mathbf{r}, \mathbf{r}_1)\, d\mathbf{r}_1;$$
$$I_0(\mathbf{r}-\mathbf{r}_1) = \frac{1}{(2\pi)^3}\int \frac{e^{i\mathbf{k}(\mathbf{r}-\mathbf{r}_1)}}{\Delta(\mathbf{k})}\, d\mathbf{k}, \quad D_{\alpha m} = \frac{\partial}{\partial x_\alpha\, \partial x_m} - \nabla^2\delta_{\alpha m} - k_0^2 \varepsilon_{\alpha m}.$$

From this expression there can be obtained general/common/total expression for tensor $\varepsilon^{\text{эф}}_{ij}(\omega, \mathbf{k})$. In view of the complexity of general/common/total expression we were bounded to the case when it is possible to disregard the phenomenon of spatial dispersion, caused by the heterogeneities of medium. This case is valid with

$$k_1 l,\ k_2 l \ll 1, \qquad (3)$$

where l - radius of the correlation of fluctuations ΔN; k_1, k_2 - the wave numbers of unusual and ordinary waves.

In this case operator $\hat{\varepsilon}_{ij}^{\text{эф}}(\omega)$ is degenerated into tensor $\varepsilon_{ij}^{\text{эф}}(\omega)=\varepsilon_{ij}^{0}(\omega)+\Lambda_{ij}(\omega)$, where

$$\Lambda_{ij}=\frac{k_0^2}{2(2\pi)^3}e_{k\alpha\beta}e_{pmn}(\varepsilon_{ip}^0-\delta_{ip})(\varepsilon_{kj}^0-\delta_{kj})\frac{\overline{\Delta N^2}}{N_0^2}\int\Gamma_N(\mathbf{r}-\mathbf{r}')D_{\alpha m}D_{\beta n}I_0(\mathbf{r}-\mathbf{r}_1)\,d\mathbf{r}_1.$$

Further calculation leads to the following expression for $\varepsilon_{ij}^{\text{эф}}(\omega)$:

$$\varepsilon_{ij}^{\text{эф}}=\begin{Vmatrix}\varepsilon^{\text{эф}} & -ig^{\text{эф}} & 0\\ ig^{\text{эф}} & \varepsilon^{\text{эф}} & 0\\ 0 & 0 & \eta^{\text{эф}}\end{Vmatrix},$$

where

$$\begin{aligned}\varepsilon^{\text{эф}}&=\varepsilon+A_1+B_1(k_0l)^2+iC_1(k_0l)^3+iC_1'(k_0l)^3,\\ \eta^{\text{эф}}&=\eta+A_2+B_2(k_0l)^2+iC_2(k_0l)^3+iC_2'(k_0l)^3,\\ g^{\text{эф}}&=g+A_3+B_3(k_0l)^2+iC_3(k_0l)^3+iC_3'(k_0l)^3.\end{aligned}\tag{4}$$

Here coefficients A_1, A_2, A_3, which do not depend on (k_0l), are polarizational numbers. Coefficients C_1, C_2, C_3 (C'_1, C'_2, C'_3) describe fading middle field due to the scattering into the unusual (ordinary) wave.

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All coefficients in (4) are represented in the form of simple integrals on the angle θ of the expressions, which contain the refractive indices of ordinary and extraordinary waves. Is investigated a special case of weak gyrotropy.

IMPEDANCE BOUNDARY CONDITIONS ON THE SURFACE OF A PLASMA WITH SHARPLY VARYING PARAMETERS.

Yu. I. Orlov, V. A. Permyakov, Ye. N. Vasil'yev.

Are obtained the approximate boundary conditions on the surface of the plasma whose dielectric constant sharply varies along the normal to the interface. During the conclusion of boundary conditions was examined the task about oblique incidence in plane electromagnetic wave on plane interface with the inhomogeneous plasma. It was assumed that the electronic concentration varies linearly along the normal to the interface and collision frequency does not depend on coordinates. Are examined the cases of TE and TM polarization. In both cases boundary conditions are obtained by the resolution of the corresponding solutions of wave equation in the series/row according to the degrees of $1/a$ (a - the gradient of dielectric constant - the high parameter of the problem). If we are bounded to the first term of expansion, then the obtained impedance will not depend on the angle of incidence in the plane wave:

$$Z = i\sqrt{\frac{\mu_0}{\varepsilon_0}} \cdot \frac{\Gamma(1/3)}{3^{1/3}\,\Gamma(2/3)} \cdot \left(\frac{K_0}{a}\right)^{1/3}. \qquad (1)$$

In formula (1) $K_0, \varepsilon_0, \mu_0$ - the parameters of free space. The account of the following terms of expansion gives the dependence of

impedance on the angle of incidence. For both polarizations are obtained the estimations of the applicability of boundary conditions (1). It is physically clear that the boundary conditions can be generalized in the cases of nonplanar waves and interfaces with a small curvature.

SOME QUESTIONS OF QUASI-OPTICS.

B. Z. Katsenelenbaum.

1. Appearance of open resonators and open (lens and mirror) transmission lines generated interest in special case of propagation of waves, with which field forms thin long beam. The transverse size/dimension of beam a is great in comparison with the wavelength $2\pi/k$, so that the correctly normal condition for the applicability of the geometric optic/optics $ka>>1$, but, furthermore, the length of beam L is great in comparison with a, $L/a>>1$, and the relation of these two high parameters of the task

$$C = \frac{ka^2}{L}$$

is, generally speaking, a finite number. At a great distance diffraction phenomena play very large role, and mathematical analysis leads to the peculiar asymptotic tasks of the theory of diffraction. The analog of these tasks is the Debye theory of the focal spot of lens. With final C the discussion deals with Fresnel diffraction on the large bodies.

The basic element of resonator or line - bent mirror, the combination of several mirrors, lens - are phase corrector.

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Its role lies in the fact that by introduction into the beam of the phase difference, different at different points of section, to convert divergent beam into that converging. The existing theories make it possible to compute the structure of the fields of natural oscillations (their own waves) and radiation losses with different phase-correcting elements/cells (see survey in UFN, 83, No 1, 81-105, May 1964).

2. Development occurs in several directions. Thoroughly are investigated different phase correctors (periscopic mirror systems, composite lenses). Is made more precise the theory of correctors themselves; they are calculated at present in the simplest geometric-optical approximation/approach, and diffraction on the edge of mirror or lens is considered very approximately. In this connection are done the attempts to develop the asymptotic theory of diffraction on the final (large) dielectric body and the mirror. Are computed losses during the transformation of some transmission modes into others, caused by divergence in position or form of phase correctors, and are established/installed allowances for the manufacturing precision. Is done the attempt to create the efficient

theory of exciters for the open lines. Are developed the antenna aspects of theory.

3. Most complicated and unclear at present question consists of creation of mathematical apparatus which would make it possible to use methods of geometric optic/optics. With large $\dot{C}$ the fields even far from the correctors actually/really can be described in the terms of ray theory, moreover essential is the concept of the caustic surface, which limits the pencil of rays. But with finite C, namely finite $C \sim 2\pi$ are of greatest interest for the millimeter waves, straight/direct use/application of ray theory for describing entire field is impossible. However, it can seem that its small modification will make it possible to create the apparatus, with which the calculation of the most essential parameters will not require each time of the solution of complicated diffraction problems. If this succeeded, then it would be possible to rapidly and efficiently create the sufficiently complete theory of quasi-optical devices/equipment.

4. Analogous situation was created in another group of quasi-optical tasks, which appear during the study of laws of propagation of waves in very wide waveguides. Usual waveguide methods are unacceptable with a large number of extending waves, and the character of the devices/equipment - mirrors used, the prisms -

prompts the need for applying the geometric-optical calculation methods. However, are usually of interest of loss to the transformation into the parasitic transmission modes, i.e., the values, not considered by ray theory and which carry diffraction character. Basic task is here the creation of sufficiently simple and common mathematical apparatus; however, in this direction there are only several preliminary considerations.

EXCITATION OF OPEN RESONATORS.

L. A. Veynshteyn.

At present are in sufficient detail investigated (with the help of the asymptotic methods of the theory of diffraction) the free oscillations/vibrations of the open resonators, in particular the resonators, formed by the well reflecting mirrors which are arranged/located against each other in the free space (flat/plane mirrors, spherical mirrors, etc.). Here it is possible to obtain (for the series/row of concrete/specific/actual systems) the simple approximations for the eigenfunctions of free oscillations/vibrations, at least for those from them, which have small radiation losses.

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These eigenfunctions in the simplest case satisfy the scalar wave equation

$$\Delta\Phi_s + k_s^2 \Phi_s = 0 \qquad (1)$$

in entire space, with exception of the volume of the mirrors, limited by surface of S, to the boundary condition

$$\Phi_s = 0 \quad \text{for } S \qquad (2)$$

and to radiation condition at infinity

$$\Phi_s = g_s(\vartheta, \varphi)\frac{e^{ik_sR}}{R} \quad \text{with} \quad R \to \infty. \qquad (3)$$

Complex functions $\Phi_s(x, y, z)$ determine real wave field $\mathrm{Re}(\Phi_s e^{-i\omega_s t})$, $k_s = \frac{\omega_s}{c}$. Due to the radiation/emission free oscillations/vibrations attenuate in the time, $k_s = k_s' - ik_s''$ and $k_s'' > 0$, therefore, according to formula (3), $|\Phi_s|$ grows with $R \to \infty$.

The calculation of natural oscillations does not exhaust by itself those tasks, which are of interest for the open resonators. Of the more complex problems which can be placed for these systems, we will consider the task about the excitation of monochromatic oscillations/vibrations and the Cauchy problem. The task about the excitation is reduced to the solution of the nonhomogeneous wave equation

$$\Delta\Phi + k^2\Phi = -4\pi\rho \qquad (4)$$

under the boundary condition

$$\Phi = 0 \quad \text{on } S \qquad (5)$$

and the condition for the absorption

$$\lim_{R\to\infty} R\Phi = 0 \quad \text{with} \quad \operatorname{Im} k > 0, \tag{6}$$

where $\rho(x,y,z)$ - the assigned function, sufficiently which rapidly decreases with $R\to\infty$. Cauchy is reduced to the solution of the unsteady wave equation

$$\Delta\Phi - \frac{1}{c^2}\frac{\partial^2\Phi}{\partial t^2} = 0 \tag{7}$$

under the same boundary condition (5) and initial conditions

$$\Phi = \Phi^0, \quad \frac{1}{c}\frac{\partial\Phi}{\partial t} = \Phi^1 \quad \text{with} \quad t = 0, \tag{8}$$

where $\Phi^0(x, y, z)$ and $\Phi^1(x, y, z)$ - assigned functions, sufficiently which rapidly decrease with $R\to\infty$.

The general solution of similar tasks can be obtained with the help of the eigenfunction expansion of the continuous spectrum which we designate through $\Phi_{\tau,\varkappa}(x, y, z)$ (τ - discrete/digital index, which substitutes two usual indices m and n; $\varkappa$ - continuous parameter) is determined as follows: functions $\Phi_{\tau,\varkappa}$ satisfy the wave equation

$$\Delta\Phi_{\tau,\varkappa} + \varkappa^2\Phi_{\tau,\varkappa} = 0, \tag{9}$$

to the boundary condition

$$\Phi_{\tau,\varkappa} = 0 \quad \text{on} \quad S \tag{10}$$

and they take with R→∞ the following form:

$$\Phi_{\tau,\varkappa} = g^0_{\tau,\varkappa}(\vartheta,\varphi)\frac{e^{-i\varkappa R}}{R} + g_{\tau,\varkappa}(\vartheta,\varphi)\frac{e^{i\varkappa R}}{R}, \quad (11)$$

where

$$g^0_{\tau,\varkappa}(\vartheta,\varphi) = \Gamma_\tau(\varkappa)\, g_{\tau,\varkappa}(\vartheta,\varphi). \quad (12)$$

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Functions $g_{\tau,\varkappa}$ are eigenfunctions of operator S^{-1}, and $\Gamma_\tau(\varkappa)$ - corresponding eigenvalues. Here S - operator, who connects the angular distribution of the divergent (scattered) wave with the angular distribution of the convergent (incident) spherical wave; S - symmetrical and unitary operator. It is easy to show that with real positive ones $\varkappa$ of function $\Phi_{\tau,\varkappa}$ satisfy the condition of the orthogonality

$$\frac{1}{4\pi}\int \Phi_{\tau,\varkappa}\Phi_{\tau',\varkappa'}\, dV = D_\tau(\varkappa)\,\delta_{\tau\tau'}\delta(\varkappa-\varkappa'), \quad (13)$$

where the integration is produced on entire space out of S. Therefore known functions it is possible to expand according to eigenfunctions $\Phi_{\tau,\varkappa}$, for example,

$$\Phi^0 = \sum_\tau \int_0^\infty c^0_\tau(\varkappa)\,\Phi_{\tau,\varkappa}\, d\varkappa,\quad c^0_\tau(\varkappa) = \frac{\int \Phi^0\Phi_{\tau,\varkappa}\, dV}{4\pi D_\tau(\varkappa)}, \quad (14)$$

but unknown functions, for example function Φ in the Cauchy problem, to seek in the form

$$\Phi = \sum_\tau \int_0^\infty C_\tau(\varkappa, t)\,\Phi_{\tau\varkappa}\, d\varkappa. \quad (15)$$

Simply it is possible to connect unknown function $C_\tau(\varkappa, t)$ with

known functions $c_\tau^0(\varkappa)$, $c_\tau^1(\varkappa)$ and thus to solve the Cauchy problem, just as the task about the excitation. However, these solutions have formal character, since functions $\Phi_{\tau,\varkappa}$ cannot be efficiently found.

Nevertheless in the integrals of form (15) it is possible to isolate the "resonance part" which is expressed clearly through eigenfunctions Φ_s and in the majority of the cases is of basic practical interest. For this way of integration it is displaced into lower half-plane $\operatorname{Im}\varkappa < 0$ and are computed deductions at the points where $D_\tau(\varkappa) = 0$ and $\Gamma_\tau(\varkappa) = 0$, i.e. at points $\varkappa = k_s$. In the calculation of deduction is essential value

$$N_s = \frac{i}{2\pi}\frac{dD_\tau(k_s)}{d\varkappa}, \qquad (16)$$

which we call the norm of function Φ_s; it it is possible to also present in the form

$$N_s = \lim_{R\to\infty e^{i\gamma}} \frac{1}{4\pi}\int_{V_R} \Phi_s^2\, dV, \qquad (17)$$

where V_R is a volume of the sphere of radius R, and angle γ is chosen so that the limit would exist, in spite of increase Φ_s according to formula (3).

The final solution of the problem about the excitation takes the form

$$\Phi = \sum_\Delta C_s\Phi_s + \tilde{\Phi}, \qquad (18)$$

where the coefficients

$$C_s = -\frac{\int \rho\Phi_s dV}{2k(k-k_s)N_s} \qquad (19)$$

essence complex amplitudes, with which are excited natural oscillations (1)-(3), in which k_s lie/rest at the region Δ, adjacent to the real axis. During the proper selection of region Δ sum (18) gives the resonance part of the field for calculating which it is necessary to know only Φ_s and k_s, and component $\tilde{\Phi}$ - the nonresonant "background", weakly depending on the frequency of excitation.

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The solution of the problem of Cauchy with t>0 takes the form

$$\Phi = \sum_{\Delta} C_s^+ \Phi_s e^{-ik_s ct} + \Phi^+, \qquad (20)$$

where

$$C_s^+ = \frac{3}{8\pi_s V_s} \int \left(\Phi^0 + \frac{i}{k_s} \Phi^1\right) \Phi_s \, dV \qquad (21)$$

and where the addition is produced on s, which satisfy condition $0 < k_s'' < \delta$, while Φ^+ decreases with t→∞ as $e^{-\delta ct}$. Therefore with large t series/row according to eigenfunctions Φ_s is principal part of the solution.

Relative to the concrete/specific/actual systems are given the simple approximations for calculating the norm, determined by formulas (16) and (17). Results easily are generalized to the case of other boundary conditions and in the case of inhomogeneous medium. Special work is dedicated to the solution of the same problems in the

electrodynamics when scalar wave equation is substituted by the equations of Maxwell in the inhomogeneous media with the absorption and by frequency dispersion.

SOME QUESTIONS OF THE THEORY OF OPEN RESONATORS.

V. S. Buldyrev, E. Ye. Fradkin.

1. Is examined boundary-value problem for electromagnetic field in open resonator with source within resonator, Leontovich boundary conditions on mirrors of resonator and conditions for radiation/emission at infinity. It is shown, with what geometries of resonator can be carried out the separation of field to TM and TE waves (disregarding by conditions on the edges/fins of mirrors). Is posed heterogeneous scalar problem for the appropriate component of Hertz's vector

$$\Delta u + k^2 u = -f(x, y, z) \qquad (1)$$

under the condition of radiating/emitting Sommerfield, the boundary conditions

$$\frac{\partial u}{\partial n_j} - ik\beta_j u = 0 \qquad \text{on} \qquad S_j\ (j = 1,2), \qquad (2)$$

where $\beta_j = \sqrt{\frac{\mu_j}{\varepsilon_j}\frac{\varepsilon}{\mu}}$ for TM waves and $\beta_j = \sqrt{\frac{\varepsilon_j}{\mu_j}\frac{\mu}{\varepsilon}}$ for TE waves, and the requirement of limitedness u on the edges/fins of mirrors. For the determination of field within the resonator it is assumed that u=0 and du/dn=0 on the external surfaces of mirrors. This assumption

contains two approximations/approaches: first, it is considered that the wave, which reached through the mirror the external surface, many times of weaker than the wave on the internal surface (mirrors are close to the ideal ones), second, is considered carried out the approximation/approach of Kirchhoff - neglect of the effects of the flowing in of current on the external surfaces of plates.

In the approximations/approaches indicated is obtained the system of two nonhomogeneous integral equations for field distribution on the internal surfaces of the mirrors of resonator.

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The corresponding uniform system can be named the system of the integral equations of the open resonator

$$2\pi u_i(M_i) = \sum_{j=1}^{2} \int_{s_j} \left(ik\beta_j \frac{e^{ikR_{ij}}}{R_{ij}} - \frac{\partial}{\partial n} \frac{e^{ikR_{ij}}}{R_{ij}} \right) u_j(N_j)\, dS \quad (j = 1,2). \qquad (3)$$

where R_{ij} - distance between point M_i, of that arranged/located on mirror s_i, and point of integration N_j, on mirror s_j. For the ideal mirrors with certain approximations/approaches, justified for the open optical resonators, equations (3) coincide with the integral equations of Fox and Lee.

2. It is shown, how spectrum of complex values of wave numbers $k_{nmq} = k'_{nmq} - ik''_{nmq}\ (k'_{nmq} \gg k''_{nmq} > 0)$ can be obtained from spectrum of

eigenvalues of system (3).

3. In approximation/approach of Fox and Lee are examined integral equations of open optical resonators with spherical mirrors. On the basis of the properties of the symmetry of such integral equations it is determined, what combinations of geometric parameters determine complex spectrum $k_{nmq}^{\cdot}$. For some resonators are considered values $k_{nmq}^{\cdot}$, the determining diffraction losses of the best-quality types of the natural oscillations of resonator.

DIFFRACTION AND THERMAL RADIATION.

M. L. Levin, S. M. Rymov.

In 1952-1953 was developed the phenomenological correlation theory of thermal agitations in electrodynamics [1, 2], which is based on the representation about the random outside sources of fluctuation field, for example, outside electrical and magnetic currents j^{e} and j^{m}. Within the absorbing medium electromagnetic field is subordinated, thus, to the nonhomogeneous equations of Maxwell, and any task about the thermal agitations proves to be actually edge/boundary electrodynamic task. Within the limits of phenomenological electrodynamics this general/common/total approach does not in any way limit the relationship/ratio between the sizes/dimensions of bodies and the wavelength and it makes it possible to contain in the unified theory all thermal fluctuation phenomena in the electrodynamics - from the classical theory of thermal radiation (approximation/approach of geometric optic/optics) to the theory of noises in the electrical circuits (quasi-stationary approximation/approach).

Besides the equations of Maxwell, the basic element/cell of this

theory are the space-time or spatial-spectral correlation function of outside currents. Initially they were obtained by generalizing Nyquist theorem [2], and then they were derived from the so-called fluctuation-dissipation theorem of Callen and co-authors [3-5]. The solution of boundary-value problem (considering all diffraction effects) and the subsequent averaging of the values, bilinear it is relative the obtained strengths of fields, make it possible to find any moments of the second order, which characterize thermal fluctuation field, the density of energy, its flow, different correlation function, etc.

As it was shown subsequently [6], the use of an electrodynamic reciprocity theorem significantly simplifies the solution of fluctuation problems.

This theorem reduces the task about the determination of the moments/torques of the second order to the calculation of heat losses of the auxiliary diffraction field, created by two point sources of dipole form. Thus, for instance, for the evenly hot bodies

$$\overline{E_{l_1}(\mathbf{r}_1)\, E^*_{l_2}(\mathbf{r}_2)} = \frac{2}{\pi}\,\varepsilon(\omega, T)\, Q_{12},$$

where E_l - projection of the Fourier component of the electrical fluctuation field to the direction of unit vector 1, $\varepsilon(\omega, T)$ - the medium energy of planck oscillator, Q_{12} - mixed heat losses of the diffraction field, created by point ones by electric dipoles

$p_1=l_1/i\omega$, $p_2=l_2/i\omega$, arranged/located at points r_1 and r_2 respectively.

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Moreover, reciprocity theorem in combinatio with the principle of the symmetry of kinetic coefficients makes it possible to obtain independent of Callen theorem and very correlation functions of outside fluctuation currents, moreover in the most general case of heterogeneous and anisotropic media both described by local material equations and possessing spatial dispersion.

Thus, for the determination of different average/mean bilinear characteristics of thermal field it suffices to know the solution of the auxiliar diffraction problems about the field of point source in the presence of body. Thus, for instance, for the absorbing half-space all characteristics of fluctuation field both near the boundary and far from it, immediately escape/ensue from the solution of the problem of Sommerfield.

In many instances of loss Q of diffraction field it is possible to find by different approximation methods. Thus, for well conductive bodies Q it is possible to compute according to the formulas of stron skin-effect, on the basis of the solution of diffraction problem for

the ideally conductive body. For the case when the distance of observation point from the body and the characteristic scales of body (sizes/dimensions, radii of curvature, etc.) are great in comparison with the wavelength, and furthermore, is great the optical thickness of body, for determination Q it is possible to use reflecting formulas with the local Fresnel coefficients. On the contrary, for the small bodies losse can be found from the quasi-static formulas. If us interests only thermal radiation/emission in the distant zone, then Q immediately is expressed as the diameter of absorption in the field of plane wave.

From that presented it is clear that the solution of any diffraction problem during irradiation by wave from the elementary source (in particular, by plane wave) includes and the solution of the definite mission about the thermal radiation.

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FORMULATION OF THE PROBLEM OF SCATTERING AT A FINITE DISTANCE.

R. G. Barantsev.

In the report is considered a question about the possibility of the distribution of arriving and outgoing flows at the final distance from the scatterer and is formulated the corresponding formulation of the problem. Is examined stationary elastic scattering on the arbitrary potential with the finite noncentral part.

The task of scattering actually consists in, knowing the flow, which arrives into the region of scattering, finding of the flow, which exits from it. At the final distance and with the finite wave number k only the possible approximate distribution of flows, moreover accuracy level is determined by the value of interference terms. Further, since the flow is received only on the pad whose sizes/dimensions not are lower than the wavelength, usual expression for the radial flow at a distance of r should be neutralized on the solid angle of order $(kr)^{-2}$. As a result the flow is characterized by

the final set of numbers. From the infinite series, which is obtained with the expansion of wave function of the spherical harmonics, remains the final sum, a number of components/terms/addends in which is equal to $\sum_{l=0}^{N}\sum_{m=-l}^{l}=(N+1)^2$, and in axisymmetric case $\sum_{l=0}^{N}=N+1$, where N=0(kr).

Asymptotic expansions of the solutions of radial equation with the the large kr>l give the possibility to find the order of interference terms and to determine arriving and exiting flows at the final distance. The task of scattering on the assigned potential is reduced to the determination of a finite number of amplitudes of the outgoing waves on the final set of the amplitudes of incident waves from the final system of linear algebraic equations.

The proposed approach offers the following possibilities:

1) The transference of the element/cell of degree of approximation from the solution into the setting, which brings task closer to physical conditions and its simplifying mathematical solution due to the neglect of the values, which do not have physical sense, even in the stage of the formulation of the problem.

2) Account of interference between the scattered wave and the passed part of the incident wave. With final kr the effect of this

interference is noticeable at the small scattering angles even in the case of the plane incident wave.

3) Uniform transition to the classical limit of $k \rightarrow \infty$. The presence of the pre-limit formulation of the problem with final k and r makes it possible to complete passage to the limit $k \rightarrow \infty$, $r \rightarrow \infty$ in different order. The usual formulation of the problem of scattering allows/assumes the infinite increase in k only after r, as a result of which are discovered the effects of asymptotic nonuniformity. So, in the task of scattering the plane wave by ideally solid sphere for the total scattering cross-section in limit of $k \rightarrow \infty$, as is known, is obtained the result, two times exceeding classical value. But if we k increase more rapidly than r, then this paradox does not appear.

4) The estimation of the conditions, under which the collective interaction is dismembered to the sequence of paired collisions, and the refined solution of this task (in the unsteady version).

ANALYTICITY OF DEPENDENCE ON THE PARAMETER AND DIFFERENT FORMULATIONS OF THE TASKS OF DIFFRACTION FOR THE HELMHOLTZ EQUATION.

G. D. Malyuzhinets.

1. Radiation principle [1-5] and principle of extinguishability [6-10][1].

FOOTNOTE [1]. Name principle of extinguishability was proposed in the work [9(1)]; formulation with its aid of the task of diffraction in the arbitrary region with the proof of uniqueness - in [9(2)], [9(3)]. In some works with the formulation of more specific problems is used the name: the principle of the maximum absorption (for example, see [10]). ENDFOOTNOTE.

Task of diffraction for the scalar equation of Helmholtz

$$\Delta u + k^2 u = 0 \qquad (1)$$

as the typical example, which makes it possible to consider the role of dependence on the parameter. Formulations of the simplest tasks in the free space with the help of the principle of extinguishability and radiation principle. Regions of the applicability of these principles. Task of diffraction for the equation of Helmholtz as the

task about forced oscillations. The analyticity of dependence on the complex parameter k as the basic property, which differs forced oscillations (resolvent) from its own ones (confirmation, that the requirement of analiticity of solution with respect to k enters into the determination of forced oscillations, described by the equation of Helmholtz, it was expressed to the author I. M. Gelfand in 1949).

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Available proofs of analyticity for special cases (Eydus, Olympius). Connection/communication with wave equation.

2. Questions of dispersion[1].

FOOTNOTE [1]. With respect to wave properties, for example, the plasma is the typical dispersive medium, true, described not by one, but by the system of equations of Helmholtz. However, in the sense of the analyticity of dependence on the parameters the case of systems of equations is very close to the case of one equation, examined/considered in the report. ENDFOOTNOTE.

Introduction instead of wave number k of the more convenient in this case propagation constant m=-ik, utilized in the theory communication along wires, and also in the theory of diffraction [8, 9]. The

generality of the equation of Helmholtz-Neumann

$$\Delta u - m^2 u = 0, \qquad (2)$$

consisting in the fact that, besides the simplest wave equation, to it is reduced also the broad class of wave equations for the media with the dispersion, which contain, besides Laplace's operator, the even linear operators, who are polynomials from $\frac{\partial}{\partial t}$ with the coefficients, time-independent. Electrical model-grids [11]. Information of such equations by means of substitution $\frac{\partial}{\partial t} = -i\omega$ to the equation of the form

$$\Delta u - \alpha(\omega, a, b, c, \ldots) u = 0, \qquad (3)$$

where α is the rational integral function of frequency ω (generally speaking, complex) and of substantially positive parameters a, b, c,..., which are coefficients in the initial wave equation for the medium with the dispersion. Definition "is sufficient general/common/total dispersive media", for which α, as the function ω in question, does not have poles and zero with $0<|\omega|<\infty$; $-\pi/2\leq\arg\alpha\leq\pi/2$. The proof of that fact that, any physical real medium (with the dispersion or without the dispersion) from the class pointed out above is the limiting case at least of one of sufficient α value m, necessary for reducing of equation (3) to the equation of Helmholtz (2), according to the formula

$$m = -ik = \sqrt{|\alpha|}\, e^{i/_2 \arg\alpha}.$$

Construction with the help of the principle of extinguishability (limitedness at infinity in the presence of dissipation) of an

example of the solution of the problem of the radiation/emission into the dispersive medium, which shows inaccuracy in the general case of radiation principle (outgoing waves), although which is coordinated with radiation condition. Examination of the possibility of eliminating this deficiency/lack in the radiation principle.

3. Questions of analytical continuation concerning parameter k (or m). Radiation conditions as the non-self-adjoint third-order boundary condition on the infinite sphere. New form of stüdy condition for the arbitrary complex values of the parameter. Its use during the derivation of the formula of Helmholtz for the exterior. The example of the simplest exterior problem of diffraction, which shows that without the imposition of the requirement of analyticity on k the generalized condition for radiation/emission allows/assumes, besides those forced, also continuous spectrum for the natural oscillations. Proof of the uniqueness of the solution of exterior problem. Resemblance and the difference between the exterior problem of diffraction and the internal task in the finite domain on boundary of which is assigned the non-self-adjoint third-order boundary condition.

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Advantages of eigenfunction expansions of the complex spectrum for

the asymptotic examinations of the behavior of solution (principal parts) in the vicinity of complex poles and for the physical interpretations. Examples: diffraction gratings, open resonators.

4. Diffraction in arbitrary region filled with inhomogeneous medium, and principle of extinguishability. Determination of distance between two points in the arbitrary region [9 (2)]. Infinite region. Various forms of the condition of extinguishability. Local condition of extinguishability [9 (1)]. Condition of extinguishability at infinity and uniform limitedness. Connection/communication with the accessory/affiliation of solution with L^2 and questions of the existence of solution. Formulation of task with the help of the condition of extinguishability at infinity for the case of the positive refractive index and smooth boundary with the third boundary condition. Principle of maximum and uniqueness theorem [9 (2)]. The type of tasks about forced oscillations with formulation of which the right side enters only into the condition of extinguishability at infinity. Theorem about the independence of the solution of this problem from the addition to this of the right side of the arbitrary function, liquidated at infinity. Use of this theorem for imparting the desirable form to the incident wave.

Formulation of task with the help of the requirement of uniform limitedness. Introduction for the assigned to region double limitless

region, comprised of two copies of initial [9 (2)]. Simultaneous examination in this double region of the tasks of Neumann and Dirikhle. Uniqueness theorem. Generalization for the case of complex refractive index.

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SOME GENERALIZATIONS OF THE RADIATION PRINCIPLE OF SOMMERFIELD.

L. G. Magnaradze.

1. Short survey/coverage of known proofs of principle and its generalizations.

2. On some new generalizations of principle and amplification of its known conditions.

ONE BOUNDARY-VALUE PROBLEM IN A REGION WITH AN INFINITE BOUNDARY.

D. M. Eydus

1. Ω - region in the space of variable/alternating $x_1, \ldots, x_n$, determined by the inequality

$$x_n - \varphi(x_1, \ldots, x_{n-1}) > 0.$$

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Function φ is determined for all $x_1, \ldots, x_{n-1}$ and has derivatives $\frac{\partial\varphi}{\partial x_k}$, k=1,..., n-1, that satisfy Gelder's condition in any are certain variable range $x_1, \ldots, x_{n-1}$.

Furthermore, $\varphi>\delta$. Constant δ we will count positive. Boundary of the region Ω let us designate through by Γ. Rassmotry the following boundary-value problem in Ω:

$$\Delta u + zu = f; \quad (1)$$

$$u|_\Gamma = 0, \quad (2)$$

$u \in L_2(\Omega)$, where z - complex number, and function f is such, that

$$N_1(f) \equiv \int_\Omega x_n^{3+\alpha} |f|^2 dx < \infty, \quad (3)$$

$\alpha>0$. It is known that this task is unambiguously solved with Im $z\neq0$. It is proven, that for all z of such, that Im $z\neq0$, Re $z\leq1$, occurs

inequality

$$\int_{\Gamma}\left|\frac{\partial u}{\partial \nu}\right|^2 |\cos(\nu, x_n)|\, d\Gamma + \int_{\Omega}\left[\frac{|u|^2}{x_n^{3+\alpha}} + \frac{1}{x_n^{1+\alpha}}\left|\frac{\partial u}{\partial x_n}\right|^2\right] dx \leqslant C N_1(f), \tag{4}$$

where the constant C depends only on l, δ, α. Let us designate through $u_z(x)$ the solution of task (3), (4), where Im $z \neq 0$. From (4) it follows that for any $\lambda<0$ there is a solution $u_\lambda^+(x)$ of equation (1), in which $z=\lambda$, that satisfies condition (2) and being the limit of certain sequence u_{z_m}, where $z_m \to \lambda$, $\operatorname{Im} z_m > 0$, the convergence occurring in any final subregion $\omega\ (\omega \subseteq \Omega)$ in sense of $L_2(\omega)$. Analogously there is a solution $u_\lambda^-(x)$ such, that $u_{z_m} \to u_\lambda^-$ when $z_m \to \lambda$, $\operatorname{Im} z_m < 0$.

In this case

$$\int_{\Omega}\left[\frac{|u_\lambda^\pm|^2 + |\nabla u_\lambda^\pm|^2}{x_n^{3+\alpha}} + \frac{1}{x_n^{1-\alpha}}\left|\frac{\partial u_\lambda^\pm}{\partial x_n}\right|^2\right] dx < \infty. \tag{5}$$

2. Let us superimpose further conditions on Ω and f. Let the function φ be such, that

$$0 < \delta \leqslant \varphi \leqslant c_1, \quad \left|\frac{\partial \varphi}{\partial x_k}\right| \leqslant \frac{c_2}{|x_k|}, \quad k = 1, \ldots, n-1, \tag{6}$$

while function f satisfies the condition

$$N(f) \equiv \int_{\Omega} x_n^{3+\alpha}\left(1 + \sum_{k=1}^{n-1} x_k^2\right)|f|^2\, dx < \infty. \tag{7}$$

Let function f' be such, that $N(f')<\infty$. Let us designate through u'_z the solution of task (3), (4), where function f is substituted on f'. With the help of (4) it is proven, that for all z of such, that Im

$z \neq 0$, $a \leq \text{Re } z \leq b$ $(a>0)$, occurs the inequality

$$\left|\int_{\Omega} u_z u_z' \, dx\right| \leqslant C \sqrt{N(f) N(f')}, \tag{8}$$

where constant C depends only on δ, c_1, c_2, α, a, b.

Lemma. Let $\mathscr{H}_0$ — subspace in Hilbert space $\mathscr{H}$, $\mathscr{P}$ — the operator of design on $\mathscr{H}_0$, G — self-conjugate operator in $\mathscr{H}$, $\mathscr{R}_z$ — its resolvent, η — region in the complex plane z, determined by inequalities Im $z>0$, $a \leq \text{Re } z \leq b$.

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Let for certain element/cell $f \in \mathscr{H}$ and for each $\psi \in \mathscr{H}_0$ in region occur the inequality

$$|(\mathscr{R}_z f, \mathscr{R}_z \psi)| \leqslant c_0,$$

where c_0 does not depend on z from η (but it can depend on ψ). Then for any z_1, z_2 of η occurs the inequality

$$\|\mathscr{P}\mathscr{R}_{z_2} f - \mathscr{P}\mathscr{R}_{z_1} f\| \leqslant C |z_2 - z_1|.$$

With the help of this lemma and inequality (8) is proven the following confirmation:

Theorem 1. Let the function φ satisfy conditions (6), and function f - condition (7). Then for any $\lambda > 0$ $u_z \to u_\lambda^+$ with $z \to \lambda$, Im $z>0$, the convergence occurring in any ω in sense of $L_2(\omega)$. Limit

function u_λ^- is the solution of equation (1), where z=λ, and satisfies condition (2). Analogously occurs convergence $u_z \to u_\lambda^-$ when $z \to \lambda$, Im z<0. Functions $u_\lambda^\pm$ satisfy condition (5). Furthermore, for any λ_1, λ_2, that belong to gap/interval $a \leqslant \lambda \leqslant b\ (a > 0)$ occurs the inequality

$$\| u_{\lambda_2} - u_{\lambda_1} \|_{L_2(\omega)} \leqslant c \, | \lambda_2 - \lambda_1 |.$$

3. Let us consider now in region Ω mixed problem

$$\frac{\partial^2 w}{\partial t^2} - \Delta w = e^{-i\sqrt{\lambda} t} f(x)$$
$$w|_\Gamma = 0$$
$$w|_{t=0} = \frac{\partial w}{\partial t}\Big|_{t=0} = 0.$$

With the help of theorem 1 and inequality (4) is proven the following theorem (principle of limiting amplitude).

Theorem 2. Let the function φ satisfy conditions (6), and function f - condition (7). Then when $t \to +\infty$ $e^{i\sqrt{\lambda} t} w \to u_\lambda^+$, $e^{i\sqrt{\lambda} t} \frac{\partial w}{\partial t} \to -i\sqrt{\lambda} u_\lambda^+$, the convergence occurring in any ω in sense of $L_2(\omega)$.

FORMULATION OF THE PROBLEM OF DIFFRACTION IN REGIONS WITH AN INFINITE BOUNDARY.

A. G. Ramm.

1. Let us consider in flat/plane region D[1] whose boundary asymptotically approaches boundary Γ_0 of angle D_0 so that

$$\rho(s, \Gamma) = O\left(\frac{1}{1+|s|^{1+a}}\right), \quad a > 0, \tag{1}$$

where $\rho(s, \Gamma)$ — distance from point $s \in \Gamma$ to Γ_0, task about construction of resolvent of Schroedinger's operator:

$$[\Delta + (k+i\varepsilon)^2 - p(x)]\, G(x, y, k+i\varepsilon) = \delta(x-y)\, bD; \tag{2}$$

$$G(x, y, k+i\varepsilon)\Big|_{x\in\Gamma} = 0; \tag{3}$$

$$G(x, y, k+i\varepsilon) \in L_2(D), \quad \varepsilon > 0 \tag{4}$$

under the assumption

$$|p(x)| = O\left(\frac{1}{1+|x|^{2+b}}\right), \quad b > 0. \tag{5}$$

FOOTNOTE [1]. Results of p. 1 are transferred to the case of the three-dimensional regions whose boundary approaches a boundary of cone. ENDFOOTNOTE.

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We will consider that out of the circle with the arbitrarily large, but fixed/recorded radius, radius-vector, intersecting boundary Γ, it remains within D region.

Theorem 1. Under the done assumptions lacks positive discrete spectrum the operator of Schroedinger Dirichlet problem. Task (1)-(3) has, and besides only, solution. This solution possesses the following properties:

a) when $\varepsilon \to 0\, G(x, y, k + i\varepsilon)$ is even relative to x, y, k, which vary on the compact subsets, approach the limiting values of G(x, y, k);

b) function G(x, y, k)=G(y, x, k) satisfies equation (2) when $\varepsilon = 0$, to boundary condition (3) when $\varepsilon = 0$ and condition for the radiation/emission

$$\lim_{R\to\infty}\int_{\Sigma_R}\left|\frac{\partial G}{\partial n}-ikG\right|^2 d\Sigma = 0; \qquad (6)$$

c) $$|G(x,y,k)|<C|\ln|x-y||, \quad |\nabla G(x,y,k)|<\frac{C}{|x-y|},$$

$$|\nabla^2 G(x,y,k)|<\frac{C}{|x-y|^2} \ \overset{(1)}{\text{при}}\ |x-y|\to 0,$$

$$|G(x,y,k)|+|\nabla G(x,y,k)|+|\nabla^2 G(x,y,k)|<\frac{C}{|x-y|^{1/2}} \ \overset{(1)}{\text{при}}\ |x-y|\to\infty,$$

$$|G(x,s,k)|<\frac{C\rho(s,\Gamma)}{|x-s|^{1/2}} \ \overset{(1)}{\text{при}}\ s\in\Gamma,$$

$$\left|\frac{\partial}{\partial k}G(x,y,k)\right|<C\frac{(|x|+|y|)}{k^{1/2}|x-y|^{1/2}} \ \overset{(1)}{\text{при}}\ |x-y|\geqslant 1;$$

d) $$G(x,y,k) \underset{\substack{|y|\to\infty\\ \arg y=\omega}}{=} \frac{1}{4i}\sqrt{\frac{2}{\pi k|y|}}\, e^{i\left(k|y|-\frac{\pi}{4}\right)} u(x,\omega,k)(1+o(1)),$$

Key: (1). where.

where the variable/alternating ω varies within the angle.

Functions u(x, ω, k) satisfy equation (2) and boundary condition (3) when $\varepsilon=0$. They play the role of plane waves for D region. These functions are the solutions of the problem of scattering on potential p(x) and boundary Γ;

e) the spectral function of Schroedinger's operator is an integral operator, strongly differentiated on λ, moreover $\frac{d\mathscr{E}_\lambda}{d\lambda}$ — integral operator with kernel $\frac{1}{\pi}\operatorname{Im} G(x,y,\sqrt{\lambda})$.

Theorem 2. Occur the inverse formulas:

$$f(\bar{x}) = \frac{1}{2\pi}\int_{D_0} \hat{f}(\bar{k})\, u(\bar{x}, \bar{k})\, d\bar{k} + \sum_{p=1}^{n} c_p \varphi_p(\bar{x}),$$
$$\hat{f}(\bar{k}) = \frac{1}{2\pi}\int_{D} f(\bar{x})\, \overline{u(\bar{x}, \bar{k})}\, d\bar{x}, \quad c_p = \int_{D} f(\bar{x})\, \overline{\varphi_p(\bar{x})}\, d\bar{x}, \tag{7}$$

integrals we accept as limits on the average $\varphi_p(x)$ — functions of the discrete/digital negative spectrum, $\bar{k}=\bar{k}\omega$.

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2. In region, examined in p. 1, for operator of Schroedinger Dirichlet problem occurs principle of limiting amplitude.

For solving the task

$$u_{tt} + \mathcal{L}u = f(x)\, e^{i\omega t}, \text{ где } \mathcal{L}u = -\Delta u + p(x)\, u, \tag{8}$$

$$u|_{t=0} = 0, \quad u_t|_{t=0} = 0, \quad u|_\Gamma = 0, \tag{9}$$

is correct the relationship/ratio

$$\lim_{t\to\infty} u(x, t)\, e^{-i\omega t} = v(x, \omega), \tag{10}$$

where

$$v(x, \omega) = \int_D G(x, y, \omega)\, f(y)\, dy \tag{11}$$

- solution of stationary boundary-value problem.

It is assumed that f(x) satisfies estimation (5).

When $D=E_3$, we can consider the speed of tendency toward the limiting amplitude. Specifically,, if f(x) and p(x) are finite or exponentially decrease, and $\nabla f, \nabla^2 f$ satisfy estimation (5) with b>1, then for solving task (8)-(9) is accurate the estimation

$$u(x, t) = e^{i\omega t} v(x, \omega) + O(e^{-\gamma t}), \qquad (12)$$

where $\gamma>0$.

If $|f(x)| = O\left(\frac{1}{1+|x|^{n+2+a}}\right)$, $|p(x)| = O\left(\frac{1}{1+|x|^{2n+1+b}}\right)$, $a>0$, $b>0$; $\nabla f(x)$, $\nabla^2 f(x)$

satisfies estimation (5) with b>1, then

$$u(x, t) = e^{i\omega t} v(x, \omega) + O\left(\frac{1}{1+t^n}\right). \qquad (13)$$

Under some assumptions about the resolvent of the abstract operator L it is possible to give the necessary and sufficient conditions so that for the operator L would be carried out the principle of limiting amplitude.

3. Let us consider question about analytical properties of solution of equation

$$\mathcal{L}u + p^2 u = f(x) \quad \text{b } D \qquad (14)$$

for parameter p, where $\mathcal{L}$ is determined by expression (8).

Let $D=E_3$. If f(x) and p(x) are finite, then the solution of equation (14) u(x, p) allows/assumes analytical continuation as

meromorphic function to entire complex plane by the variable/alternating p, moreover with Re $p>-a, a>0$, $u(x, p)$ - it is analytical.

If $p(x)$ and $f(x)$ exponentially decrease, then $u(x, p)$ is continued as meromorphic function for the variable/alternating p into half-plane Re $p>-b$, where $b>0$, moreover with Re $p>-a$, $a>0$ $u(x, p)$ - it is analytical.

If $p(x)$ and $f(x)$ decrease exponentially and $f(x)$ satisfies some conditions of smoothness, then $u(x, p)$ is differentiated on p, up to axis Re $p=0$.

The method of analytical continuation is based on the theorem obtained by the author, in which are given the necessary and sufficient conditions of one or the other smoothness of the solution of the operational equation $A(p)u=g(p)$ in the banach space.

In the flat/plane case analytical continuation is realized to the plane with the section/cut. By this is explained the diffusion of waves in the flat/plane case.

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Hence it follows that the remainder in formula (12) or (13) takes form $0(1/t)$, moreover the order of decrease indicated, generally speaking, cannot be improved, raising smoothness and speed of decrease $f(x)$ and $p(x)$.

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RADIATION CONDITIONS FOR ELLIPTICAL OPERATORS OF HIGHER ORDERS AND THE GENERALIZED PRINCIPLE OF MAXIMUM ABSORPTION.

B. R. Vaynberg.

In the present report will be found the conditions to infinity, which isolate unique solution for the broad class of elliptic equations in entire space. These conditions are analogous to the known conditions of radiating/emitting Helmholtz ($\Delta u+k^2u=f$). Furthermore, for the equations in question will be proved the validity of the principle of maximum absorption and certain generalized principle of maximum absorption. Will be indicated connection/communication between the conditions at infinity and the generalized principle of maximum absorption.

Through $x=(x_1,\ldots,x_n)$ we will designate the points of n-,dimensional real space $R_n(x)$, $r=(\Sigma x_i^2)^{1/2}$, $\omega=(\omega_1,\ldots,\omega_n)$— unit vector, $\omega_i=\frac{x_i}{r}$.

Let us consider first elliptical operator $P(i\frac{\partial}{\partial x})=P(i\frac{\partial}{\partial x_1}, \ldots, i\frac{\partial}{\partial x_n})$ with the constant coefficients of order $2m$, $P(\sigma)=P(\sigma_1,\ldots,\sigma_n)$ — its characteristic polynomial. Let operator

$P(i\frac{\partial}{\partial x})$ satisfy the following three conditions:

1) the dimensionality of the real zero polynomials $P(\sigma)$ is equal to n=1,

2) grad $P(\sigma)\neq 0$ in real zero $P(\sigma)$,

3) surface curvature $P(\sigma)=0$ (σ - it is real) not at one point is equal to zero.

Let us designate through K_j, $j=1,2,\ldots,t$ the singly connected surfaces into which falls surface $P(\sigma)=0$. From the ellipticity of polynomial $P(\sigma)$ and conditions 1-3 it follows that K_j — these are the locked convexs surface. Let us select for each K_j by arbitrary form the orientation (let us assign the direction of standard/normal n). This can be made 2^t methods. Then on each surface K_j there is exactly one point $\sigma_j(\omega)=(\sigma_{1j}(\omega),\ldots,\sigma_{nj}(\omega))$, in which vector n and ω are parallel and coincide in the direction. Through $\mu_j(\omega)$ let us designate the value of projection $\sigma_j(\omega)$ on vector ω: $\mu_j(\omega)=(\sigma_j(\omega),(\omega)$.

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Theorem 1. If $P(i\frac{\partial}{\partial x})$ — elliptical operator, who satisfies conditions 1-3 and f(x) - the arbitrary finite function (perhaps,

generalized), then there is a unique solution of the equation

$$P\left(i\frac{\partial}{\partial x}\right) u(x) = f(x) \qquad (1)$$

in any of the following classes of functions W_k, $k = 1,2,\ldots,2^t$: $u(x) \in W_k$, if this function is represented in the form of the sum of the functions: $u(x) = \sum_{j=1}^{t} u_j(x)$, for which in the vicinity of infinity take the place either of the inequality

$$|u_j(x)| < Cr^{\frac{1-n}{2}}; \quad \left|\frac{\partial u_j(x)}{\partial r} + i\mu_j(\omega) u_j(x)\right| < Cr^{n/2},$$

or the inequality

$$\int_{R<r<2R} |u_j(x)|^2 dx < CR;$$
$$\int_{R<r<2R} \left|\frac{\partial u_j(x)}{\partial r} + i\mu_j(\omega) u_j(x)\right|^2 dx < CR^{-1}.$$

Observation 1. Since the orientation of surface of $P(\sigma)=0$ can be selected 2^t by different methods, then theorem 1 gives to us 2^t different classes W_k.

Observation 2. In theorem 1 as a special case, are contained the normal conditions for radiation/emission for Helmholtz's operator.

From conditions 1-3 it follows that polynomial $P(\sigma)$ is represented in the form: $P(\sigma)=T(\sigma)P_1(\sigma)$, where $T(\sigma)$ - polynomial with the real coefficients, but polynomial $P_1(\sigma)$ does not have real zero. Let us designate through K^+ the surface, which consists of several surfaces K_j, and through K^- - remaining surfaces K_j.

Lemma 1. There is this polynomial $Q(\sigma)$ with the real coefficients, such that $Q(\sigma)>0$ on K^+ and $Q(\sigma)<0$ on K^-, moreover the operator

$$P_\varepsilon\left(i\frac{\partial}{\partial x}\right)\equiv\left[T\left(i\frac{\partial}{\partial x}\right)+\varepsilon Q\left(i\frac{\partial}{\partial x}\right)\right]P_1\left(i\frac{\partial}{\partial x}\right)$$

is elliptical with all $\varepsilon=\varepsilon_1+i\varepsilon_2$, if $\varepsilon_2\neq 0$.

Obviously, $P_\varepsilon(\sigma)\neq 0$ when $\varepsilon_2\neq 0$, and, which means, equation $P_\varepsilon\left(i\frac{\partial}{\partial x}\right)u_\varepsilon(x)=f(x)$ with any finite function f(x) has the unique solution decreasing at infinity. Specifically, it we will designate through $u_\varepsilon(x)$. We will indicate that $\varepsilon\to+0$, if $\varepsilon\to 0$ so that $\varepsilon_1\to+0$ and $\left|\frac{\varepsilon_2}{\varepsilon_1}\right|>\delta>0$.

Theorem 2. When $\varepsilon\to+0$ function $u_\varepsilon(x)$ descends in the weak sense to solution of u(x) of equation (1), to belonging one of the classes W_k. If $f(x)\in L_2$ and f(X)=0 with r>a, then for any limited region Ω

$$\|u(x)\|_{W_2^{2m}(\Omega)}\leqslant C(a,\Omega)\|f\|_{L_2},\quad \|u_\varepsilon(x)-u(x)\|_{W_2^{2m-s}(\Omega)}\leqslant C(\varepsilon)C(a,\Omega)\|f\|_{L_2};$$

$$\lim_{\varepsilon\to+0}C(\varepsilon)=0, \tag{2}$$

where s=0, if the order of operator $Q\left(i\frac{\partial}{\partial x}\right)$ does not exceed the order of operator $T\left(i\frac{\partial}{\partial x}\right)$ and s=1 in other case.

Observation 1. If $\mu_j=\operatorname{sign}_{\sigma\in k_j}Q(\sigma)$, the u(x) belongs to this class

W_k, during construction of which the orientation of surfaces K_j it is assigned by vector $\mathbf{n}_j = \mu_j \operatorname{grad} T(\sigma)$. Thus, depending on selection $Q\left(i\frac{\partial}{\partial x}\right)$ we obtain in the limit solution from any class W_k. In particular, if $Q(\sigma) \equiv \pm 1$, are obtained solutions from those classes W_k, for which n=grad T(σ) with all j, or, accordingly with all n=-grad T(σ). These two classes we will call natural.

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Let us consider now the equations

$$P\left(x, i\frac{\partial}{\partial x}\right) u(x) \equiv P\left(i\frac{\partial}{\partial x}\right) u(x) + \lambda R\left(x\ \ i\frac{\partial}{\partial x}\right) u(x) = f(x); \qquad (3)$$

$$P_\varepsilon\left(x, i\frac{\partial}{\partial x}\right) u_\varepsilon(x) \equiv \left[T\left(i\frac{\partial}{\partial x}\right) + \varepsilon Q\left(i\frac{\partial}{\partial x}\right)\right] P_1\left(i\frac{\partial}{\partial x}\right) u_\varepsilon(x) + + \lambda R\left(x, i\frac{\partial}{\partial x}\right) u_\varepsilon(x, = f(x), \qquad (4)$$

and $f(x) = 0$

where $f(x) \in L_2$ when $r > a$, $P\left(i\frac{\partial}{\partial x}\right)$ just as it is above, elliptical operator, who satisfies conditions 1–3, $Q\left(i\frac{\partial}{\partial x}\right)$ is of the order not higher than the order of operator $T\left(i\frac{\partial}{\partial x}\right)$ and satisfies the conditions of lemma 1, and $R\left(x, i\frac{\partial}{\partial x}\right)$ — any operator of order is not above 2m with the finite coefficients (coefficients of operator $R\left(x, i\frac{\partial}{\partial x}\right)$ equal to zero with r>b). It is assumed that the coefficients of operator $R\left(x, i\frac{\partial}{\partial x}\right)$, standing with the derivatives of order j, have j of continuous derivatives. Let us designate through D the set of those values λ (complex), at which operator $P\left(x, i\frac{\partial}{\partial x}\right)$ is elliptical. Through D_0 let us designate that connected the components of set D, who contains point λ=0. Index $\varkappa_k(\lambda)$ of operator (3) is called difference $\varkappa_k(\lambda) = n_k^1(\lambda) - n_k^2(\lambda)$, where $n_k^1(\lambda)$ — this number of solutions of

equation (3) with $f(x) \equiv 0$, those belonging W_k, and $n_k^2(\lambda)$ — this is a number of conditions to right side of f(x), which must be superimposed so that equation (3) would have a solution from W_k.

Theorem 3. With all λ from $D\varkappa_k(\lambda) = 0$. With all λ from D_0, with exception perhaps certain isolated/insulated set Λ_k, in class W_k there is a unique solution of equation (3), i.e., when $\lambda \in D_0 \setminus \Lambda_k$ it will be: $n_k^1(\lambda) = n_k^2(\lambda) = 0$.

It is not difficult to give the example when set Λ_k is not empty. Now we will give one sufficient condition of the fact that $\lambda_0 \notin \Lambda_k$.

Theorem 4. Let each of the irreducible factors of polynomial P(σ) have real zero and with $\lambda = \lambda_0$ operator (3) is formally symmetrical and equation $P(x, i\frac{\partial}{\partial x})\, u(x) = 0$ does not have finite solutions. Then for any finite f(x) from L_2 equation (3) has in natural classes W_k unique solution.

Theorem 5. Let with $\lambda = \lambda_0$ equation (3) have in class W_k the unique solution of u(x) and operator $Q(i\frac{\partial}{\partial x})$ corresponds to class W_k (see observation 1 to theorem 2). Then with sufficiently small $|\varepsilon|$ and $\varepsilon_2 \neq 0$ equation (4) has solution $u_\varepsilon(x)$, unique exponentially decreasing at infinity moreover for any limited region Ω

$$\| u(x) \|_{W_2^{2m}(\Omega)} \leqslant C(a, b, \Omega) \| f \|_{L_2},$$

$$\| u_\varepsilon(x) - u(x) \|_{W_2^{2m}(\Omega)} \leqslant C(\varepsilon) C(a, b, \Omega) \| f \|_{L_2};$$

$$\lim_{\varepsilon \to +0} C(\varepsilon) = 0.$$

The theorem, close to theorem 1, was obtained independently by B V. Grushin.

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A PRIORI EVALUATIONS OF SOLUTIONS AND SOLVABILITY OF GENERAL BOUNDARY PROBLEMS FOR THE LINEAR ELLIPTICAL EQUATIONS OF ARBITRARY ORDER WITH DISCONTINUITY COEFFICIENTS.

Z. G. Sheftel'.

1. In report are investigated general boundary problems for linear elliptic equations on the order of 2 m with coefficients, which have first-order discontinuities on some smooth surfaces, with assignment on these surfaces of specified "conditions for coupling" for limiting values of unknown functions (boundary conditions and conditions for coupling are assigned in these tasks by general/common/total linear differential operators). From of this type by tasks collide, for example, during the construction of mathematical theory the diffractions. In the report these tasks are studied with the help of the a priori estimations in norm L_p and in Hoelder norms. These estimations give the possibility to demonstrate the existence of the generalized (strong) solution and its smoothness up to boundary and surface of discontinuity of the coefficients.

2. Let limited region G of n-dimensional Euclidean space with

boundary Γ be decomposed into two parts of G_1 and G_2 by surface γ, which does not have from Γ common points. Let us introduce direct sum of S. L. Sobolyev's spaces $W_p^l(G_1)+W_p^l(G_2)=W_p^l$ ($l\geqslant 0$ — whole, p>1) with norm $\|u\|_{l,p}$. Each element/cell $u\in W_p^l$ can be represented in the form $u(x)=u_1(x)+u_2(x)$, $u_r\in W_p^l(G_r)$. We will use also spaces of fractional order $W_p^{l-\frac{1}{p}}(S)$ [1,2] and space $C^{l+\alpha}(\overline{\gamma})$ with Hoelder norms $|u|_{l+\alpha}^{\gamma}$, $0<\alpha<1$. Let us assume still $C^{l+\alpha}(G)=C^{l+\alpha}(G_1)+C^{l+\alpha}(G_2)$ with norm $|u|_{l+\alpha}^{G}=|u_1|_{l+\alpha}^{G_1}+|u_2|_{l+\alpha}^{G_2}$.

Let us assign the elliptical differential operator A with the disruptive complex coefficients

$$(Au)(x)=\begin{cases}(A^1u)(x), & x\in G_1,\\(A^2u)(x), & x\in G_2,\end{cases}\quad A^r=\sum_{|\mu|\leqslant 2m}a_\mu^r(x)D^\mu, \tag{1}$$

and also boundary operators (to Γ) and the operators of coupling (on γ):

$$B_k^r=\sum_{|\mu|\leqslant m_k^r}b_{k\mu}^r(x)D^\mu,\quad (r=1,2,3;\quad k=1,\ldots,i_r,\quad i_1=i_2=2m,$$
$$i_3=m;\quad m_k^1=m_k^2=m_k), \tag{2}$$

where complex functions $b_{k\mu}^1(x)$, $b_{k\mu}^2(x)$ are determined on γ, $b_{k\mu}^3(x)$ — on Γ.

Theorem 1. Let us assume $l_1=\max\{2m, m_k^r+1\}$, $l>l_1$ — whole; we will assume that γ and Γ — class C^l, $a_\mu^r(x)\in C^{l-2m}(G_r)$, $b_{k\mu}^r(x)\in C^{l-m_k^r}(\gamma)(\Gamma)$ $(r=1,2,3)$. Let $u\in W_p^{l_1}$, while

$Au \in W_p^{l-2m}$, $[B_k u]_\gamma = B_k^1 u_1 - B_k^2 u_2|_\gamma \in W_p^{l-m_k-\frac{1}{p}}(\gamma)$, $B_k^3 u|_\Gamma \in W_p^{l-m_k^3-\frac{1}{p}}(\Gamma)$. In that and only that case, if operator A is correctly elliptical, and operators (2) cover[1] it, the solution of "u" enters in W_p^l and

$$\|u\|_{l,p} \leqslant K\left(\|Au\|_{l-2m,p} + \sum_{k=1}^{2m} \|[B_k u]\|_{l-m_k-\frac{1}{p},p} + \sum_{j=1}^{m} \|B_j^3 u\|_{l-m_j^3-\frac{1}{p},p} + \|u\|_{0,p}\right), \tag{3}$$

where the constant K>0 does not depend on u.

FOOTNOTE [1]. The determination of covering for the case of discontinuity coefficients see in [3.4]. ENDFOOTNOTE.

Theorem 2. Let us assume $l_0 = \max\{2m, m_k^r\}$, $l > l_0$, $0 < \alpha < 1$; we will assume that γ and Γ — class $C^{l+\alpha}$, $a_\mu^r(x) \in C^{l-2m+\alpha}(G_r)$, $b_{k\mu}^r(x) \in C^{l-m_k^r+\alpha}(\gamma)(\Gamma)$ $(r = 1, 2, 3)$. Let $\in C^{l+\alpha}(G)$, while $Au \in C^{l-2m+\alpha}(G)$, $[B_k u]_\gamma \in C^{l-m_k+\alpha}(\gamma)$, $B_j^3 u|_\Gamma \in C^{l-m_j^3+\alpha}(\Gamma)$.

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In that and only that case, if operator A is correctly elliptical, and operators (2) are covered his, $u \in C^{l+\alpha}(G)$ and

$$|u|_{l+\alpha}^G \leqslant K\left(|Au|_{l-2m+\alpha}^G + \sum_{k=1}^{2m} |[B_k u]|_{l-m_k+\alpha}^\gamma + \sum_{j=1}^{m} |B_j^3 u|_{l-m_j^3+\alpha}^\Gamma + |u|_0^G\right), \tag{4}$$

where constant K>0 does not depend on u.

Both these theorems are proven with the help of explicit

integral formulas derived in [5] of the representation of solutions for the regions of the special type and operators with the constant and piecewise constant coefficients. The methodology of proof is close to that used in [2].

3. Everywhere subsequently we will consider that $m_k^r \leqslant 2m-1$, r=1, 2, 3. Let us designate through W_p^{2m} (gr) a set of such $u \in W_p^{2m}$, for which

$$[B_k u]_\gamma = B_k^1 u_1 - B_k^2 u_2|_\gamma = 0, \quad k = 1, \ldots, 2m, \qquad (5)$$
$$B_j^3 u|_\Gamma = 0, \quad j = 1, \ldots, m.$$

Is obvious, W_p^{2m} (gr) - subspace W_p^{2m}, dense in $L_p(G)$. By considering A as operator in L_p with the domain of definition $\mathfrak{D}(A) = W_p^{2m}$ (gr), let us introduce the adjoint operator A*, which functions in $L_q\left(\frac{1}{p}+\frac{1}{q}=1\right)$. Let N(N*) - the set of those $u \in \mathfrak{D}(A)$ $(v \in \mathfrak{D}(A^*))$, for which Au=0(A*v=0).

Theorem 3. Let A - correctly elliptical operator and operators (2) cover him; γ and Γ — surface of class C^{2m}, $a_p^r(x) \in C(G_r)$, $b_{kp}^r(x) \in C^{2m-m_k^r}(\gamma)(\Gamma)$. Then a) A - the locked operator with the locked in L_p range of values $\mathcal{R}(A)$; b) of subspace N and N* are finite-dimensional. In other words, a is Φ- operator (according to the terminology of I. Ts. Gokhberg and M. G. Kreyk [6]).

Proof is based on estimation (3) (with l=2m). In this case the

finite dimensionality of N* is proven with the help of Riesz theorem about the completely continuous operators by the construction of "local solutions" and estimation of their norms with the use of theorems of insertion and inequality (3).

Corollary. In addition to the conditions of theorem 2γ and Γ — class C^{2m+s} ($s>0$ — whole), $a^r_\mu(x) \in C^s(G_r)$, $b^r_{k\mu}(x) \in C^{2m-m^r_k+s}(\gamma)\,(\Gamma)$. The boundary-value problem

$$Au = f,\ u \in W^{2m}_p\,(\mathrm{rp}) \qquad (6)$$

has a solution and $u \in W^{2m}_p\,(\mathrm{qr}) \cap W^{2m+s}_p$ in that and only that case, if $f \in W^s_p$, $(f, N^*) = 0$, where subspace $N^* \subset L_q$ is finite-dimensional.

Hence with the help of the theorems of insertion and estimations (4) it is derived/concluded.

Theorem 4. Let A - correctly elliptical operator and operators (2) cover him; γ and Γ — class $C^{2m+s+\alpha}$ ($s>0$ — whole, $0<\alpha<1$), $a^r_\mu(x) \in C^{s+\alpha}(G_r)$, $b^r_{k\mu}(x) \in C^{2m-m^r_k+s+\alpha}(\gamma)\,(\Gamma)$ $(r=1,2,3)$; let us consider task (6) with $f \in C^{s+\alpha}(G)$. If $\int f\bar{v}dx=0$ for all $\in N^*$ (where N* is finite-dimensional), then task (6) has a solution $u \in C^{2m+s+\alpha}(G)$, i.e. classical solution.

4. Let operators (2) be normal (see [3, 4]).

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In this case (if only γ, Γ and coefficients are sufficiently smooth) the domain of definition of the adjoint operator A^* is determined by conditions of type (5), A^* coinciding with formally adjoint operator A^+. This gives the possibility to substantially make more precise the formulations of theorem 4 and corollary 3.

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HETEROGENEOUS ELLIPTICAL PROBLEMS WITH DISCONTINUITY COEFFICIENTS AND A LOCAL INCREASE IN THE SMOOTHNESS OF THE GENERALIZED SOLUTIONS UP TO THE BC'INDARY OF THE REGION AND THE DISCONTINUITY SURFACE OF THE COEFFICIENTS.

Ya. A. Roytberg.

Let G - limited region of space E_n with boundary Γ; G_1 - subregion G with the boundary γ, which does not have from Γ the common points, $G_2 = G \setminus \overline{G}_1$. In G is examined correctly elliptical [1, 2] expression with the disruptive complex coefficients:

$$(\mathscr{L}u)(x) = \begin{cases}(\mathscr{L}^1u)(x), & x \in G_1,\\ (\mathscr{L}^2u)(x), & x \in G_2\end{cases} \quad \mathscr{L}^i = \sum_{|\mu| \leqslant 2m} a^i_\mu(x) D^\mu \quad (i = 1, 2)$$

$$\left(\mu = (\mu_1, \ldots, \mu_n);\ |\mu| = \mu_1 + \ldots + \mu_n;\ D^\mu = D_1^{\mu_1}, \ldots, D_n^{\mu_n}, D_k = \frac{1}{i}\frac{\partial}{\partial x_k}\right).$$

On γ it is assigned 2m pairs of differential expressions $B^i_j = \sum_{|\mu| \leqslant m_j} b^i_{j\mu}(x) D^\mu$ $(x \in \gamma,\ m_j \leqslant 2m - 1;\ j = 1, \ldots, 2m;\ i = 1, 2),$ while on $\Gamma - m$ expressions $B^3_k = \sum_{|\mu| \leqslant m^3_k} b^3_{k\mu}(x) D^\mu (x \in \Gamma;\ m^3_k \leqslant 2m - 1;\ k = 1, \ldots, m).$ Boundary expressions B^i_j, B^3_k are assumed to be normal ones and those covering differential expression $\mathscr{L}$ with discontinuity coefficients [1, 2]. Let us designate through $u_i(x)\ (x \in \gamma,\ i = 1, 2)$ the limiting value of u(x) from the side G_i; $[(B_ju)(x)] = (B^1_ju_1)(x) - (B^2_ju_2)(x)\quad (x \in \gamma,\ j = 1, \ldots, 2m).$

Let us consider the boundary-value problem

$$(\mathcal{L}u)\ (x)=f(x)\,(x\in G);\ \ [(B_j u)\,(x)]=\psi_j(x)\,(x\in\gamma);\ \ (B_k^3 u)\,(x)=\varphi_k(x)\,(x\in\Gamma)\ \ (1)$$
$$(j=1,\ldots,2m;\ k=1,\ldots,m).$$

function u(x) is called the strong generalized solution of task (1), if there is a sequence u_ν of functions, it is sufficient smooth ones in $\bar{G}_i\ (i=1,2)$, moreover in appropriate norms $u_\nu\to u$; $\mathcal{L}u_\nu\to f$; $[B_j u_\nu]\to\psi_j$; $B_k u_\nu\to\varphi_k\ (j=1,\ldots,2m;\ \ k=1,\ldots,m)$. In the work are described the spaces, between which closing/shorting representation $u\to(\mathcal{L}u,\ B_1^3 u,\ldots,B_m^3 u,\ \ [B_1 u],\ldots,[B_{2m}u])$ is homeomorphism. Thereby is solved a question about the strong generalized solvability of task (1).

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Function u is called the weak generalized solution of task (1), if it satisfies certain integral identity, obtained with the help of Green's formula. It is proven, that the weak generalized solution will be strong, and strong - weak.

Let u - generalized solution of task (1) and let G_0 - subregion G, which adjoins piece Γ_0 of surface Γ and which contains the piece γ_0 of surface γ. If into $G_0\cup\Gamma_0$ the coefficients of differential expressions, and also function f, ψ_j, φ_k are more smooth, then u will be in $G_0\cup\Gamma_0$ with respect to smoother, in particular, under appropriate conditions of smoothness u it will be the classical solution of task

(1).

Let us give more precise formulations of the described confirmations. Let

$$W_2^s(G) = W_2^s(G_1) \oplus W_2^s(G_2);$$
$$K_s = W_2^s(G) \oplus \sum_{k=1}^{m} W_2^{2m-m_k^3-\frac{1}{2}+s}(\Gamma) \oplus \sum_{j=1}^{2m} W_2^{2m-m_j-\frac{1}{2}+s}(\gamma);$$

and $\widetilde{W}_2^s$ — the closure of a set of the functions, determined in $\bar{G}_1 \cup \bar{G}_2$, be sufficient smooth ones in each $\bar{G}_i$, according to the norm

$$|||u|||_s^2 = \|u\|^2_{W_2^s(G)} + \sum_{k=1}^{2m} \left\| \frac{\partial^{k-1}}{\partial \nu^{k-1}} u \right\|^2_{W_2^{s-k+\frac{1}{2}}(\Gamma)} + \sum_{k=1}^{2m} \sum_{i=1}^{2} \left\| \frac{\partial^{k-1} u_i}{\partial \nu^{k-1}} \right\|^2_{W_2^{s-k+\frac{1}{2}}(\gamma)}$$

(ν — normal to the surface, s - arbitrary whole).

In the simpler case of the absence of defect is valid theorem 1. Let us consider representation $\Lambda: u \rightarrow (\mathcal{L}u; B_1^3 u, \ldots, B_m^3 u; [B_1 u], \ldots, [B_{2m} u])$ as operator, who functions from $\widetilde{W}_2^s$ in K_{s-2m}. Then under appropriate conditions of smoothness closing/shorting $\bar{\Lambda}$ operator Λ establishes/installs the homeomorphism between these spaces (with s≥2m homeomorphism establishes/installs Λ).

Theorem 1 is valid also in the case of the presence of defect, is necessary only instead of $\widetilde{W}_2^s$ and K_{s-2m} to examine their appropriate subspaces.

Theorem 2. Let $u \in \widetilde{W}_2^s$ (s — integer) - the strong generalized

solution of task (1). Let G_0 - subregion G, which adjoins piece Γ_0 of surface Γ; Γ_1 — boundary G_0, $\Gamma_1 \cap \Gamma = \Gamma_0'$, $G_0 \cap \gamma$ — it is empty; let $\chi(x) \in C^\infty(\bar{G})$ be nullified in certain vicinity in $\bar{G}$ of set $\bar{G} \setminus \bar{G}_0$, and near Γ_0 $\frac{\partial\chi}{\partial\nu} = 0$. Then, if $\chi f \in W_2^{s-2m+1}(G_0)$, $\chi\varphi_k \in W_2^{s-m_k+\frac{1}{2}}(\Gamma_1)$ (k=1,... ,m) and the coefficients of differential expressions $\mathcal{L}$, $B_k^0 (k = 1, \ldots, m)$ are sufficiently smooth into $G_0 \cup \Gamma_0$, that $\chi u \in \widetilde{W}_2^{s+1}(G_0)$. Analogous confirmation is correct in the case when G_0 contains the piece γ_0 of surface γ. Choosing γ(X) identical to equal to 1 on part of G_0, we obtain confirmation about a local increase in the smoothness of the generalized solutions. Let us note, that for a local increase in the smoothness it is significant that in theorem 1, homeomorphisms are established/installed for all the wholes s.

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2. APPLICATION OF THE SPECTRAL THEORY OF OPERATORS.

THE SPECTRAL THEORY OF OPERATORS AND SOME QUESTIONS OF DIFFRACTION.

V. A. Marchenko.

ANALYTICAL PROPERTIES OF RESOLVENTS OF ONE BOUNDARY-VALUE PROBLEM.

Ye. Ya. Khruslov.

In the joint operation with V. A. Marchenko we examined a question about the propagation of the oscillations/vibrations through the system of obstructions $\{S_n\}$ in the form of surfaces S_n, located in the vicinity of Lyapunov's certain fixed/recorded surface S, on which is assigned zero boundary condition.

With the definite requirements, assigned on $\{S_n\}$, the oscillations/vibrations are described approximately by the solution of the equation of Helmholtz

$$\Delta u + k^2 u = 0 \tag{1}$$

in the region, which lies out of surface of S, with following boundary conditions on S:

$$u^+(x) = u^-(x) = u(x),$$
$$\frac{\partial u^+(x)}{\partial n} - \frac{\partial u^-(x)}{\partial n} = f(x)\,u(x). \tag{2}$$

Here: f(x) - the assigned on surface of S continuous positive function. Signs + and - noted the limiting values of functions and their normal derivatives in accordance with one and other side of surface. Normal to the surface of S is directed to the side, noted by sign +.

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In the report is examined boundary-value problem (1)-(2). Primary attention is given to the study of the analytical properties of resolvent (Green's function) of task as the functions of parameter k. Surface S is assumed to be that limited.

Green's function task (1) - (2) with Jm k>0 can be represented in the form

$$G(x, y; k) = \frac{1}{4\pi}\frac{e^{ik|x-y|}}{|x-y|} - \int\limits_S \frac{e^{ik|x-\xi|}}{|x-\xi|}\rho(\xi, y; k)\, ds_\xi,$$

where x-y - distance between points x and y of three-dimensional space, $\rho(\xi, y; k)$ - certain density of the simple layer, distributed over surface of S, which satisfies the integral equation

$$\rho(x, y; k) + \int\limits_S \frac{e^{ik|x-\xi|}}{|x-\xi|} f(x)\rho(\xi, y; k)\, ds_\xi = \frac{f(x)}{4\pi}\frac{e^{ik|x-y|}}{|x-y|}.$$

Research of the resolvent of this equation makes it possible to establish/install the following properties of Green's function.

1. Function of Green G(x, y; k) of task (1) - (2) - holomorphic function of parameter k at $\mathrm{Jm}\, k > 0$ can be analytically continued into lower half-plane. The analytical continuation of Green's function is no longer resolvent of task (1)-(2)since it increases with $x \to \infty$.

In the lower half-plane k function G(x, y; k) cannot have other special features/peculiarities, except poles. Thus, G(x, y; k) - the meromorphic function of parameter k.

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2. Important property of function G(x, y; k) is the fact that its poles cannot unlimitedly approach real axis, it is more precise: there is this negative even function $Y=F(X)$, which satisfies condition $Y\to-\infty$ with $X\to\infty$ that all poles G(x, y; k) can lie/rest at plane $k(X+iY)$ only lower than curve, assigned by equation $Y=F(X)$. If G(x, y; k) has poles, then they are arranged/located symmetrically relative to imaginary axis.

The following property of function G(x, y; k) occurs when surface S is locked.

3. In boundary condition (2) let us place $f(x)=\lambda\varphi(x)$. Then with the sufficiently large λ function G(x, y; k) has poles which with $\lambda\to\infty$ approach the poles of Green's function internal relative to surface of S boundary-value problem under the zero boundary condition.

The case of large ones λ is most interesting, since it

corresponds to the sufficiently dense distribution of obstructions near surface of S. In this case they will have a considerable effect on the propagation of oscillations/vibrations.

The formulated properties of Green's function are used to the research of the behavior of solution u(x, t) of thé wave equation $\Delta u = d^2u/dt^2$ with $t \to \infty$, which satisfies to certain initial data and boundary conditions (2) on the locked surface of S.

It is shown that with any finite initial data u(x, t) fades not more slowly than $e^{-\beta_0 t}$, and there are always such initial data, with which the fading will occur accurately as $e^{-\beta_0 t}$, where β_0 - distance from the real axis to the nearest to it pole of Green's function task (1)-(2).

Analogous research is carried out for the space of two measurements. In this case there is an essential difference: resolvent is a meromorphic function of parameter k in the complex plane with the section/cut along the negative alleged semi-axis. Therefore fading the solution of wave equation it occurs exponentially, but according to the power law.

BOUNDARY-VALUE PROBLEMS WITH THE PARAMETER UNDER THE BOUNDARY CONDITION (WAVEGUIDES, OSCILLATION OF A VISCOUS FLUID).

S. G. Kreyn, G. I. Laptev.

The number of questions of vibration theory leads to the examination of boundary-value problems whose unknown parameter λ (eigenvalue) enters both into the equation and into the boundary conditions.

In the report based on the example of two tasks, one of the theory of waveguides, another of the vibration theory of viscous fluid, shows the need for research of the spectrum of the operational equations, which nonlinearly depend on the spectral parameter. To the study of some classes of such equations is devoted the basic content of report.

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1. As is known, process of propagation of waves in periodic cylindrical waveguides can be described by differential equation

$$\Delta u + \frac{\omega^2}{c^2(x, y, z)} u = 0, \qquad (1)$$

examined/considered in tube domain G= [0, 1] xS, where S -

two-dimensional section of waveguide, and by boundary conditions

$$u(l, y, z) = \rho u(0, y, z);$$
$$u'_x(l, y, z) = \rho u'_x(0, y, z) \qquad (2)$$

on bases G and

$$u\Big|_\Gamma = 0 \text{ or } \frac{\partial u}{\partial n}\Big|_\Gamma = 0 \qquad (3)$$

on lateral surface Γ.

Function c (x, y, z) is periodical of x with period 1.

Equation (1) and conditions (3) can be treated as a special case of the differential equation

$$\frac{d^2u}{dx^2} = Au - \omega^2 B(x) u, \quad (0 \leqslant x \leqslant l) \qquad (4)$$

in the Hilbert space $L_2(S)$ with the self-adjoint non-negative operators A and B (x) operator A is not limited).

For equation (4) can be constructed the theory of boundary-value problems with the same completeness as for the ordinary scalar differential equation. The use/application of this theory to the task, which corresponds to boundary conditions (2), makes it possible to compose the integral equation of task which can be considered as operational equation in function space from x with the values in $L_2(S)$. This operational equation linearly depends on parameter ω^2 and

it is nonlinear from the parameter ρ.

Research of the equation indicated gives the possibility to do a series/row of the qualitative conclusions/outputs from which let us note the following: 1) with each that fixed/recorded $\rho \neq 0$ their own and associated functions of task (1)-(2)-(3) form complete system in G region; 2) with that fixed/recorded ω multipliers ρ are symmetrically arranged/located relative to unit circle, but circle/circumference themselves them it is located not more finite number, and the set of multipliers can have only two condensation points: 0 and ∞.

These results supplement research of M. G. Krein and G. Ya. Lyubarskiy (see PMM, Vol 25, Iss. 1, 1961).

2. Task about small oscillations of heavy viscous incompressible fluid in open container (without taking into account surface tension) reduces to system of equations of Navier-Stokes

$$\frac{\partial u}{\partial t} = \nu \Delta u - \nabla p_1, \ \mathrm{div}\, u = 0, \qquad (5)$$

in question in in question in region G with liquid, with boundary conditions

$$u = 0 \text{ на}^{(1)} \text{ стенке } \Gamma_1 \text{ сосуда}^{(2)} \text{ и};$$
$$\frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x} = 0, \ \frac{\partial u_y}{\partial z} + \frac{\partial u_z}{\partial y} = 0, \ \frac{\partial}{\partial t}\left(p_1 - 2\nu \frac{\partial u_z}{\partial z}\right) = g u_z \qquad (6)$$

Key: (1). on the wall. (2) container and.

on floating surface Γ_0 of liquid.

Here $\nu = \frac{\mu}{\rho}$ — kinematic modulus of viscosity, $p_1 = \frac{p}{\rho} + gz$, where p - pressure, ρ - density of liquid, g - acceleration of gravity, μ - coefficient of viscosity/ductility/toughness.

For normal oscillatory mode of the form

$$\mathbf{u}(t, x, y, z) = e^{-\lambda t}\mathbf{v}(x, y, z) \text{ and } p_1(t, x, y, z) = e^{-\lambda t} q_1(x, y, z)$$

is obtained the boundary-value problem whose parameter λ enters into equation and boundary conditions. With the use of theorems of the insertion of Sobolyev and the Korn inequality the task is reduced to the research of the solutions of the operational equation

$$\nu\mathbf{v} = \lambda P\mathbf{v} + \frac{g}{\lambda} Q\mathbf{v}, \qquad (7)$$

where P and Q - self-adjoint non-negative completely continuous operators in the Hilbert space $L_2(G)$.

This equation was studied in the work of N. G. Askerov and lecturers (DAN USSR, Vol. 155, No 3, 1964).

Research of equation (7) leads to the following qualitative

conclusions: there is a denumberable number of normal oscillatory mode, all normal oscillatory mode are aperiodic motions, with exception, perhaps, a finite number of damped oscillations; with the sufficiently high viscosity there are no oscillatory motions. Normal oscillatory mode and solutions connected to them form complete in complete in vector-function system, their vertical components forming complete system of functions on floating surface Γ_0.

Both theorems about the completeness in points/items 1 and 2 are obtained on the basis of the familiar Keldysh theorem (DAN USSR, 1951, 77, No 1).

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SOME QUESTIONS OF THE THEORY OF LINEAR OPEN SYSTEMS.

M. S. Livshits.

It is known that the tasks of the propagation of waves in the linear closed systems lead to the self-adjoint operators and they are solved by resolution in the series (or integral) of Fourier in terms of the natural oscillations of these systems. Thereby is given the simple and fruitful model of closed system in the form of the set (discrete/digital or continuous) of the not interacting between themselves oscillators. However, for the systems, which interact with the external world, this model is applicable only in the rare cases. Arises the important question about the resolution of open-circuited system into the simplest composite/compound component parts.

Let us agree the physical system, connected with the external world by any communication channels, to call open system.

Examples of the open systems are numerous. They include mechanical, electrical and electromechanical multiterminal networks, different radio engineering communications - antenna, waveguides with

the branchings off and the heterogeneities, cavity resonators with the deriving/concluding devices/equipment, etc. In quantum physics these systems appear in the form of the compound nuclei, which are formed during the collisions of different particles. Furthermore, in many instances different parts of this closed system can be considered as the open systems, connected with each other by the communication channels. The important role of open-circuited systems in different regions of physics causes the necessity for the separate theory, which considers interaction of system with the external world.

Report is dedicated to the explanation of some characteristic features of the theory of the linear open systems. Naturally, in this theory must play large role the non-self-adjoint operators. It proves to be, however, that the concept, adequate to the open system, is not the operator, but more complicated mathematical formation/education - operational complex or node/unit.

In many questions the set of the communication channels can be described by linear differential equations or in the partial derivatives. Thus, for instance, as the equations of channels for the electrodynamic systems can serve the equations of Maxwell in the empty space, and in quantum physics - wave equations of free particles. Each type of the communication channels generates a

defined class of those allied to these channels of the open systems.

We will be occupied by the following two tasks:

1. Are assigned signals at the input of system F, it is necessary to find the oscillations of system itself and signals on the output.

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2. Observer, who is located out of system, sends by communication channels signals. These signals are processed by system and are returned to the observer. It is necessary to construct the model of system, equivalent to system itself from the point of observer's indicated view. It is also necessary to find the dependence between the motion of model and motion of system F. In this case must be previously indicated, what elements/cells can be used for the construction of model.

One of the main results lies in the fact that under specific conditions each open system is decomposed/expanded into the chain/network of the connected elementary (one-dimensional) systems, moreover a number of communication channels between the components/links of chain/network is equal to a number of channels,

which combine entire system with the external world. The study of these chains/networks, included within the open systems and the external communication channels appearing as continuation, proves to be useful in different questions.

Let us note that in the case of closed system this resolution is converted in the known resolution in terms of the independent natural oscillations of system. A number of communication channels in this case is equal to zero. Are examined also some tasks of analysis and the synthesis of the open or closed systems with the help of the assigned elementary systems and such elementary operations as cohesion/coupling and splitting/fission of systems, and also the opening of new ones and the closure of the old communication channels.

NONSELF-ADJOINT OPERATORS IN THE THEORY OF ELECTRICAL CIRCUITS.

A. G. Rutkas.

Are examined the transmitting reactive/jet multiterminal networks with the lumped parameters (replacement scheme of the waveguide articulation of complex configuration, filters, etc.). Their oscillations are studied from the point of view of the open systems of M. S. Livshits.

On the diagram is constructed the linear non-self-adjoint operator T, characteristic matrix/die - function of which coincides with the transmission matrix/die of multiterminal network. With the help of T it is possible to find also the internal state of multiterminal network from the assigned input; to indicate equivalent chain of the simplest multiterminal networks; to use for studying the oscillations spectral analysis of the non-self-adjoint operators.

1. Let multiterminal network F contain N of concentrated elements/cells (inductance L_j and capacities/capacitances C_k), m external input branches and m output ones (2m=2). Currents $I_\alpha^{(-)}$ and stresses/voltages $U_\alpha^{(-)}$ input branches we consider the components of input vector $\varphi^{(-)}=(U_1^{(-)},\ldots,U_m^{(-)};I_m^{(-)},\ldots,I_1^{(-)})$. At output Φ respectively is

constructed output vector $\varphi^{(+)}=(U_1^{(+)},\ldots,U_m^{(+)};\ I_m^{(+)},\ \ldots,\ I_1^{(+)})$. The vector of internal states is defined as

$$\phi=(\sqrt{L_1}J_{L1},\ \ldots,\ \sqrt{C_N}\,U_{CN}),$$

so that the square of its norm is equal to the doubled energy of system.

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The basic task - of supplying in the conformity to multiterminal network Φ the non-self-adjoint operator T so that would be fulfilled the relationships/ratios:

$$\frac{T-T^*}{i}\phi=\sum_{\alpha,\beta=1}^{n}(\phi,\ e_\alpha)\,J_{\alpha\beta}\,e_\beta; \qquad (1)$$

$$(T-\omega I)\,\phi=\sum_{\alpha=1}^{n}\varphi_\alpha^{(-)}\,e_\alpha; \qquad (2)$$

$$\varphi^{(+)}=\varphi^{(-)}-i\sum_{\alpha=1}^{n}(\phi,\ e_\alpha)\,J\,a_\alpha, \qquad (3)$$

where vectors e_α, with the help of they were which it is established a correspondence, are called canal; matrix/die $J=\|J_{\alpha\beta}\|$ possesses properties $J=J^*$, $J^2=I$; $\varphi_\alpha^{(-)}$ - components the vectors $\varphi^{(-)}$, $a_1=(1,0,\ldots,0),\ldots,\ a_n=(0,\ 0,\ \ldots,\ 0,\ 1)$.

The posed problem is solved by theorem 1. To multiterminal network Φ can be supplied in the conformity the only operator T, satisfying conditions (1), (2), (3) then and only then when Φ accomplishes single transmission at the infinite frequency, and the set of capacitive and input branches are partial tree/wood[1], the set of

capacitive and output branches - also partial tree/wood.

FOOTNOTE [1]. Partial tree/wood is called the tree/wood, which is the part of the graph/count (multigraph) and which contains all its apexes/vertexes. ENDFOOTNOTE.

For the recording of operator T, canal vectors e_a and matrix/die J according to the form of multiterminal network it is possible to give the standard rules, which do not require calculations. The self-adjointness of operator T occurs with some special geometric relationships/ratios.

Theorem 2. If multiterminal network Φ does not satisfy the conditions of theorem 1, then by the addition of a finite number of concentrated elements/cells it is possible to correct this position (it is structural/constructural), moreover the electrical state of obtained multiterminal network Φ_1 how conveniently differs little from the state of initial Φ.

2. Analogous results are established/installed for circuits with mutual inductance. Is used the apparatus for theory of graphs.

3. Constructed operator T is applied for following purposes:

a) representation of multiterminal network Φ in the form of chain/network of simplest multiterminal networks (realized or not realized);

b) calculation of frequency characteristics of circuits. Thus, the transmission matrix/die $S(\omega)$ of multiterminal network Φ, which satisfies the conditions of theorem 1, can be represented by the formula:

$$S(\omega) = I - i\| ((T - \omega I)^{-1} e_{\alpha},\ e_{\beta}) \| J;$$

c) the study of oscillations $\phi(t), \varphi^{(+)}(t)$ with arbitrary input signal $\varphi^{(-)}(t)$.

EXPANSION OF THE GROWING FUNCTIONS OF THE EIGENFUNCTIONS OF THE NON-SELF-ADJOINT OPERATOR AND TASK OF DIFFRACTION.

Yu. I. Grosberg.

In the present work is examined the scalar, stationary task of diffraction on the limited body in the space of any number of measurements. The difficulties, which appear during the mathematical research of this task, are partially connected with the continuity of its spectrum and the unlimitedness of the resolving operator resultant. This difficulty can be surmounted by introduction to the scalar product of the properly selected weight factor.

1. Let us consider in space n of variable/alternating locked, sufficiently smooth (n-1) -dimensional diversity (Γ) and region (Ω), external for it. Let us designate through H the Hilbert function space, assigned on (Ω), with the scalar product

$$(u, v) = \int_{(\Omega)} u\bar{v}\rho d\omega. \qquad (1)$$

Weight function $\rho>0$ decreases at infinity as $r^{-\sigma}$, $1<\sigma\leq 2$ (r - di.tance from fixed/recorded point $O \in (\bar{\Omega})$).

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Let L[u] - closing/shorting the operator, determined by the formula

$$L[u] \equiv -\frac{\Delta u + k^2 u}{\rho} \qquad (2)$$

on the functions from H, twice continuously differentiated, which have $L[u] \in H$, that satisfy on (Γ) the boundary condition

$$\left[\cos\alpha \frac{\partial u}{\partial n} + \sin\alpha \cdot u\right]_{(\Gamma)} = 0 \qquad (3)$$

and the conditions of radiating/emitting Sommerfield in the form

$$\frac{\partial u}{\partial r} - iku = o\left(r^{-\frac{n-1}{2}}\right). \qquad (4)$$

It is assumed that $\lim\limits_{r\to\infty} k > 0.$ During the proper limitations to the real functions ρ, k, α is valid the following theorem. Operator L[u] - J- self-adjoined has purely discrete spectrum, arranged/located in the lower half-plane. Equation L [u] =w is solved with any $w \in H$, and inverse operator is completely continuous (according to the norm, generated (1)).

In metric H it is convenient to consider (through the norm of operator L^{-1}) the errors, which appear with approximate solution of the tasks of diffraction.

2. If (Γ) - sphere of radius a, k and α are constant, then conveniently to take ρ=r^{-2} and to introduce spherical coordinates. Then operator L [u] has eigenvalues

$$\lambda_{p,q} = \left(\frac{n-2}{2}\right)^2 + \nu_q - \mu_p^2; \; p, q = 1, 2, \ldots, \qquad (5)$$

where ν_q — eigenvalue of Laplace's operator on the sphere, and μ_p — root of the equation

$$\cos\alpha\cdot\frac{d}{da}\left[a^{\frac{2-n}{2}}H_{\mu}^{(1)}(ka)\right]+\sin\alpha\, a^{\frac{2-n}{2}}H_{\mu}^{(1)}(ka)=0. \qquad (6)$$

The corresponding eigenfunctions will be

$$u_{p,q}=Y_q r^{\frac{2-n}{2}}H_{\mu_p}^{(1)}(kr), \qquad (7)$$

where Y_q— spherical function. Radial eigenfunctions $v_p=r^{1-\frac{n}{2}}H_{\mu_p}^{(1)}(kr)$ were introduced by Sommerfield [1].

3. B. M. Levitan and I. S. Sargsyan [2] showed that under specific conditions in Fourier series according to eigenfunctions of operators on $[0, \infty)$ it is possible to expand functions, which do not belong $L^{(2)}$ and even increasing by infinity. Analogous facts occur with respect to expansions in terms of functions v_p. This makes it possible to use to the tasks of diffraction the method of eigenfunctions in the form, proposed by G. A. Girnberg [3].

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Let, for example, (Γ) — sphere of radius a, and k and α be constant. It is necessary to solve the equation

$$\Delta u+k^2u=w \qquad (8)$$

under the conditions (3) on (Γ) and the condition at infinity of the form

$$\frac{\partial (u-u_0)}{\partial r} - ik(u-u_0) = o\left(r^{-\frac{n-1}{2}}\right), \tag{9}$$

where u_0 - assigned "incident wave". Solution can be represented in the form $u = \sum_p X_p v_p$, where X_p depend only on angular coordinates. For determining these coefficients is obtained the equation

$$\tilde{\Delta} X_p - \left[\left(\frac{n-2}{2}\right)^2 - \mu_p^2\right] X_p = A_p \int_0^\infty w v_p r^{n-1} dr - \lim_{r\to\infty} A_p r^{n-1} v_p \left(\frac{\partial u_0}{\partial r} - iku_0\right) \tag{10}$$

and the condition of smoothness on the entire sphere. In this formula $\tilde{\Delta}$ - Laplace's operator on the sphere, and A_p— normalizing constants. Limit in right side (10) can prove to be the generalized function. If the incident wave is flat/plane, then the obtained resolution coincides with the fact which usually is obtained with the help of the transformation of Watson.

Analogous formulas can be obtained for the case when (Γ) — ellipsoid.

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EXPANSIONS IN TERMS OF THE FUNCTIONS CONNECTED WITH THE WATSON TRANSFORM.

V. F. Lazutkin.

In the work are examined the eigenfunction expansions of the following task:

$$y'' + \left(1 - \frac{\nu^2 - \frac{1}{4}}{x^2}\right) y - q(x)\, y = 0 \qquad (1)$$

on the semi-axis $0<a\leq x<+\infty$ with the conditions

$$y(a) = 0, \; y(x) \underset{x\to\infty}{=} e^{ix}(1 + o(1)). \qquad (2)$$

The role of the spectral parameter plays ν^2. $q(x)$ it is limited it is of the order $O\left(\frac{1}{x^{1/3+\varepsilon}}\right)$, $\varepsilon > 0$.

Resolutions of this form are encountered with the solution of the tasks of diffraction on the infinite elliptical (in particular, circular) cylinder, sphere, etc.

In the work is proven the completeness of the system of the eigenfunctions of task in Hilbert space $\mathcal{H}$ of the functions, quadratic-integrated in by weight $1/x^2$ on [a, +∞].

In spite of completeness, it proves to be that for "good"

functions, generally speaking, there is no theorem of resolution. Is given the example:

$$f(x) = e^{\frac{(x-x_0)^2}{(x-x_0)^2-\delta^2}} \overset{(1)}{\text{при}} \; |x - x_0| \leqslant \delta \overset{(2)}{\text{и}} f(x) = 0 \overset{(1)}{\text{при}} \; |x - x_0| > \delta$$

Key: (1). with. (2). and.

(finite, infinitely differentiated function).

Fourier coefficients f(x) approach infinity, and series/row diverges already in the sense of mean convergence.

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Let us introduce into $\mathcal{H}$ operator L, assigned by the differential expression

$$ly = x^2 [y'' + (1 - q(x)) y] + \frac{1}{4} y \tag{3}$$

on the functions from $\mathcal{H}$, which satisfy the conditions:

1) y and y' is absolutely continuous,

$$\begin{aligned} &2)\ ly \in \mathcal{H}, \\ &3)\ y(a) = 0, \\ &4)\ \lim_{x\to\infty} W(y_1, y) = 0, \end{aligned} \tag{4}$$

where W - Wronskian, a y_1 - solution of equation (1), which satisfies radiation condition; $y_1(x) = e^{ix}(1 + o(1))$ with $x \to \infty$.

The resolvent of operator L

$$R_\lambda f = \int_a^\infty G(r, \xi, \lambda) \frac{f(\xi)\, d\xi}{\xi^2} \qquad (5)$$

is meromorphic function λ. G(x, ξ, λ) - Green's function of operator (3), (4).

The pole of resolvent(the eigenvalues of task) they have the following asymptotic behavior:

$$\nu_n = \frac{i\pi n}{\ln i\pi n}\left(1 + O\left(\frac{\ln \ln n}{\ln n}\right)\right);\ \nu^2 = \lambda. \qquad (6)$$

If we from the plane λ reject the vicinities of the poles whose boundaries are assigned by the expression

$$\nu \ln \frac{2\nu}{la} = \nu_n \ln \frac{2\nu_n}{la} + re^{i\varphi},\ \ r < \pi/2,\ \ 0 < \varphi \leqslant 2\pi,$$

then in the remaining part of the plane are valid the following estimations of the norm of the resolvent, considered as operator from $\mathcal{H}$ and C.

$$\|R_\lambda\|_c = \sup_{\substack{a < x < +\infty \\ \|f\|_{\mathcal{H}} = 1}} \left| \int_a^\infty G(x, \xi, \lambda) \frac{f(\xi)}{\xi^2}\, d\xi \right| \qquad (7)$$

With Imλ<0

$$\|R_\lambda\|_c = O(\lambda^{-1/4} \ln^{1/16} \lambda).$$

With Imλ>0 in the field, included between the negative part of the real axis and the line of poles.

$$\|R_\lambda\|_c = O\left(e^{-i\pi\nu + 2\nu \ln \frac{2\nu}{la}} \lambda^{-1/4} \ln^{1/4} \lambda\right). \qquad (8)$$

In the remaining part of the upper half-plane

$$\|R_\lambda\|_c = O(e^{-i\pi\nu} \lambda^{-1/4} \ln^{3/4} \lambda). \qquad (9)$$

Given estimations give the possibility to demonstrate the completeness of their own and associated functions L in a following manner.

Let us introduce on the domain of definition L operator T(t) (t>0) from the formula

$$T(t)f = f + \frac{1}{2\pi i}\int_{-i\sigma-\infty}^{-i\sigma+\infty} e^{i\lambda t} R_\lambda L f \frac{d\lambda}{\lambda}. \qquad (10)$$

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It is assumed that all special features/peculiarities lie/rest above the duct/contour of integration. Are proven the following two facts:

1) $\lim\limits_{t\to 0} \| T(t) f - f \|_C = 0$

($\| \|_C$ - designates the maximum of modulus/module);

2) $\frac{1}{2\pi i}\int_{-i\sigma-\infty}^{-i\sigma+\infty} e^{i\lambda t} R_\lambda L f \frac{d\lambda}{\lambda} = -f(x) + \sum_n \operatorname*{Res}_{\lambda=\lambda_n} e^{i\lambda t} R_\lambda f(x),$

moreover series/row on the deductions descends evenly on x.

SPECTRAL THEORY OF NON-SELF-ADJOINT DIFFERENTIAL OPERATORS AS APPLIED TO THE STUDY OF PROCESSES IN A ONE-DIMENSIONAL MEDIUM.

V. F. Zhdanovich.

Let the process, which occurs in the one-dimensional medium, be described by the system of the differential equations

$$\sum_{k=0}^{m} \frac{\partial^k}{\partial t^k} L_k u(x, t) = 0, \qquad (1)$$

where $L_k u(x, t) = \sum_{s=0}^{m_k} A_{ks}(x) \frac{\partial^s}{\partial x^s} u(x, t)$ $(k = 0, 1, \dots m)$— ordinary differential operators with the complex matrix coefficients above the n-dimensional vector function u(x, t). We will distinguish two cases: 1) medium the limited and space coordinate x is changed on segment [0, 1] and 2) medium not limited and x is changed either on entire real axis ($-\infty<x<+\infty$) (case 2a), or on semi-axis ($0\leqslant x<+\infty$) (case 2b).

In case 1) let are assigned the boundary conditions of the form

$$\sum_{k=0}^{p} \frac{\partial^k}{\partial t^k} [M_{0k}\, \Gamma u(0, t) + M_{1k} \Gamma u(1, t)] = 0, \qquad (2)$$

where p can be any whole non-negative number (in particular, it can be $p > m$), M_{ik} $(i = 0, 1;$ k=0, 1, ... p) - constant matrices/dies with the complex elements/cells, $\Gamma u(0, t)$ and $\Gamma u(1, t)$— value at points x=0 and x=1 (respectively) the differential operator

$$\Gamma u(x, t) = \left\{ \frac{\partial^{q-1} u(x, t)}{\partial x^{q-1}}, \frac{\partial^{q-2} u(x, t)}{\partial x^{q-2}}, \dots u(x, t) \right\},$$

translating the n-dimensional vector function u(x, t) into nq-dimensional vector function $\Gamma u(x, t)$, $q = \max m_k$ $(k = 0, 1, \ldots m)$.

In the case 2a) let are assigned asymptotic "boundary conditions". For example, let occur one of the following cases: for each t⩾0 function $\frac{\partial^k}{\partial t^k} u(x, t)$ $(k = 0, 1, \ldots, m-1)$ 1) it is quadratically summarized, 2) are limited, 3) have the previously prescribed asymptotic behavior, etc.

In the case of 2) are combined asymptotic boundary conditions with the boundary condition of form (2) with x=0.

In all cases the process is managed by the assignment of the initial conditions

$$\frac{\partial^k u(x, 0)}{\partial t^k} = f_k(x) \quad (k = 0, 1, \ldots, m-1). \tag{3}$$

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The mathematical task whose solution is considered in the report, lies in the fact that to represent the process (function u(x, t)described above) in the form of the sum of the waves of the following form: $y_k(x) t^k e^{\lambda t}$ $(k = 0, 1, \ldots)$, where $y_k(x)$ — complex-valued

function from x and λ - complex number.

Without limiting generality, it is possible to consider that m=1, since always this it is possible to achieve by the introduction of new unknown functions. Then, supplying function $u(x,t)=y_0(x)e^{\lambda t}$ (or function $u(x,t)=y_0(x)t^k e^{\lambda t}$ with k>0) equation (1) and boundary conditions (2), we obtain for determining of $y_0(x)$ and λ (or $y_k(x)$ and λ with k>0) spectral task, i.e., the task to eigenvalues and eigenfunctions, which we will examine in certain banach space, to a considerable degree determined by the character of boundary conditions. To research and solution of spectral problem it is possible to apply the changed theory of the non-self-adjoint operators, developed in such works as work [1], [2], etc. In accordance with this theory is constructed the operator-significant function E(Ω), argument of which are the subsets Ω of the complex plane Z, and by values k - projection (i.e. such, that $E^2(\Omega)=E(\Omega)$) operators, who function in the banach space about which it was mentioned above, and certain nilpotent operator N from the same space. In this case by nilpotent operator is understood the bounded operator, for which there is an integer k_0 such, that $N^{k_0}=0$ and $N^{k_0-1}\neq 0$. Function E(Ω) proves to be additive, i.e., for the nonintersecting sets Ω_1 and Ω_2 $E(\Omega_1 U \Omega_2)=E(\Omega_1)+E(\Omega_2)$, which gives the possibility to consider as the its generalization of measure of a set and to construct for it the generalization of Stieltjes's integral.

Then representation of function u(x, t) required in the task in the form of the sum of the waves of form indicated above (if, of course, it occurs) is given by the formula

$$u(x,\ t)=\sum_{k=0}^{k_0-1}\int_Z N^k\frac{t^k}{k!}e^{\lambda t}dE\ (\Omega)\, f_0\,(x), \qquad (4)$$

where Z - complex plane. But it is necessary to say that there are the processes for which problem stated above has a solution not for any initial conditions, but even if has solution, then in certain generalized sense (for example, improper integral (4) it can diverge and then for obtaining with formula (4) of function u(x, t) it is necessary to use one or another the method of the addition of the divergent integrals). Lecturer carried out research of task when equation (1) and boundary conditions (2) satisfy further conditions. If $0 \leqslant x \leqslant 1$, then task is investigated, for example, in the following cases: 1) m=1, m_0=1, m_1=0, p=1 (these research are partially published in the work [31]); 2) m=1, m_0=1, m_1=0; 3) m=1, m_1=0, p=0; 4) m=1, m_0=0, m_1=1, p=1, etc. If $-\infty<x<+\infty$ (or $0 \leqslant x<+\infty$), then task was investigated with m=1, m_1=0 and during the further limitations to the coefficients of equation (1). For example, with m_0=1 it was assumed that coefficient $A_{10}(x)$ the constant, and coefficients $A_{01}(x)$ and $A_{00}(x)$ belong to space $L(-\infty, +\infty)$ (or $L(0, \infty)$ in the case 2c), or it was assumed that coefficient $A_{10}(x)$ constant, and coefficients $A_{01}(x)$ and $A_{00}(x)$ are periodic, etc.

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NUMERICAL ESTIMATIONS OF EIGENVALUES OF THE NON-SELF-ADJOINT LINEAR DIFFERENTIAL OPERATOR OF THE SECOND ORDER WITH PERIODIC COEFFICIENTS.

M. I. Serov.

Is examined the equation

$$y'' = p(x)y' + [\lambda - q(x)]\, y = 0, \qquad (1)$$

where p(x) and q(x) - complex periodic functions the real variable x: p (x+1)=p(x), q(x+1)=q(x). Coefficients are such, that the invariant of equation (1)

$$Q(x) = q(x) - \frac{p^2(x)}{4} - \frac{p'(x)}{2}$$

- the piecewise-continuous function with the bounded variation in the period. Eigenvalue of operator (1) we will call similar λ, with which equation (1) has at least one limited along the entire real x axis solution. Set of eigenvalues let us name the spectrum.

In [1] and [2] it is proved that if

$$\int_0^1 \mathrm{Re}\, p(x)\, dx \neq 0, \qquad (2)$$

then the spectrum consists of a finite number of limited components, each of which is homeomorphic circle/circumference, and one unlimited

component, which asymptotically flows together with the parabola whose axis is parallel to real axis λ - plane. The infinite branch of this parabola is directed to the side of the positive part of the real axis. Hence follows the existence of at least one real eigenvalue of the task in question. In addition to those being in [1] and [2] qualitative by research and asymptotic estimations let us give some numerical estimations.

Without resorting to generality, it is possible to consider that

$$\int_0^1 Q(x)\,dx = 0. \tag{3}$$

Let us designate $\lambda=k^2$. Characteristic equation can be registered in the form

$$\cos\left[t - \frac{i}{2}\int_0^1 p(x)\,dx\right] - \cos k =$$

$$= \frac{1}{2}\sum_{n=2}^{\infty}\frac{1}{k^n}\int_0^1 \dots \int_0^{x_{n-1}} Q(x_1)\sin[k(1 - x_1 + x_n)] \times$$

$$\times \left\{\prod_{j=2}^{n} Q(x_j)\sin[k(x_{j-1} - x_j)]\,dx_j\right\} dx_1. \tag{4}$$

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Here t - real parameter, $0\leqslant t\leqslant 2\pi$. Series/row in right side (4) descends evenly and absolutely within any band complex k - plane, parallel to its real axis.

Using different majorants for this series/row, it is possible to obtain numerical estimations.

Without giving linings/calculations, let us point out upper limit maximum real eigenvalue under the conditions (2) and (3):

$$\lambda_{\text{neu}} < 4\left[\frac{\int_0^1 |Q(x)|\,dx}{\int_0^1 \mathrm{Re}\, p(x)\,dx}\right]^2. \tag{5}$$

Let us point out also estimation for the alleged part of eigenvalue in the case when

$$\int_0^1 \mathrm{Re}\, p(x)\,dx = 0.$$

In this case the spectrum consists of a denumberable number of segments of certain analytical curve, which asymptotically approaches under condition (3) a real axis λ - plane. The spectrum is limited then to the left, on top and from below. As shown in [1], and [2], $\max|\mathrm{Im}\,\lambda| \leqslant \sup|\mathrm{Im}\,Q(x)|$. Using this estimation and after selecting the appropriate majorant for series/row (4), we will obtain as upper bound for $|\mathrm{Im}\lambda|$ the function, which monotonically vanishes with increase $\mathrm{Re}\lambda$.

Designations: v - total variation Q(x) in the period, $M = \sup|Q(x)|$, $m = \sup|\mathrm{Im}\,Q(x)|$, $r = \mathrm{Re}\,\lambda$. Then with $r>2v\,\mathrm{ch}(m/r)$ we have

$$|\operatorname{Im}\lambda| < 4\frac{\sqrt{Mv}\,\operatorname{ch}\frac{m}{r}}{\sqrt[4]{r}}\sqrt{\frac{r+\frac{m^2}{r^2}}{r-2v\,\operatorname{ch}\frac{m}{r}}}. \qquad (6)$$

Estimations (5) and (6) allow/assume decrease. However, the order of tendency toward zero functions in the right side of the estimation of type (6) can be changed only during the imposition of the further conditions of smoothness on Q(x).

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3. Integral equations.

On the methods of solving integral first-order equations.

A. N. Tikhonov.

Paired integral equations and solution of some problems of the theory of wave diffraction.

Yu. V. Gandel'.

Many boundary-value problems for the equation of Helmholtz are reduced to the paired integral equations the form

$$\int_0^\infty S(\lambda) J_\nu(\lambda r)(\lambda^2 - k^2)^\beta \, d\lambda = f(r) \quad (0 < r < a),$$

$$\int_0^\infty S(\lambda) J_\nu(\lambda r) \, d\lambda = g(r) \quad (r > a),$$

where integer $\nu > 0$, and also numbers k>0 and β $(0 < \beta^2 < 1$, in the tasks $\beta=-1/2$ interesting us) are assigned. Somewhat simpler equations (k=0) are encountered in the theory of potential.

To the theory of these equations are devoted sufficiently many works [1, 2, 7]. It is characteristic that large role in the works indicated play some of N. Ya. Sonin's formulas [3]. From theoretical-functional apparatus here found use, on one hand, [1, 2, 5] the generalized integral representations of Wiener-Paley, with another [7] - the methods, connected with the problem of Gilbert/Hilbert and finally technician Wiener-Hopf.

The tasks to which this apparatus is applied, are sufficiently

special (infinitely thin shield with the circular opening/aperture or the slot, the presence of the specific symmetry of field).

However, in these cases the method of the paired integral equations leads to the very efficient results.

In the report is stated the method of solving of paired equations or their reducing to the regular equation of Fredholm for the function, through which the unknown function $S(\lambda)$ is expressed with the help of Wiener-Paley's generalized formula. For example, in the case of diffracting the sound on the circular disk, which reduces to the equations

$$\int_0^\infty C(\lambda) J_0(\lambda r)\lambda d\lambda = 1 \quad (0 < r < a),$$

$$\int_0^\infty C(\lambda) J_0(\lambda r)\frac{\lambda d\lambda}{\sqrt{\lambda^2 - k^2}} = 0 \quad (r > a),$$

the unknown function $C(\lambda)$ is represented in the form

$$C(\lambda) = \sqrt{\frac{2}{\pi}}\int_0^\infty h(s)\sin(s\sqrt{\lambda^2 - k^2})\,ds,$$

where $h(s)$ satisfies the equation of Fredholm

$$h(r) - \frac{i}{\pi}\int_0^a \left\{\frac{\operatorname{sh} k(s+r)}{s+r} - \frac{\operatorname{sh} k(s-r)}{s-r}\right\} h(s)\,ds = \sqrt{\frac{2}{\pi}}\frac{\operatorname{sh} kr}{k}.$$

This integral equation was for the first time obtained in works [5, 2] and recently by completely different method in work [4].

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The same equation is encountered in work [5] with construction of first approximation in the vector task of diffraction which earlier was examined in work [6].

In the report is given new solution of the problem of diffracting the electromagnetic waves in circular opening/aperture in the flat/plane shield, moreover also by reducing to the same to integral equation which is necessary to decide under two different right sides and some further conditions. These conditions project/emerge here instead of the conditions of Mayksner and they completely naturally appear with the solution of problem with the help of the paired integral equations.

In conclusion are analyzed different approaches with the help of the paired integral equations to the task of diffraction in slot, including solution in the locked form, obtained recently in work [7].

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One task of diffracting the acoustic wave for the multiply connected and laminar regions.

D. Z. Avazashvili.

Let there be in the infinite space the nonintersecting regions T_1, T_2, ..., T_n, limited by surfaces of S_1, S_2, ..., S_n, moreover each and regions T_1, T_2, ..., T_n is filled with homogeneous isotropic medium. Let these media be characterized by respectively constant parameters k_1, k_2, ..., k_n; the part of the infinite space T_0, external with respect to regions T_j $(j = 1, 2, ..., n)$, it is also uniform and isotropic, having parameter k_0. We will consider that the driver is placed in all media.

The propagation of the wave processes, connected with the phenomenon of diffraction for the above-determined regions, is led to the following mathematical task.

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To find such function $\varphi(m)$, which satisfies the conditions:

$$\left.\begin{array}{l}
1)\ \Delta\varphi + k_0^2\varphi = -f_0 \overset{(1)}{\text{в}}\ T_0; \\
2)\ \Delta\varphi + k_j^2\varphi = -f_j \overset{(1)}{\text{в}}\ T_j\ (j = 1, 2, \ldots, n); \\
3)\ (\varphi)_j = (\varphi)_0,\ \frac{1}{k_j^2}\left(\frac{\partial\varphi}{\partial n}\right)_j = \frac{1}{k_0^2}\left(\frac{\partial\varphi}{\partial n}\right)_0 \overset{(2)}{\text{на}}\ S_j\ (j = 1, 2, \ldots, n); \\
4)\ \frac{\partial\varphi}{\partial r} - ik_0\varphi = e^{ik_0r} o\,(r^{-1})\ \text{на}\ \overset{(3)}{\text{бесконечности}},
\end{array}\right\} \quad (1)$$

Key: (1). in. (2). on. (3). at infinity.

where $\Delta \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$; k_j (Re $k_j > 0$, Im $k_j \geqslant 0$) — complex wave number; f_j — assigned function, which characterizes source, it is continuously differentiated to the second order inclusively; r - distance between two points; $ro\,(r^{-1}) \to 0$ with $r \to \infty$; $(\varphi)_j$, $(\varphi)_0$, $\left(\frac{\partial\varphi}{\partial n}\right)_j$, $\left(\frac{\partial\varphi}{\partial n}\right)_0$ respectively designate the limiting values of function φ and not normal derivative from within and from without on surface S_j; it is assumed that all surfaces $S_j\ (j = 1, 2, \ldots, n)$ of Lyapunov's type.

It is proven, that task (1) has a unique solution and a solution represented in the form of the following loaded integral equation of Fredholm's type the second kind:

$$\frac{1}{k^2(M)}\varphi(M) = \frac{1}{4\pi}\sum_{j=1}^{n}\left\{\frac{k_j^2 - k_0^2}{k_j^2}\int_{T_j}\varphi(N)\frac{e^{ik_0 r(M, N)}}{r(M, N)}\,d\tau_N + \right.$$
$$\left. + \left(\frac{1}{k_j^2} - \frac{1}{k_0^2}\right)\int_{S_j}\varphi(N)\frac{\partial}{\partial n_N}\left(\frac{e^{ik_0 r(M, N)}}{r(M, N)}\right)dS_N\right\} + \frac{1}{4\pi}\sum_{j=0}^{n}\frac{1}{k_j^2}\int_{T_j} f_j(N)\frac{e^{ik_0 r(M, N)}}{r(M, N)}\,d\tau_N, \quad (2)$$

where

$$\frac{1}{k^2(M)} = \begin{cases} \frac{1}{k_0^2}, & M \in T_0, \\ \frac{1}{k_j^2}, & M \in T_j, \\ \frac{1}{2}\left(\frac{1}{k_j^2} + \frac{1}{k_0^2}\right), & M \in S_j. \end{cases}$$

Task (1) is examined also for the laminar regions and it is solved analogously with two-dimensional problem of diffracting electromagnetic waves [1]. Let there be in the infinite space n of the consecutively/serially inserted in each other regions, limited by respectively in pairs nonintersecting surfaces $S_j\ (j = 1, 2, \ldots, n)$, which are filled with the homogeneous and isotropic medium, which is characterized by respectively constant parameters $k_j\ (j = 1,2 \ldots, n)$. The region, limited by surface S_j (under the assumption of the absence of all subsequent regions), let us designate through T_j; external surface - through S_1 and the external (infinite) region - through T_0 which is also uniform isotropic, having a parameter k_0. Let us designate through $T_j^{\xi} - T_{j+1} = T_{j,j+1}$ the region, included between S_j and S_{j+1}; in this case let $T_{0j} \equiv T_0$ and $T_{n,n+1} \equiv T_n$. For such regions task (1) is formulated as follows.

To find such function $\varphi(M)$, which satisfies the conditions:

$$\left.\begin{array}{l}1)\ \Delta\varphi + k_0^2\varphi = -f_0 \text{ в}^{(1)}\ T_0;\\ 2)\ \Delta\varphi + k_j^2\varphi = -f_j \text{ в}^{(1)}\ T_{j,\,j+1};\\ 3)\ (\varphi)_j = (\varphi)_{j-1},\ \dfrac{1}{k_j^2}\left(\dfrac{\partial\varphi}{\partial n}\right)_j = \dfrac{1}{k_{j-1}^2}\left(\dfrac{\partial\varphi}{\partial n}\right)_{j-1} \text{ на}^{(2)}\ S_j;\\ 4)\ \dfrac{\partial\varphi}{\partial r} - ik_0\varphi = e^{ik_0r}o(r^{-1}) \text{ на}^{(3)} \text{ бесконечности.}\end{array}\right| \qquad (3)$$

Key: (1). in. (2). on. (3). at infinity.

It is proven, that task (3) has the unique solution, which satisfies the following loaded integral equalization of Fredholm's type second kind:

$$\frac{1}{k^2(M)}\varphi(M) = \frac{1}{4\pi}\sum_{j=1}^{n}\left\{\frac{k_0^2(k_j^2 - k_{j-1}^2)}{k_j^2 k_{j-1}^2}\int_{T_j}\varphi(N)\frac{e^{ik_0 r(M,N)}}{r(M,N)}d\tau_N + \right.$$
$$\left. + \left(\frac{1}{k_j^2} - \frac{1}{k_{j-1}^2}\right)\int_{S_j}\varphi(N)\frac{\partial}{\partial n_N}\frac{e^{ik_0 r(M,N)}}{r(M,N)}\,dS_N\right\} + \frac{1}{4\pi}\sum_{j=0}^{n}\frac{1}{k_j^2}\int_{T_{j,\,j+1}} f_j(N)\frac{e^{ik_j r(M,N)}}{r(M,N)}d\tau_N, \qquad (4)$$

where

$$\frac{1}{k^2(M)} = \begin{cases}\dfrac{1}{k_0^2}, & M \in T_0,\\ \dfrac{1}{k_j^2}, & M \in T_{j,\,j+1},\\ \dfrac{1}{2}\left(\dfrac{1}{k_j^2} + \dfrac{1}{k_{j-1}^2}\right), & M \in S_j.\end{cases}$$

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Solution of the axisymmetric contact problem of steady-state oscillations of half-space.

V. N. Zakorko, N. A. Rostovtsev.

Is examined the elastic half-space, in which were established/installed the oscillations, caused by the action of force $Qe^{-i\omega t}$. This action is transmitted through circular cross-section rigid weightless flat-based die/stamp, which is indented without the inclination/slope. Sufficiently large static load P ensures the contact of die/stamp with the elastic medium. Shearing stresses in the region of contact and outside it are absent. Is placed the task of determining the amplitude $\hat{p}(r)$ of pressure under the die/stamp from the known amplitude $\hat{w}(r)=w_0$ residues/settlings in the region of contact.

It is possible to demonstrate that for $\hat{p}(r)$ we have the integral equation

$$\hat{w}(r) = -\frac{k_2^2}{\mu}\int_0^a \rho\hat{p}(\rho)\int_0^\infty \frac{s\sqrt{s^2-k_1^2}}{F(s)} I_0(rs)\, I_0(\rho s)\, ds, \qquad (1)$$

where a - radius of die/stamp; k_1, k_2 - the wave numbers ($k_1^2=\frac{\omega^2}{c_1^2}$, $k_2^2=\frac{\omega^2}{c_2^2}$; $c_1^2=\frac{\lambda+2\mu}{\delta}$; $c_2^2=\frac{\mu}{\delta}$; δ - the density of elastic medium; λ, μ - elastic constant Lames),

$$F(s)=(2s^2-k_2^2)^2-4s^2\sqrt{s^2-k_2^2}\sqrt{s^2-k_1^2}. \qquad (2)$$

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Changing to the dimensionless quantities

$$\left.\begin{array}{l} x^2=\frac{r}{a},\ y=\frac{\rho}{a},\ \sigma=\frac{s}{k_2};\ \prod(x)=\frac{\hat{p}(r)}{\mu};\ W=\frac{w}{a}; \\ h^2=\frac{k_1^2}{k_2^2}=\frac{\mu}{\lambda+2\mu}=\frac{1-2\nu}{2(1-\nu)};\ 1-h^2=\frac{1}{2(1-\nu)};\ \alpha=ak_2 \end{array}\right\} \qquad (3)$$

and selecting in the kernel of equation (1) the term, which corresponds to the pole of integrand at infinity, we give equation (1) (on the assumption that $W=W_0$ - constant number) to the form

$$\mathrm{H}(x)=c+\frac{2}{(1-\nu)\pi}\int_0^1\mathrm{H}(y)\,\mathrm{K}(x,y)\,dy, \qquad (4)$$

where

$$\mathrm{H}(x)=\Pi(x)\sqrt{1-x^2},\quad c=\frac{2W_0}{(1-\nu)\pi}; \qquad (5)$$

$$\mathrm{K}(x,y)=\frac{y}{\sqrt{1-y^2}}\int_0^\infty G(\sigma)\left\{\cos x\sigma+\sqrt{1-x^2}\,\alpha\sigma\int_x^1\frac{\sin x\sigma t}{\sqrt{t^2-x^2}}dt\right\}I_0(\alpha y\sigma)\,d\sigma; \qquad (6)$$

$$G(\sigma)=\frac{\alpha}{2(1-h^2)}+\frac{\alpha\sigma\sqrt{\sigma^2-h^2}}{(2\sigma^2-1)^2-4\sigma^2\sqrt{\sigma^2-h^2}\sqrt{\sigma^2-1}}. \qquad (7)$$

$G(\sigma)$ has the simple pole in interval $(1, \infty)$, which corresponds to the speed of Rayleigh waves. Therefore integral (6) is computed in the sense of principal value. For this from $G(\sigma)$ clearly is selected the term, which corresponds to pole. Equation (4) is solved by successive approximations.

Unsteady processes in wire antennas.

L. N. Lutchenko.

The proposed method of study makes it possible to study transient processes in the thin wire antennas both locked (single-turn), and extended, that are located in the free space. Let the locus of the centers of the cross sections of lead/duct (axis of lead/duct) form the line $\rho=\rho(\varphi)$, which lies at one plane, moreover radius of curvature R at each point of line always is much more than "b" of the thickness of the cross section of lead/duct and distance between two points of the order of the length of the lead/duct between them.

It is assumed also that the loads, connected with the antenna, can be considered as the local heterogeneities processes in which with the sufficient degree of accuracy can be considered quasi-stationary. Ohmic losses in the lead/duct are not considered.

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The conditions for inversion into zero tangential component of

electric field on the surface of conductor, besides the connection points of loads and sources, make it possible to obtain equation for the one-dimensional function of current distribution along the lead/duct of the antenna:

$$\frac{\partial}{\partial s}\left(\frac{\partial I}{\partial s}\chi\right)-\frac{1}{c^2}\frac{\partial^2 I}{\partial t^2}\chi=-\frac{2}{c}\frac{\partial E_{\text{стр}}}{\partial t}-\frac{2}{c}\frac{\partial}{\partial t}\sum_{i=1}^{N}Z_iI(st)\,\delta(s-s_i)-$$

$$-\frac{1}{2\pi}\sqrt{\frac{\mu}{\varepsilon}}\left[\frac{\partial}{\partial s}\int_0^{2\pi}\int_{-L}^{L}\frac{\left[\frac{\partial I}{\partial s'}\right]-\frac{\partial I}{\partial s}\cos(\nu-\nu')}{2\pi r}ds'd\psi-\right.$$

$$\left.-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\int_0^{2\pi}\int_{-L}^{L}\frac{[I(s't)]-I(st)}{2\pi r}\cos(\nu-\nu')\,ds'd\psi\right], \qquad (1)$$

where S - coordinate, calculated along the lead/duct of antenna; $(\nu-\nu')$ — the angle between the tangents to the axis of lead/duct at observation points and by that flowing; r - distance between these points; s_i — the place of activation of load; Z_i— operator, depending on the form of load and equal to multiplication by resistance, if it is active;

$\chi=\frac{1}{2\pi}\sqrt{\frac{\mu}{\varepsilon}}\int_0^{2\pi}\int_{-L}^{L}\frac{\cos(\nu-\nu')}{r}ds'd\psi$ — the matched impedance of heterogeneous long line.

Equation is solved approximately with the use of expansion in terms of the small parameter 1/ln(8L/b). For the zero approximation is chosen the function of current distribution in the asymptotically thin antenna. The solution of equation (1) in the first

approximation, makes it possible to take into account the final thickness of lead/duct, the concrete/specific/actual geometric form of antenna and radiation loss. Is considered the time of the establishment of process and it is noted that the resonance length of antenna is changed not only caused by the radiation/emission, but also because of the local reflections, caused by a change in the matched impedance.

Radiation of currents and charges, which fly with constant velocity near ideally conductive bodies (problems, which allow exact solution).

B. M. Bolotovskiy, G. V. Voskresenskiy.

With the uniform motion of the sources of electromagnetic field in the free space the radiation/emission is absent; to this corresponds the presence only of the damped harmonics in field expansion of source into the Fourier integral. However, if near the trajectory of charges there are optical heterogeneities (shields or other obstructions), then with the flight/span of sources past them appear the radiations/emissions (appear undamped harmonics in Fourier-representation of field). This radiation/emission can be represented as the diffraction of the field of the moving/driving source on the met heterogeneity.

In the theory of the diffraction of electromagnetic waves is worked out a strict method of the solution of problems for some scattering structures, which uses reducing of task to the system of the paired integral equations, which admit solution by the method of Wiener - Hopf - Foch [1]. In the report is given the survey/coverage

of the works in which the method indicated is applied for the determination of a precise representation of the electromagnetic radiation field of the elementary current distribution and the charges, which evenly fly near from the metallic semi-bounded obstruction: half-plane, the open end/lead of the flat/plane and circular waveguide, diffraction grating, formed by the periodic system of the half-planes of waveguide branching off. In each case are obtained field expressions of radiation/emission into the free space (continuous spectrum) and the excitation coefficients of waves in the regions, limited by the metallic surfaces (discrete/digital set of waveguide harmonics). Are investigated radiation losses.

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Diffraction of electromagnetic waves on the plasma cylinder.

G. I. Makarov, V. V. Novikov.

In the unbounded medium, characterized by constants ε_1, μ_1, is arranged/located the circular plasma cylinder of radius a whose properties are described by function $\varepsilon_2(r, z)$, $\mu_2 = \mu_1 = \text{const}$. At distance r=b from the axis of cylinder is arranged/located the vertical electric dipole harmonically oscillating with the frequency ω. It is assumed that function $\varepsilon_2(r, z)$ possesses the following properties:

$$\left|\frac{\partial \varepsilon_2(r, z)}{\partial r}\right| = O\left(\frac{\varepsilon_2(r, z)}{z_1}\right); \quad (1)$$

$$kz_1 < 1, \quad k = \frac{\omega}{c}; \quad (2)$$

$$\left|\frac{\partial \varepsilon_2(r, z)}{\partial z}\right| = O\left(\frac{1 + |\varepsilon_2(r, z)|}{z_2}\right); \quad (3)$$

$$kz_2 \gg 1; \quad (4)$$

$$\varepsilon_2(a, z) = \varepsilon_1; \quad (5)$$

$$\varepsilon_2(0, z) = 0, \quad (6)$$

i.e. function $\varepsilon_2(r, z)$ slowly varies to scale of the wavelength of the falling/incident disturbance/perturbation in direction z and it is rapid in direction r.

If we introduce in a special manner potentials, then it appears

that the solution of the equations of Maxwell can be expressed with the help of two scalar functions. The construction of solution in the exterior of cylinder does not present difficulties and it is carried out by standard methods. However, as far as the interior of cylinder is concerned, here task is reduced to the system of two ordinary differential equations of the second order with the variable coefficients. This system is simplified, if we use the method of standard equations; however, nevertheless its solution is not expressed as known special functions. In the work the solution of system is reduced to the system of two integral equations of Fredholm second kind. It proves to be that in the case

$$ka < 1 \qquad (7)$$

it is possible to use a method of successive approximations and to express the solution through the degenerate hypergeometric functions. As a result it is possible to find electromagnetic field in exterior and interior of cylinder. It proves to be, in particular, that the presence of the point where ε_2 becomes zero, leads to the appearance of the waves whose polarization is different from the polarization of the incident field.

Diffraction of plane waves on slot and strip[redacted].

M. D. Khaskind [deceased], L. A. Weinstein.

To the classical task about the diffraction of plane wave on the continuous ~~belt~~ strip and on the slot in the infinite plane is devoted an extremely large number of article; however,, until now, is absent such asymptotic expression for the diffraction field in the distant zone, which would satisfy both the reciprocity principle (7) and to boundary conditions (8) and (9) and at the same time it would be useful for any directions. In this work is derived this expression, moreover is incidentally shown the inaccuracy of the solution, given in article [3].

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We use the designations, used in works [1] - [3]. On the slot $-1<x<1$ in plane $z=0$ diffracts the plane wave

$$V_0 = e^{ik(\alpha_0 x + z\sqrt{1-\alpha_0^2})}, \qquad (1)$$

falling from the lower half-space $z<0$. The complete field V satisfies the scalar wave equation

$$\Delta V + k^2 V = 0 \qquad (2)$$

and the boundary condition

V=0 with z=0 and |x|>1.

In the upper half-space $z \geqslant 0$ at the sufficiently large distances from the slot the field takes the form of cylindrical wave

$$V = \varphi(\alpha_0, \alpha) \frac{e^{i\left(kr - \frac{3\pi}{4}\right)}}{\sqrt{2\pi kr}}, \qquad (4)$$

where

$$\left.\begin{array}{l} \alpha_0 = \cos\vartheta_0, \quad \alpha = \cos\vartheta, \\ 0 \leqslant \vartheta_0 \leqslant \pi, \quad 0 \leqslant \vartheta \leqslant \pi, \end{array}\right\} \qquad (5)$$

ϑ - angle between the direction to observation point and x axis, and ϑ_0 - angle between the direction of propagation of incident wave (1) and x axis.

The same function $\varphi(\alpha_0, \alpha)$ determines diffraction field at any point of space according to the formula

$$V = V^0 + \frac{1}{2\pi i} \int_{-\infty}^{\infty} \varphi(\alpha_0, \alpha)\, e^{ik(x\alpha + |z| \sqrt{1-\alpha^2})} \frac{d\alpha}{\sqrt{1-\alpha^2}}. \qquad (6)$$

It must satisfy the reciprocity principle

$$\varphi(\alpha_0, \alpha) = \varphi(-\alpha, -\alpha_0) \qquad (7)$$

and the boundary conditions

$$\varphi(\alpha_0, \pm 1) = 0 \qquad (8)$$

and

$$\varphi(\pm 1, \alpha) = 0. \qquad (9)$$

The same function $\varphi(\alpha_0, \alpha)$ determines the field, scattered by the strip/film - 1<x<1, z=0, on surface of which is placed the boundary condition

$$\frac{\partial V}{\partial z} = 0 \text{ with } z=0,\ |x|<1, \qquad (10)$$

if on it falls the same wave (1).

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According to works [1] and [2] the function $\varphi(\alpha_0, \alpha)$ can be represented in the form

$$\varphi(\alpha_0, \alpha) = \Phi(\alpha_0, \alpha)\, e^{-i\varkappa\alpha} + \Phi(-\alpha_0, -\alpha)\, e^{i\varkappa\alpha}, \qquad (11)$$

where $\varkappa = kl$ is the basic parameter of the task (dependence of functions φ, Φ and so forth on it is not written out clearly), but function Φ is a solution of the integral equation

$$\Phi = \Phi_0 + O\Phi, \qquad (12)$$

where the function

$$\Phi_0(\alpha_0, \alpha) = -\frac{\sqrt{1-\alpha_0}\sqrt{1-\alpha}}{\alpha-\alpha_0}\, e^{i\varkappa\alpha_0}, \qquad (13)$$

determines field from the isolated/insulated half-plane x>1, but operator O is clearly written out in works [1] - [3].

Equation (12) can be solved by successive approximations. We obtain

$$\Phi(\alpha_0, \alpha) = \sum_{n=0}^{\infty} \Phi_n(\alpha_0, \alpha), \quad \Phi_n = O^n \Phi_0. \tag{14}$$

In the work are investigated the general/common/total properties of functions Φ_n. If we assume

$$\varphi_n(\alpha_0, \alpha) = \Phi_n(\alpha_0, \alpha)\, e^{-i\varkappa\alpha} + \Phi_n(-\alpha_0, -\alpha)\, e^{i\varkappa\alpha}, \tag{15}$$

then it appears that all functions φ_n satisfy condition (7). However, to condition (8) function φ_n they do not satisfy, instead of this occurs the relationship/ratio

$$\Phi_n(\alpha_0, 1) = -\Phi_{n-1}(-\alpha_0, -1)\, e^{2i\varkappa} \quad (n = 1, 2, \ldots). \tag{16}$$

Functions Φ_0, Φ_1 and Φ_2 can be registered in the form

$$\Phi_n(\alpha_0, \alpha) = \Phi_0(\alpha_0, \alpha)\, Z_n^+(\alpha_0, \alpha) + \Phi_0(-\alpha_0, -\alpha)\, Z_n^-(\alpha_0, \alpha)\, e^{2i\varkappa\alpha}, \tag{17}$$

where

$$\left.\begin{aligned} &Z_0^+ = 1, && Z_0^- = 0, \\ &Z_1^+ = \Gamma(\varkappa, \alpha_0) && Z_1^- = \Gamma(\varkappa, \alpha), \\ & && Z_2^- = \Gamma(\varkappa, -\alpha_0)\,\Gamma(\varkappa, \alpha), \end{aligned}\right\} \tag{18}$$

but the uncopied/unordered function Z_2^+ is finite with finite α_0 and α and order q^2, where value

$$q = \frac{e^{i\left(2\varkappa - \frac{3\pi}{4}\right)}}{2\sqrt{\pi\varkappa}} \tag{19}$$

is low under condition $\varkappa \ll 1$. The function

$$\Gamma(\varkappa, \alpha) = \sqrt{1 - \alpha^2} \int_{\infty}^{\varkappa} H_0^{(1)}(2t)\, e^{-2it\alpha}\, dt \tag{20}$$

satisfies the conditions

$$\Gamma(\varkappa, 1) = -1, \quad \Gamma(\varkappa, -1) = 0, \qquad (21)$$

therefore it natural to consider the value of the order of one.

Since Φ_3 and Φ_4 order q^2, and $\Phi_5, \Phi_6, \ldots$ is still less, then during the construction of the asymptotic solution of stated problem naturally taking into account only Φ_0, Φ_1 and that part of function Φ_2, which is proportional Z_1^2. We obtain, thus, for function (11) the asymptotic expression

$$\varphi^{(1)}(\alpha_0, \alpha) = \Phi_0(\alpha_0, \alpha) T^{(1)}(\alpha_0, \alpha) e^{i\varkappa\alpha} + \Phi_0(-\alpha_0, -\alpha) T^{(1)}(-\alpha_0, -\alpha) e^{-i\varkappa\alpha}, \qquad (22)$$

where

$$T^{(1)}(\alpha_0, \alpha) = [1 + \Gamma(\varkappa, \alpha_0)][1 + \Gamma(\varkappa, -\alpha)]. \qquad (23)$$

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It approximates exact expression with the relative error

$$|q|^2 = \frac{\lambda}{8\pi d}, \qquad (24)$$

it accurately satisfies conditions (7), (8) and (9) and has the same structure, as the approximation formula of Ufimtsev [4], [5].

Instead of the simple method of successive approximations, which leads to series/row (14), it is possible to use Fredholm first

theorem. Fredholm's denominator in the first approximation, is equal to

$$D = 1 - q^2 \text{ with } |\varkappa| \gg 1). \tag{25}$$

Equation D=0 determines the complex natural frequencies of this system.

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One approximate method of solving the integral equation of the diffraction of electromagnetic waves on a band of finite width.

G. Ya. Popov.

Is proposed the approximate method of solving the integral equation

$$\int_0^a H_0^{(2)}(k|x-y|)\,\varphi(y)\,dy = Ae^{-ikx} + Be^{ikx} = g(x), \quad x > 0, \tag{1}$$

to which is led [1] two-dimensional problem about the diffraction of electromagnetic waves on the ideally conducting infinitely thin band of final width.

This method, significantly based on the use of the following (apparently, the for the first time discovered in the work author [2]) the property of Laguerre's polynomials $L_m^{\alpha}(x)$

$$\frac{1}{\pi}\int_0^{\infty} \frac{K_0(|x-y|)}{\sqrt{y}e^{y}} L_m^{-1/2}(2y)\,dy = \mu_m e^{-x} L_m^{-1/2}(2x)$$
$$(0 < x < \infty,\ m = 0, 1, 2, \ldots;\ \sqrt{2}\,2m!!\,\mu_m = \sqrt{\pi}\,(2m-1)!!), \tag{2}$$

consists of the following.

Integral equation (1) by using the known relationship/ratio

between the Hankel function $H_0^{(2)}(z)$ and MacDonald $K_0(z)$ is reduced to the form

$$\frac{1}{\pi}\int_0^a K_0(|\xi-\eta|)\,\chi_a(\eta)\,d\eta=f(\xi) \quad \left(f(\xi)=g\left(\frac{\xi}{ik}\right)\right), \qquad (3)$$

moreover occurs connection/communication

$$2\varphi(x)=k\chi_{ika}(ikx). \qquad (4)$$

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Further, following G. A. Greenberg [3], is introduced the key equation

$$\int_0^a K_0(|\xi-\eta|)\,\varphi(\rho,\eta)\,d\eta=K_0(\xi+\rho),\ \rho>0 \qquad (5)$$

and it is proven, that

$$\chi_a(\xi)=\chi_\infty(\xi)+\int_0^\infty \chi_\infty(a+t)\,\varphi(t,a-\xi)\,dt, \qquad (6)$$

where $\chi_\infty(\xi)$ — the solution of integral equation (3) with $a=\infty$.

The solution of key equation is found by the formula

$$2\varphi(\rho,a-\xi)=\psi^+(\rho,\xi)-\psi^-(\rho,\xi), \qquad (7)$$

where $\psi^\pm(\rho,\xi)$ — the essence of the solution of the following integral equations:

$$\psi^\pm(\rho,\xi)\mp\int_0^\infty \psi^\pm(t,\xi)\,v(\rho,a+t)\,dt=v(\rho,\xi)\pm v(\rho,a-\xi). \qquad (8)$$

In turn, the kernel of the latter is to find from the following

integral equation:

$$\int_0^\infty K_0(|\xi - \eta|)\, v(\rho, \eta)\, d\eta = K_0(\xi + \rho), \quad \rho > 0. \tag{9}$$

In order to obtain the solution of integral equations (8), the right side of equation (9) is decomposed/expanded in the series/row on Laguerre's polynomials:

$$\frac{1}{\pi} K_0(\xi + \rho) = \sum_{m=0}^{\infty} a_m(\rho)\, e^{-\xi} L_m^{-1/2}(2\xi), \quad \rho > 0. \tag{10}$$

Subsequent use (2) gives

$$v(\rho, \xi) = \sum_{m=0}^{\infty} \frac{a_m(\rho)}{\mu_m} \frac{e^{-\xi}}{\sqrt{\xi}} L_m^{-1/2}(2\xi). \tag{11}$$

Here

$$\frac{a_m(t)}{\sqrt{2}} = \frac{\Gamma(m + 1/2)}{\pi} \sqrt[4]{\frac{2}{t}}\, W_{-1/4-m,\, -1/4}(2t),$$

where $\Gamma(z)$ — Euler's gamma function; $W_{\lambda,\mu}(z)$ — Whittaker function.

Substitution (11) in (8) reduces the problem in question to the infinite system of algebraic equations.

But if we are be satisfied by approximate solution, for example to hold down/retain in expansion (10) a finite number of terms, then in accordance with (9) problem will be reduced to the resolution of integral equations (8) with the degenerate kernel. Use/application to the solution of the latter of certain special method made it possible

to obtain the solution of the integral equation of diffraction (1) in connection with the case when in the resolution (10) is retained n+1 term in the following form:

$$2\varphi(x) = Ak\chi_n(ikx;\ ika) + Bke^{ika}\chi_n(ika - ikx;\ ika), \qquad (12)$$

where

$$\chi_n(\xi;\ \alpha) = \sqrt{\frac{2}{\pi}}\frac{e^{-\xi}}{\sqrt{\xi}} + \sum_{m=0}^{n}\left[\frac{C_m^{(n)}L_m^{-1/2}(2\xi)}{e^{\xi}\sqrt{\xi}} + \frac{D_m^{(n)}e^{\xi}L_m^{-1/2}(2a - 2\xi)}{e^{a}\sqrt{\alpha - \xi}}\right]. \qquad (13)$$

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In this case, which is substantial for conducting the calculations, for coefficients $C_m^{(n)}$ and $D_m^{(n)}$ are obtained the recurrent formulas, which connect them with analog coefficients $C_m^{(n-1)}$ and $D_m^{(n-1)}$ of the previous approximation/approach and which, just as (13), do not contain quadratures.

The greater the number ka, the less the terms it is necessary to retain in expansion (10) for obtaining the sufficiently accurate result and, that means the simpler formula (13).

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Diffraction of electromagnetic waves on the anisotropic step in a [redacted] plane waveguide.

P. S. Mikazan, Ya. Ya. Zush

In all known to the author works on the diffraction of the electromagnetic waves, solved with the help of the method of Wiener - Hopf - Foch, was produced the factorization of even function. For the purpose of the expansion of the circle of the tasks, solved by this method, is worked out the method of the factorization of the function of the following form:

$$L(w)=\frac{\sqrt{k^2-w^2}\,(1+a_1w)+b_1w^2+c_1}{\sqrt{k^2-w^2}\,(1+a_2w)+b_2w^2+c_2}$$

according to the formula

$$L(w)=(h_1-w)(h_2+w)L^+(w)L^-(w),$$

where $L^+(w)$ - the function, holomorphic with Im $w \geqslant 0$ and which does not have there zero; $L^-(w)$ - the function, which satisfies the same conditions with Im $w \leqslant 0$, but with $w = h_1 - h_2$ function $L(w)$ becomes zero.

For the factorization is introduced new function $\eta_1(w)$, not having zero and poles

$$\eta_1(w)=\frac{(g_1-w)(g_2+w)L(w)}{(h_1-w)(h_2+w)},$$

where g_1 and g_2 - points of the poles of function $L(w)$.

Since in the tasks in the diffraction of electromagnetic waves always occur the relationships/ratios

$$a_1 = aa_2,$$
$$b_1 = ab_2,$$

then

$$\lim_{|w|\to\infty} \eta_1(w) = a$$

On the basis of this property with the help of the Cauchy integrals it is obtained

$$\eta_1^{\pm}(w) = \sqrt{\eta_1(w)}\, e^{\pm \frac{1}{2\pi i} \int\limits_0^\infty \frac{w \ln \eta_1(u)\,\eta_2(u) + u \ln \frac{\eta_1(u)}{\eta_2(u)}}{u^2 - w^2} du},$$

where

$$\eta_2(u) = \eta_1(-u).$$

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Then

$$L^+(w) = \frac{\eta_1^+(w)}{g_1 + w},$$

$$L^-(w) = \frac{\eta_1^-(w)}{g_1 - w}.$$

This method of the factorization of function, which is not even, will help to solve series of problems in the diffraction of electromagnetic waves as, for example, the diffraction of electromagnetic waves in the flat/plane waveguide with the

anisotropic step.

Are solved two problems in the diffraction of electromagnetic waves in the flat/plane waveguide. It is assumed that one wall of waveguide ideally conducting, and on the second wall is replaced the thin anisotropic layer, which has step with z=0. In the first task TE the wave attacks from the side z=-∞ to the step. Magnetic permeability is expressed by tensor, and $\varepsilon \approx 1$. In the second task attacks TM the wave, dielectric constant is expressed by tensor, and $\mu \sim 1$. In both tasks anisotropic layer is magnetized by transverse magnetic field.

Hertz's potential is assigned in the form:

for the first task

$$\Psi_y = \int_c e^{iwz} \cos v\,(a-x)\,F(w)\,dw$$

and for the second task

$$\Psi_z = \int_c e^{iwz} \sin v\,(a-x)\,F(w)\,dw,$$

where $v = \sqrt{k^2 - w^2}$, $F(w)$ — unknown function, and k - wave number.

Are used the approximate boundary conditions, obtained by N. A. Kuz'min. These boundary conditions take the form:

for the first task

$$E_y = -k\delta\left(\frac{\mu_2}{\mu_1}H_x - i\mu_\perp\mu_1 H_z\right); \quad \begin{matrix} z<0, & x=0, \\ z>0, & x=0, \ \delta\to\delta_1 \end{matrix}$$

and for the second task

$$E_z = \delta\frac{1}{\varepsilon_{11}}\left(\frac{\partial E_x}{\partial z} - \varepsilon_{13}\frac{\partial E_z}{\partial z}\right) - ik\delta H_y; \quad \begin{matrix} z<0, & x=0, \\ z>0, & x=0, \ \delta\to\delta_1, \end{matrix}$$

where δ, δ_1 - thickness of anisotropic layer with $z<0$ and $z>0$ respectively.

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With the help of the boundary conditions are obtained the functional equations:

for the first task

$$\int_c e^{iwz} L(w) I(w)\, dw = 0; \quad z<0$$
$$\int_c e^{iwz} I(w)\, dw = 0; \quad z>0$$

and for the second task

$$\int_c e^{iwz} N(w) G(w)\, dw = 0; \quad z<0$$
$$\int_c e^{iwz} G(w)\, dw = 0; \quad z>0$$

where I(w) and G(w) - unknown functions, and

$$L(w)=\frac{\sin va-w\delta\frac{\mu_2}{\mu_1}\sin va+v\delta\mu_{+}\mu_1\cos va}{\sin va-w\delta_1\frac{\mu_2}{\mu_1}\sin va+v\delta_1\mu_{+}\mu_1\cos va}$$

and

$$N(w)=\frac{v\sin va-\delta\frac{1}{\varepsilon_{11}}w^2\cos va+i\delta\frac{\varepsilon_{13}}{\varepsilon_{11}}wv\sin va+k^2\delta\cos va}{v\sin va-\delta_1\frac{1}{\varepsilon_{11}}w^2\cos va+i\delta_1\frac{\varepsilon_{13}}{\varepsilon_{11}}wv\sin va+k^2\delta_1\cos va}$$

the functions which must be factored.

During the solution of the characteristic equation of the first task it was discovered, that with an increase δ instead of the slow wave appears the special rapid wave.

The theory of dielectric waveguide with the conducting diaphragm.

S. S. Kalmykova, V. I. Kurilko.

In the work is examined the theory of scattering the surface electromagnetic waves, which extend along the anisotropic dielectric cylinder ($r \leqslant a$ $-\infty<z<+\infty$), on the jump of the tensor of dielectric constants this waveguide ($\hat{\varepsilon}(z<0,\ z\leqslant a) \neq \hat{\varepsilon}(z>0,\ r\leqslant a)$). In this case it is assumed that the uniform parts of the waveguide are divided by the infinitely thin conducting diaphragm ($r\leqslant a$, $z=0$).

It is shown that a strict task of finding Fourier's component stray field in this system is reduced to the solution of the following integral equation of Fredholm:

$$\psi(x) + \frac{1}{2(\pi i)^2}\int_{-\infty}^{+\infty}\frac{Z(x')\,dx'}{x'-x}\sum_{m=1}^{2}\frac{Z_m(x')}{\Delta_m(x')}\cdot\int_{-\infty}^{+\infty}\left[\frac{1}{Z(x'')}-\frac{1}{Z(x')}\right]\frac{\psi(x'')\,dx''}{x''-x'} =$$

$$= -\frac{\gamma_n}{(\pi i)^2}\int_{-\infty}^{+\infty}\frac{Z(x')\,dx'}{(x'-x)(x'^2-\gamma_n^2)}\cdot\sum_{m=1}^{2}\frac{Z_m(x')-Z(\gamma_n)}{\Delta_m(x')};$$

$$Z(x) \equiv \frac{iv(x)}{k}\cdot\frac{Q_0(x)}{Q_1(x)};\quad v(x) \equiv (x^2-k^2)^{1/2};\quad k=\frac{\omega}{c}; \qquad (1)$$

$$Q_n(x) \equiv K_0(vb)\cdot I_n(va) - (-1)^n\cdot I_0(vb)\cdot K_n(va),\quad n=0,\ 1;$$

$$Z_m(x) \equiv \frac{i\beta_m}{k\varepsilon_z^m}\cdot\frac{J_0(\beta_m a)}{J_1(\beta_m a)};\ \beta_m(x) \equiv \left[\frac{\varepsilon_z^m}{\varepsilon_\perp^m}(\varepsilon_\perp^m k^2 - x^2)\right]^{1/2};$$

$$\Delta_m(x) \equiv Z(x) - Z_m(x)$$

(b - radius of jacket/case/housing, γ_n — the wave number of incident wave).

The solution of this equation is found with the method of iteration for the plasma-type waveguide in the magnetic field in the extreme case $\delta_m \ll a < b \ll \lambda$, $\delta_m = c/\omega|\varepsilon_{\cdot}^{m}|$ (scattering ground wave on the jump of the density of plasma). Are obtained analytical expressions for the transformation ratios of ground waves. Is examined a question about the effect of the conducting diaphragm to the characteristics of scattering. Is found also short-wave asymptotics of field near the edge of diaphragm. Is obtained also the electrostatic solution of this problem for case $\hat{\varepsilon}_2 \equiv \hat{\varepsilon}_1 \equiv 1$.

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For the plane geometry the equation, analogous (1), is solved in the electrostatic approximation/approach for case $\varepsilon_{\cdot}^{(1)} \to \infty$, $\hat{\varepsilon}_2 \equiv 1$ (propagation of coaxial wave in the flat/plane waveguide with the abruptly varying section). It is shown also, that in the case when the height/altitude of the conducting diaphragm is equal to half of the height/altitude of waveguide ($\hat{\varepsilon}_2 \equiv \hat{\varepsilon}_1 \equiv 1$), equation of type (1) gives the solution, obtained earlier by L. A. Weinstein [1].

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One approximation method of the solution of the problem of diffraction of electromagnetic waves at the open end of the [redacted]plane waveguide and some allied problems.

L. B. Tartakovskiy, A. I. Rubenstein.

Is proposed the approximation method of solving some diffraction tasks, which are reduced to the equations of the type of Wiener - Hopf and to the systems of such equations. Briefly it is reduced to the following.

In many tasks the structure of current is known a priori, without the solution of quite diffraction problems. So is matter concerning the flat/plane waveguide where the current is composed of the current of waveguide waves (damping and undamped) and current on the external walls of half-plane, which can be in the first approximation, considered proportional to current on the shadow side of half-plane with the incidence/drop on the latter of plane wave. Analogous structure has current in the system of two waveguides, formed by three half-planes. Method lies in the fact that, using the approximate representation of current in the form of the sum of known components/terms/addends with the undetermined coefficients, they

attain inversion into zero on the sufficiently rapid system of the points of the integrand integral of which describes the tangential component of electric field on the metal.

Use/application of a method to the task of radiation/emission from the open end/lead of the flat/plane waveguide gives a good coincidence of numerical results with the results of a strict solution of L. A. Weinstein [1].

Let us illustrate the aforesaid above. The task about is flat/plane ohm waveguide it is reduced [1] to the following functional equations:

$$\int_{-\infty}^{\infty} e^{iwz} F(w)\, dw = -e^{-ihz}; \quad z < 0; \tag{1}$$

$$\int_{-\infty}^{\infty} e^{iwz} L(w) F(w)\, dw = 0, \quad z > 0. \tag{2}$$

Let us consider the simplest case of the excitation by wave TEM Then (see [1], page 10) $h = k,\ L(w) = v(1 - e^{ivd}),\ v = \sqrt{k^2 - w^2}$.

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We seek the Fourier transform F(w) of current density in the form

$$F(w)=F_0(w)\,F_1(w)+\sqrt{\frac{2k}{k-w}}F_2(w)\equiv$$

$$\equiv\frac{k}{w-k}\prod_{n=1}^{N}\frac{1-\dfrac{k-w}{k-\hat{w}_n}}{1-\dfrac{k-w}{k-w_n}}\left(\frac{A_0}{w-k}+\sum_{m=1}^{M}\frac{A_m}{w+u_m}\right)+\sqrt{\frac{2k}{k-w}}\left(\frac{B_0}{w-k}+\sum_{m=1}^{M}\frac{B_m}{w+u_m}\right), \quad (3)$$

where

$$w_n=\frac{2\pi}{d}\sqrt{\left(\frac{d}{\lambda}\right)^2-n^2},\ \hat{w}_n=\frac{2\pi}{d}\sqrt{\left(\frac{d}{\lambda}\right)^2-\left(\frac{2n-1}{2}\right)^2};$$
$$u_n=\frac{2\pi}{d}\sqrt{\left(\frac{d}{\lambda}\right)^2-\left(\frac{2n-1}{4}\right)^2},\ n=1,2,\ldots. \quad (4)$$

Condition (1) then gives

$$\left.\begin{array}{l}F_0(-u_m)\,A_m+\sqrt{\dfrac{2k}{k-u_m}}B_m=0,\ m=1,2,\ldots,M,\\ F_0(-k)\,A_0+B_0=1.\end{array}\right\} \quad (5)$$

According to (3), (4) left side (2) can be rewritten as

$$\int_{\Gamma}e^{iwz}v\left[(1-\cos vd)\,F_0(w)\,F_1(w)-i\sqrt{\frac{2k}{k-w}}F_2(w)\sin vd\right]dw, \quad (6)$$

where duct/contour Γ connects up upper half-plane point k with infinite so that on Γ function $v=\sqrt{k^2-w^2}$ is real. From (4), (5) it follows that the expression in the brackets in (6) is converted into zero at points $w_n, \hat{w}_n, n=1,2,\ldots,N$. $F_1(w)$ and $F_2(w)$ in the upper half-plane slowly vary and, after requiring inversion into zero expressions

$$T(w)=(1-\cos vd)\,F_0(w)\,F_1(w)-i\sqrt{\frac{2k}{k-w}}F_2(w)\sin vd, \quad (7)$$

where

at points u_m, $|\sin vd|$ reaches maximum value, and in k it is possible to expect good satisfaction (2).

Solving the system of linear equations (5) and $T(k)=0$, $T(u_n)=0$, n=1, 2, ..., M, we will obtain $A_m = A_m(q;\ N, M)$, and then diffraction of coefficients of reflection and transformation on the current

$$\left.\begin{aligned}
&R_{0,0}(q, N, M) = \frac{kA_0}{2k_0} + \sum_{m=1}^{M} \frac{kA_m}{k+u_m};\\
&R_{0,n}(q;\ N, M) = \left(\frac{kA_0}{k+w_n} + \sum_{m=1}^{M} \frac{kA_m}{w_n+u_m}\right) \frac{w_n-\hat{w}_n}{\hat{w}_n-k} \prod_{\substack{l=1\\ l\neq n}}^{N} \frac{1-\dfrac{k-w_n}{k-\hat{w}_l}}{1-\dfrac{k-w_n}{k-w_l}};\\
&n = 1, 2, \ldots, N.
\end{aligned}\right\} \qquad (8)$$

The calculations for N=11 and $M=6$ carried out on computer(s), show a good coincidence of coefficients (8) with strict values [1].

The case of the antiphase excitation of two waveguides, formed by three half-planes, by wave TEM is reduced to the system

$$\left.\begin{aligned}
&\int_{-\infty}^{\infty} e^{iwz}\Phi_1(w)\,dw = -e^{-ikz}, \quad z<0;\\
&\int_{-\infty}^{\infty} e^{iwz}\Phi_2(w)\,dw = -\Omega e^{-ikz}, \quad z<0;\\
&\int_{-\infty}^{\infty} e^{iwz}[v(1+e^{i2vd})\Phi_1(w) + ve^{ivd}\Phi_2(w)]\,dw = 0, \quad z>0;\\
&\int_{-\infty}^{\infty} e^{iwz}[2ve^{ivd}\Phi_1(w) + v\Phi_2(w)]\,dw = 0, \quad z>0.
\end{aligned}\right\} \qquad (9)$$

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We assume/set

$$\Phi_1(w) = \Phi_0^+(w)\,\Phi_1^+(w) + \Phi_0^-(w)\,\Phi_1^-(w) + \sqrt{\frac{2k}{k-w}}\,\Phi_{\text{кр}}(w);$$
$$\Phi_2(w) = \Phi_0^+(w)\,\Phi_1^+(w) - \Phi_0^-(w)\,\Phi_1^-(w), \tag{10}$$

where

$$\Phi_0^-(w) = \frac{k}{w-k}\prod_{n=1}^{N}\frac{1-\dfrac{k-w}{k-\hat{w}_n}}{1-\dfrac{k-w}{k-w_n}}; \quad \Phi_0^+(w) = \frac{k}{w-\hat{w}_1}\prod_{n=2}^{N+1}\frac{1-\dfrac{\hat{w}_1-w}{\hat{w}_1-w_n}}{1-\dfrac{\hat{w}_1-w}{\hat{w}_1-\hat{w}_n}};$$

$$\Phi_1^-(w) = \frac{P_0^-}{w+k} + \sum_{m=1}^{M}\frac{P_m^-}{w+u_m}; \quad \Phi_1^+(w) = \frac{P_0^+}{w+k} + \sum_{m=1}^{M}\frac{P_m^+}{w+u_m};$$

$$\Phi_{\text{кр}}(w) = \frac{Q_0}{w+k} + \sum_{m=1}^{M}\frac{Q_m}{w+u_m}.$$

If we designate through

$$T(w) = (1+\cos vd)\,\Phi_0^+(w)\,\Phi_1^+(w) - (1-\cos vd)\,\Phi_0^-(w)\,\Phi_1^-(w) + i\sqrt{\frac{2k}{k-w}}\,\Phi_{\text{кр}}(w)\sin vd, \tag{11}$$

then equations (9) are substituted approximately to the system of the linear equations

$$\left.\begin{array}{l}
P_0^+\Phi_0^+(-k) + P_0^-\Phi_0^-(-k) + Q_0 = 1;\\
P_0^+\Phi_0^+(-k) - P_0^-\Phi_0^-(-k) = 1;\\
\left.\begin{array}{l}P_m^+\Phi_0^+(-u_m) + P_m^-\Phi_0^-(-u_m) + \sqrt{\dfrac{2k}{k+u_m}}\,Q_m = 0\\ P_m^+\Phi_0^+(-u_m) - P_m^-\Phi_0^-(-u_m) = 0,\end{array}\right\} m = 1, 2, \ldots, M;\\
T(k) = 0;\\
T(u_m) = 0, \quad m = 1, 2, \ldots, M.
\end{array}\right\} \tag{12}$$

Thus, the task about three half-planes in principle does not differ from the task about the flat/plane waveguide and it is possible to expect sufficiently accurate numerical results.

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Excitation of the ideally conducting cylinder of finite length with the distribution of the unknown magnetic field to TE and TM waves.

Ye. N. Vasil'yev, A. A. Falunin.

Electromagnetic field of arbitrary sources in the presence of the ideally conducting infinite cylinder, as is known, is divided into TE and TM waves, which do not interact with each other. However, during the excitation of the cylinder of finite length the waves indicated interact between themselves about the ends/faces of cylinder, which is evident from the distribution curves of full current on the cylinder of finite length [1].

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However, the absence of the distribution of the currents, which correspond to TE and TM waves, strongly impedes the analysis of these results. In this work the excitation of the cylinder of finite length is examined by somewhat different method, which makes it possible to divide TE and TM waves.

For the composition of integral equation relative to surface

electric current densities (H) we use Lorenz's lemma, which taking into account boundary conditions with the body surface will be registered in the form

$$\oint_{\Sigma} [\mathbf{E}_2\mathbf{H}_1]\,\mathbf{u}\,ds = \int_{V} (\mathbf{j}_1^{м}\cdot\mathbf{H}_2 - \mathbf{j}_1^{э}\cdot\mathbf{E}_2)\,dV, \qquad (1)$$

where the first field unknown, and the second - auxiliary. For the determination of the vector function H_1 with the help of scalar equation (1) it is necessary to use two linearly independent auxiliary sources. For simultaneous satisfaction of the condition for distribution of TE and TM waves as such sources is used the system z of the electrical and magnetic currents, which create either TE or TM waves. Decomposing/expanding fields and kernels of equation (1) in the series/row for the azimuth harmonics, we bring two-dimensional equation (1) to the one-dimensional.

In the axisymmetric case of field they are divided into TE and TM waves for any body of revolution.

During the asymmetric excitation we write/record the unknown and auxiliary field through the vector potentials and we integrate in parts. As a result this equation (1) relative to the complete field H on lateral and butt ends of cylinder it passes in the equation relative to fields H, created by TE and TM waves. In the section of the rounding between the ends/faces and the lateral surface of

cylinder is sought, as before, the complete field H. If in the usual solution of problem are two unknown field components H, then in the task with the distribution of waves a number of unknowns doubles; however, separate components prove to be interdependent/interconnected, and this makes it possible to leave only two unknown field components.

For the numerical solution of integral equation is used the method of Krylov - Bogolyubov, which substitutes, actually, the unknown value by the piecewise constant function. Thus, integral equation is reduced to the system of algebraic equations.

During the use of electronic computers (EVM) the solution of this system does not meet fundamental difficulties. Using the symmetry of body relative to axis r, it is possible instead of the unknown functions to find their even and odd components. As a result of this are obtained two matrices/dies with the doubly smaller order, to use with which considerably more easily.

For the solution of problem examined above were comprised the programs for the computer(s), which make it possible find the current distribution of TE and TM waves on the cylinders with length to several wavelengths.

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Excitation of body of revolution in the presence of sphere coaxial with it.

Ye. N. Vasil'yev, A. R. Seregina, V. G. Kamenev.

The electrodynamic task of exciting the ideally conductive body of rotation in the presence of sphere is reduced to the solution of integral equation relative to currents on the body of revolution. For the composition of equation is written/recorded the field of the currents, which flow along the body surface of rotation taking into account reflection from the sphere. Then to this field is assigned the requirement of satisfaction to the boundary condition:

$$\mathbf{n} \times \mathbf{H}^{\text{полн}} = \mathbf{j},$$

which reduces to the integral equation of Fredholm of the second order. Kernel of integral equation naturally falls into two components/terms/addends - one P (v, V'), which corresponds to body of revolution in the free space, and second L (v, V') considers

reflection from the sphere.

Kernel P (v, V') is expressed as function S_m, being m-th term of expansion in the Fourier series in terms of the azimuth coordinate of function $e^{-ik\rho}/k\rho$. Kernel L (v, V') is represented in the form of series/row according to the spherical functions which do not always descend sufficiently rapidly. Is conducted an improvement in the convergence of series.

The solution of integral equation is produced numerically by method of Krylov and Bogolyubov. This method reduces the integral equation to the system of linear algebraic equations.

Are given the results of calculating the current distribution on the cylinder in the presence of metallic and dielectric spheres. The effect of sphere is exhibited in the appearance of the further wave reflected on the cylinder. Are computed the radiation patterns of slot antenna on the cylinder of finite length, arranged/located about the sphere.

Application of method of regularization to the calculation of diffraction tasks.

V. I. Dmitriyev, A. N. Tikhonov.

Excitation of dielectric body of revolution.

Ye. N. Vasil'yev, L. B. Materikova.

In the work is proposed the numerical method of calculation of equivalent electrical and magnetic current distributions on the dielectric body of revolution during the excitation by its arbitrary outside sources. Method is based on the use of integral equations and the use/application to the smooth body of revolution of arbitrary form.

Basis undertakes the system of vector integral equations relative to surface electrical and magnetic currents on the body with arbitrary parameters ε and μ, obtained in [1]. System consists of two equations of Fredholm of the second order with the weakly polar kernel.

In this work the problem is placed more concretely/specifically/actually - it is examined dielectric body of revolution with the analytical surface. For the solution is introduced the orthogonal system of rotational coordinates u, v φ. The kernels and the unknown currents, entering the system of integral

equations, are represented in the form of series/rows on the azimuth coordinate φ. This makes it possible to obtain the infinite series of the systems of one-dimensional integral equations for the azimuth harmonics of currents. Each system consists of four equations relative to the appropriate harmonics of longitudinal and transverse currents on the body. The kernels of system are expressed as function S_m, being the m-th coefficient of expansion in the Fourier series Green's function $e^{-i\rho}/\rho$, and its derivatives.

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As an example is examined the case of the axisymmetric excitation of dielectric body. With m=0 the system of four equations falls into two independent systems of two equations, corresponding to waves TE and TM. Calculations are performed for the case of the excitation by wave TM. Are investigated kernels of integral equation. For solving the system is selected the method of Krylov and Bogolyubov. The essence of method consists of the replacement of the unknown function of the piecewise constant. As a result of this the system of equations is reduced to the system of algebraic equations. Definite difficulty appears during the calculation of the diagonal matrix elements of algebraic system, since separate kernels of integral equation have a special feature/peculiarity with the coincidence of observation points and integration. In these kernels

is selected singular part and it is computed especially.

As the exciting source is examined the electric dipole, arranged/located along the rotational axis.

Are given the results of calculations for several bodies of various forms in different position of source. All calculations were performed in the electronic computer. The possibilities of computer technology set limitations on the sizes/dimensions body. The maximum length of generatrix in the given examples composes the value of the order of several wavelengths.

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Application of integral equations in problems of diffraction.

V. V. Kravtsov.

Arbitrary scalar wave from the wave vector k falls on the body, limited by the smooth locked surface of s, on which is satisfied the boundary condition of the first, second or third kind. Stray field is defined as the solution of the equation of Helmholtz

$$\Delta u + k^2 u = 0$$

out of the body, which satisfies the specified boundary condition on s and the condition for radiation/emission at infinity. The solution of the boundary-value problem indicated with the help of Green's formula is reduced to the integral first-order equation of Fredholm's type. Is proved the uniqueness of the solution of this equation. Is given an example of the numerical solution of integral equation by the method of regularization, proposed by A. N. Tikhonov.

The method of solution is postponed by the case of diffracting the electromagnetic waves on the locked impedance surface.

4. Asymptotic methods and nonanalytical solutions.

L. N. Sretenskiy.

Poincare's method in the theory of radio wave diffraction.

1. In large memoir, published into XXIX volume "Rendiconti del Circolo Matematico di Palerno" in 1910, Poincare solved task about Hertz wave diffraction with terrestial globe. External electric field is created by point periodic source, which are located on the surface of sphere. The result of work was the establishment of known at present formula for the electric current density for the high frequencies of the transmitter:

$$\mu = \frac{e^{i\varepsilon_1\varphi}}{\sqrt{\varepsilon_1(1-e^{-2i\varphi})}},$$

where ε_1 is certain specific complex number, φ - latitude of observation point.

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2. This result was obtained with the help of establishment of

asymptotic formula for function, represented in the form of infinite series whose general term contains product from Bessel function to Legendre's polynomial. Studying this product as the analytic function of the index, considered as the complex variable, Poincare, arrived at the formula for the image of function μ. The method, proposed by Poincare for the establishment of asymptotic formula, caused large controversy. As a result of long discussion of question, which consisted, actually the matters, in the establishment of asymptotic formula for the function, represented as the infinite series in which each term depends on the high parameter (frequency of transmitter), by different scientists (by MacDonald, Nicholson, by Love, etc.) were confirmed conclusion by Poincare. Watson arrived at Poincare's formulas by the determination of the asymptotic estimation of some definite integrals.

3. Poincare's method for determination of asymptotic formulas for functions, represented by infinite series, can be used with great success to solution of diverse problems of hydrodynamics of wave motions of liquid, theory of gas jets and to questions of architectural acoustics.

Short-wave asymptotic behavior of coefficient of reflection.

M. V. Fedoryuk.

Let us consider the equation

$$\psi''(x) + k^2 q(x) \psi(x) = 0, \qquad (1)$$

$q(x) > 0$, $q(\pm\infty) = 1$, $k \gg 1$, $q(z)$ - analytic function, Let $\psi \sim e^{ikx}$, $x \to +\infty$, then $\psi \sim Ae^{ikx} + Be^{-ikx}$, $x \to -\infty$. Let us designate

$$S(z_1, z_2) = i \int_{z_1}^{z_2} \sqrt{q(z)}\, dz.$$

Stokes's lines (hp) are called lines Re S=c, zeroaxial q and first-order poles q. Axis Ox is included in the symmetrical region D, limited by line C and which does not contain them, S (D) - band. Let G - part of boundary D, which lies at the upper half-plane. Let us consider the following cases:

1. G contains exactly one simple zero q(z).

2. G contains exactly two simple zeros z_1, z_2. Then

$$B = c_0 (-2i \cos k\alpha + O(k^{-1})) \cdot \exp [k (S(0, z_1) + S(0, z_2))];$$
$$\alpha = iS(z_1, z_2). \tag{2}$$

The path of integration for α lies/rests on G, at the remaining cases - at D and Re $S(0, z_i) < 0$: c_0. Comparison with the exact solution for $q(x) = E + U_0 \operatorname{ch}^{-2}(\beta x),\ E > 0,\ U_0 > 0$ gives coincidence with (2).

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This case occurs for even whole q(z), which does not have pure imaginary zero. With $k \sim \pi/2\alpha\ (2n+1)$ is a resonance of the coefficient of reflection R.

3. G contains one simple zero z_1 and one simple pole z_2. Then they are connected by line C. and

$$B = c_0 \cdot 2i (1 - \cos k\alpha + O(k^{-1})) \exp (kS(0, z_1)). \tag{3}$$

With $k \sim 2\pi n/\alpha$ there is a resonance R.

Asymptotic approximations in dynamics of elastic layered inhomogeneous medium.

V. Yu. Zavadsky.

Is examined a special case of moving the elastic layered inhomogeneous medium when displacement vector U (x,y,z,t) satisfying equation of motion

$$\rho\frac{\partial^2 U}{\partial t^2} = (\lambda+\mu)\nabla(\nabla U) + \mu\nabla^2 U + \nabla\lambda(\nabla U) + \nabla\mu\times(\nabla\times U) + 2(\nabla\mu\nabla)U,$$

is located in plane xz, does not depend on coordinate y and depends on x, t accordingly multiplier $\exp[-i\omega(t - vx)]$. The density ρ of elastic medium is considered constant, and Lame's parameters λ, μ - changing only in the dependence on coordinate z as the continuous differentiated functions. Are introduced vector ψe_y and scalar φ the potentials, connected with the displacement vector by the formula

$$U = \nabla\varphi + \nabla\times(\psi e_y),$$

and also values S, T, connected with the potentials φ, ψ the expressions

$$S = (\lambda + 2\mu)(\varphi'' - \omega^2 v^2\varphi);\quad T = -\mu(\psi'' - \omega^2 v^2\psi)\quad \left(' = \frac{d}{dz}\right).$$

showed that the equation of motion for vector U is satisfied when values S, T satisfy the following system of two differential equations

$$S'' + \omega^2\chi_1(z)\,S - \omega q c_t^{-2} T + \frac{q'}{\omega + \omega^{-1}q^2}\left(T' - i\omega \nu S + \omega^{-1} q S' + i\nu q T\right) = 0;$$
$$T'' + \omega^2\chi_2(z)\,T + \omega q c_t^{-2} S - \frac{q'}{\omega + \omega^{-1}q^2}\left(S' - i\omega \nu T + \omega^{-1} q T' + i\nu q S\right) = 0,$$

where are introduced the designations: $\chi_1(z) = c_l^{-2}(z) - \nu^2,\ \chi_2(z) = c_t^{-2} - \nu^2,\ c_l = \left(\frac{\lambda + 2\mu}{\rho}\right)^{\frac{1}{2}}$ - speed of longitudinal waves; $c_t = \left(\frac{\mu}{\rho}\right)^{\frac{1}{2}}$ - speed of transverse waves; $q = 4i\nu c_t(z)\,c_t'(z)$.

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System of equations for values S, T is investigated asymptotically on the assumption that ω is great. For the determination of the asymptotic solution of this system is used Liouville's method, which consists in the fact that the unknown values S, T are represented in the form

$$S = Ae^{\omega\int \alpha(z)\,dz} + Ba(z)\,e^{\omega\int \beta(z)\,dz};$$
$$T = Be^{\omega\int \beta(z)\,dz} + Ab(z)\,e^{\omega\int \alpha(z)\,dz},$$

where A, B - arbitrary constants; $\alpha(z)$, $\beta(z)$, $a(z)$, $b(z)$ - unknown functions. As a result of differentiation of the given above

expressions for S, T and substitution into the initial equations is obtained the system of four nonlinear differential equations for the unknown functions $\alpha(z)$, $\beta(z)$, $a(z)$, $b(z)$. These functions are sought in the form of series/rows according to degrees ω:

$\alpha(z)=\alpha_0(z)+\alpha_1\omega^{-1}+\ldots$; $\beta(z)=\beta_0(z)+\beta_1(z)\,\omega^{-1}\ldots$; $a(z)=a_0(z)+a_1(z)\,\omega^{-1}+\ldots$;

$b(z)=b_0(z)+b_1(z)\,\omega^{-1}+\ldots$. Being limited to terms $O(1)$, $O(\omega^{-1})$ and throwing/rejecting the remaining terms whose contribution with $\omega\to\infty$ is small, we come to following asymptotic expressions for S, T:

$$S\cong C_1\chi_1^{-\frac{1}{4}}e^{i\omega\int\sqrt{\chi_1(z)}\,dz}+\omega^{-1}\chi_2^{-\frac{1}{4}}C_2\delta_1e^{i\omega\int\sqrt{\chi_2(z)}\,dz}+$$
$$+C_3\chi_1^{-\frac{1}{4}}e^{-i\omega\int\sqrt{\chi_1(z)}\,dz}+\omega^{-1}\chi_2^{-\frac{1}{4}}C_4\delta_1e^{-i\omega\int\sqrt{\chi_2(z)}\,dz};$$
$$T\cong C_2\chi_2^{-\frac{1}{4}}e^{i\omega\int\sqrt{\chi_2(z)}\,dz}+C_1\omega^{-1}\delta_2\chi_1^{-\frac{1}{4}}e^{i\omega\int\sqrt{\chi_1(z)}\,dz}+$$
$$+C_4\chi_2^{-\frac{1}{4}}e^{-i\omega\int\sqrt{\chi_2(z)}\,dz}+C_3\omega^{-1}\delta_2\chi_1^{-\frac{1}{4}}e^{-i\omega\int\sqrt{\chi_1(z)}\,dz},$$

where C_1-C_4 - arbitrary constants; $\delta_1=\frac{c_l^2q}{c_t^2-c_l^2}$; $\delta_2=\frac{c_t^2q}{c_l^2-c_t^2}$. On the basis of asymptotic solutions for S, T are found the asymptotic representations of scalar φ and vector of potentials, that differ from expressions given above for S, T only by the values of values δ_1, δ_2. In the latter case $\delta_1=\frac{c_l^2q}{c_l^2-c_t^2}$, $\delta_2=\frac{c_l^2q}{c_t^2-c_l^2}$.

Liouville's method cannot be used when coefficients $\chi_1(z)$, $\chi_2(z)$ have single zero at the turning points z_1, z_2; $\chi_1(z_1)=0$, $\chi_2(z)=0$. The asymptotic representation examined in the vicinity of turning points contains the divergent terms. Using an asymptotic method of standard equation, it is possible to write uniform asymptotic solution for the

potentials φ, ψ (and also for values S, T), valid both in the vicinity of turning points and far from them:

$$\varphi = P(z, \chi_1) + \omega^{-1} \frac{c_t^2 q}{c_l^2 - c_t^2} P(z, \chi_2); \quad \psi = P(z, \chi_2) + \omega^{-1} \frac{c_l^2 q}{c_l^2 - c_t^2} P(z, \chi_1),$$

where $P(z, \chi)$ - general uniform asymptotic solution of the equation of Sturm - Liouville the normal mode

$$\frac{d^2 p}{dz^2} + \omega^2 \chi(z)\, p = 0;$$

$P(z, \chi)$ it satisfies the standard equation

$$\frac{d^2 P}{dz^2} + \left[\omega^2 \chi(z) + \frac{1}{2}\{\tau(z), z\}\right] P = 0,$$

where $\{\tau(z), z\} = \frac{\tau'''}{\tau'} - \frac{3}{2}\left(\frac{\tau''}{\tau'}\right)^2$ - Schwarz's derivative of $\tau(z)$ on z; function $\tau(z)$ is determined in the form

$$\tau(z) = \left[\frac{3}{2}\int_{z_0}^{z} \sqrt{\chi(z)}\, dz\right]^{\frac{2}{3}}, \quad z > z_0, \ \chi(z) > 0;$$

$$\tau(z) = -\left[\frac{3}{2}\int_{z_0}^{z} \sqrt{-\chi(z)}\, dz\right]^{\frac{2}{3}}, \quad z < z_0; \ \chi(z) < 0; \ (\chi(z_0) = 0).$$

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Since $P(z, \chi)$ is expressed as the Airy's function - Foch w(t) in the

form $P(z,\chi) = A\,[\tau'(z)]^{-\frac{1}{4}} w[-\omega^{\frac{2}{3}}\tau(z)] + B\,[\tau'(z)]^{-\frac{1}{4}} \bar{w}(-\omega^{\frac{2}{3}}\tau]$ (A, B— constants; $\bar{w}(t)$ - the complexly conjugated/combined to $w(t)$ function; $w(t)$, $\bar{w}(t)$ - are linearly independent), then uniform asymptotic solutions for the potentials can be expressed also through function w:

$$\varphi \cong C_1\,[\tau_1']^{-\frac{1}{4}} w(-\omega^{\frac{2}{3}}\tau_1) + C_2\omega^{-1}\frac{c_l^2 q}{c_t^2 - c_l^2} w(-\omega^{\frac{2}{3}}\tau_2) +$$
$$+ C_3\,[\tau_1']^{-\frac{1}{4}} \bar{w}(-\omega^{\frac{2}{3}}\tau_1) + C_4\omega^{-1}\frac{c_l^2 q}{c_t^2 - c_l^2}[\tau_2']^{-\frac{1}{4}} \bar{w}(-\omega^{\frac{2}{3}}\tau_2);$$

$$\psi \cong C_2\,[\tau_2']^{-\frac{1}{4}} w(-\omega^{\frac{2}{3}}\tau_2) + C_1\omega^{-1}\frac{c_l^2 q}{c_t^2 - c_l^2}[\tau_1']^{-\frac{1}{4}} w(-\omega^{\frac{2}{3}}\tau_1) +$$
$$+ C_4\,[\tau_2']^{-\frac{1}{4}} \bar{w}(-\omega^{\frac{2}{3}}\tau_2) + C_3\omega^{-1}\frac{c_l^2 q}{c_t^2 - c_l^2}[\tau_1']^{-\frac{1}{4}} \bar{w}(-\omega^{\frac{2}{3}}\tau_1),$$

where C_1-C_4 - constants; $\tau_1(z)$, $\tau_2(z)$ - are determined through $\chi_1(z)$, $\chi_2(z)$, analogously with function $\tau(z)$.

In the report are examined some properties of asymptotic solutions for values φ, ψ, S, T and

their connection/communication with the behavior of longitudinal and transverse waves in the elastic layered inhomogeneous medium.

Asymptotic solution of the three-dimensional problem about the passage of the short electromagnetic waves through thin layers.

I. V. Sukharevskiy.

Are examined the thin layers from the dielectric, limited by the smooth surfaces, locked or by those exiting to infinity. Layers are assumed to be, generally speaking, equidistant; significant dimension according to the thickness δ and wave number in air k_0 are connected with relationship/ratio $k_0\ \delta \sim 1$. Layer G divides space into two air regions G_1 and G_2. In G_1 are arranged/located the sources, which generate field, is subject to determination field in any assigned point set of region G_2 (for example, on certain surface, which belongs G_2), and also in the distant zone. The solution of the corresponding electrodynamic boundary-value problem is found out in the form of the products of the oscillating exponential factors with proper phases (in each of the regions G_1, G, G_2) to the power (according to degrees δ or, which is equivalent, according to degrees $1/k_0$) asymptotic series. The coefficients of these series/rows in G_1 and G_2 are usual position functions, and into G - functions, determined in the "stretched" along the standard/normal layer (expansion of the type of boundary layers).

This approach leads to the alternating sequence of the boundary-value problems, decided (asymptotically) consecutively/serially: in G, then in G_1 and G_2, then again in G, etc. This process makes it possible to find any number of members of unknown asymptotic expansions. In particular, in the zero asymptotic approximation are derived the formulas, which express field E, H in region G_2 in the locked form through the field of primary radiations.

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At the basis of method lie/rest some heuristic principles, a strict proof of the asymptotic character of the obtained formulas in the general case is not carried out.

The proposed method is applicable not only to the uniform layers, but also to the layers with the continuous or piecewise-continuous (laminar) heterogeneity. Furthermore, relationship/ratio $k_0\delta \sim 1$ can be replaced with the more general/more common/more total: $k_0^p\delta \sim 1$ ($p > 0$ and besides is fixed/recorded).

Account of repeated diffractions with the asymptotic solution of the problems of diffraction.

A. D. Gondr, B. Ye. Kinber.

1. With solution of problems of diffraction on bodies of complex form usually is examined finite sequence of diffractions, caused by reflections, by formation of edge/boundary waves on edges, by bending of waves of shadow side of convex body and so forth and the like, phase of each member of solution satisfying Fermat's principle. An essential deficiency/lack in this method is [illegible] avalanche (exponential) increase of a quantity of waves, which participate in the diffractions of consecutive multiplicities. Meanwhile it is possible to show that many all paths, which correspond to all consecutive diffractions, are the set of all routes on certain circuit (graph/count) from the primary source of field to observation point. The determination of this circuit is the solution of purely geometric problem and is illustrated in the report based on the examples to diffraction on the strip/film, the radiation/emission from the open end/lead of the sectoral horn, diffraction on two cylinders and the radiation/emission from the optical-type antenna. In the majority of the cases this geometric equivalent diagram of

diffraction (GES) contains a final quantity of components/links and nodes/units. To nodes/units GES correspond separate diffractions (i.e. the diffractions, which correspond to simple self-similar problems - reflection, diffraction during the wedge, the diffraction during the convex body, the diffraction on the concave body). Components/links of GES correspond to oscillation loops between the consecutive diffractions. If we decompose the field, which corresponds to each component/link of GES, along the system of multipoles, then the analysis of repeated diffractions is reduced to the analysis of equivalent circuit (ES), to each component/link of which, in contrast to GES, corresponds the specific geometric trajectory, and the specific transmission mode. From the solution of the self-similar problems of diffraction for each node/unit of ES can be determined the scattering matrix whose coefficients show transformation in directions and according to transmission modes.

2. Solution of problem, i.e., determination of amplitudes of multipoles or their diffraction fields, is reduced to solution of system of linear equations

$$\begin{aligned} \Pi_\alpha^- &= A\Pi_\alpha^+ + B\Pi_\beta^+; \\ \Pi_\beta^- &= C\Pi_\alpha^+ + D\Pi_\beta^+, \end{aligned} \qquad (1)$$

where Π_α^+ - column vector of wave amplitudes, which are adequate/approach body from primary source; Π_α^- - column vector of

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waves, which arrive into observation point; Π_β^+, Π_β^- - amplitude of adequate/approaching nodes/units and outgoing from nodes/units waves, which circulate on internal components/links of ES. A, B, C, D - pieces of partitioned matrix ES, comprised of the coefficients of the scattering matrices of separate nodes/units and which correspond to the transformation of external waves into the external ones (A), internal into the external ones (B), external into the internal ones (C) and internal into the internal ones (D). Since

$$\Pi_\beta^- = \varepsilon\Pi_\beta^+, \qquad (2)$$

where ε - matrix/die with the single elements/cells, arranged/located symmetrically relative to diagonal,

$$\Pi_\beta^+ = (\varepsilon - D)^{-1} C\Pi_\alpha^+ \qquad (3)$$

and

$$\Pi_\alpha^- = [A + (\varepsilon - D)^{-1}C]\,\Pi_\alpha^+. \qquad (4)$$

The obtained construction/design of solution is meromorphic function and, naturally, describes all resonance phenomena with the diffraction.

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3. As illustration of method presented is examined asymptotic

solution of problem about radiation/emission from open end/lead of flat/plane symmetrical sectoral horn with sharp edges. To internal components/links of GES correspond the direct method between the edges of horn and the paths, which correspond to reflections in its internal cavity.

To two nodes/units they correspond to the diffraction of cylindrical waves on the edges of horn. The expansion of edge/boundary waves is conducted according to the system of multipoles with the diagrams of form $(1-\cos\psi)^m$ and $\sin\psi\,(1-\cos\psi)^m$, where m=0, 1, 2, ..., ψ - angle, calculated off the direction to the edge. In this system of multipoles asymptotic expression for the matrix coefficient, which corresponds to the conversion of modes of multipole $(1-\cos\psi)^m$ into multipole $(1-\cos\psi)^l$ on the edge of half-plane within ES, takes the form

$$a_{2l,\,2m}=\frac{e^{ikR}}{2^{l-m+1/2}}\sum_{q=0}^{l}\frac{\Gamma\left(l-q-\frac{1}{2}\right)}{\Gamma(q+{}^1/_2)\,\Gamma(l-q+1)\,\Gamma(q+1)}\times$$
$$\times\sum_{\varkappa=m}\left(-\frac{1}{2}\right)^{\varkappa}\Gamma(\varkappa+{}^1/_2)\left(\frac{i}{kR}\right)^{\varkappa+1/2}\tilde{\Lambda}_1(\varphi,\varphi_0,q,\varkappa-m), \qquad (5)$$

where

$$\Lambda_1=\sum_{\tau=0}^{q}C_{\tau}^{(q)}\left[\left(\cos\frac{\varphi-\varphi_0}{2}\right)^{-2(\varkappa-m+\tau)-1}\mp\left(\cos\frac{\varphi+\varphi_0}{2}\right)^{-2(\varkappa-m+\tau)-1}\right];$$
$$C_{\tau}^{(q+1)}=C_{\tau-1}^{(q)}(\varkappa-m+\tau-{}^1/_2)(\varkappa-m+\tau)-$$
$$-C_{\tau}^{(q)}\left[(\varkappa-m+\tau+{}^1/_2)^2-(q+{}^1/_2)^2\right]C_0^{(0)}=1.$$

Plus sign corresponds to Neumann's problem, minus sign to the Dirichlet problem, R, φ_0 - coordinates of multipole, φ - direction to

observation point.

Expression (5) is inconvenient for calculating the field at observation point, since does not make it possible to cross the boundary the light/world - shadow. Therefore expression for the radiation pattern of half-plane, excitable edge/boundary wave by multipole $(1-\cos\psi)^m$, is written/recorded in the form

$$\left(1+i\frac{\partial}{\partial kR}\right)^m \{u(\varphi-\varphi_0) \mp u(\varphi+\varphi_0)\},$$

$$u(\alpha, l) = \frac{e^{-i\frac{\pi}{4}}}{\sqrt{\pi}} e^{-il\cos\alpha} \int\limits_{\infty\,\mathrm{sign}\cos\frac{\alpha}{2}}^{\sqrt{2l}\cos\frac{\alpha}{2}} e^{it^2}\,dt. \qquad (6)$$

$$\left(1+i\frac{\partial}{\partial l}\right)^m u = (1+\cos\alpha)^m u + \frac{\cos\frac{\alpha}{2}}{\sqrt{2l}\,\pi} e^{i\left(l-\frac{\pi}{4}\right)} \sum_{q=1}^{m} i^q \binom{q}{m} \times$$

$$\times \sum_{\tau=1}^{q} \binom{\tau}{q} (-i\cos\alpha)^{q-\tau} \sum_{t=0}^{\tau-1} C_{\tau-1}^{t} \left(i2\cos^2\frac{\alpha}{2}\right)^{\tau-t-1} (-1)^t \frac{\Gamma(t+1/2)}{l^t}. \qquad (7)$$

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Are given some results of calculating the radiation patterns of horn on EVTsM [ЭВЦМ - electronic digital computer].

4. In example examined above were used asymptotic expansions for edge/boundary waves with diffraction on half-plane. For the illustration of the applicability of method in other cases is

examined the method of calculation of matrix coefficients in the task of diffracting the plane wave on two cylinders and the account of the multiple reflections of edge/boundary waves on the concave surface of optical-type antenna.

Asymptotic solution of the equations of Maxwell near caustics.

Yu. A. Kravtsov.

As is known, the asymptotic solution of the equations of Maxwell for the method of the geometric optic/optics becomes meaningless near caustics, on which the amplitudes of zero approximation go to infinity. In this work is shown how should be modified the usual method of geometric optic/optics, in order to reduce the divergence of the amplitudes of zero approximation on the simple caustic curve, after preserving at the same time ray representations. Specifically, the solution of the equations of Maxwell is sought in the form

$$\mathbf{E}=\frac{1}{2}\left[(\mathbf{M}+\mathbf{N})\,w\,(k^{2/3}\psi_1)-\frac{ik^{-1/3}}{\sqrt{-\psi_1}}(\mathbf{M}-\mathbf{N})\,w'\,(k^{2/3}\psi_1)\right]e^{ik\psi_2};$$

$$\mathbf{H}=\frac{\sqrt{\varepsilon}}{2}\left[(\mathbf{P}+\mathbf{Q})\,w\,(k^{2/3}\psi_1)-\frac{ik^{-1/3}}{\sqrt{-\psi_1}}(\mathbf{P}-\mathbf{Q})\,w'\,(k^{2/3}\psi_1)\right]e^{ik\psi_2}, \qquad (1)$$

where w(t) - the Airy's function, which satisfies equation;

$$w''(t)-tw(t)=0,$$

w'(t) - its derivative, and M, N, P, Q, ψ_1 and ψ_2 - unknown values.

After assuming $\mathbf{M}=\mathbf{M}_0+\frac{1}{k}\mathbf{M}_1+\frac{1}{k^2}\mathbf{M}_2+\ldots$, $\mathbf{N}=\mathbf{N}_0+\frac{1}{k}\mathbf{N}_1+\frac{1}{k^2}\mathbf{N}_2+\cdots$ and so

forth, it is possible to show that solvability conditions of the equations of zero approximation are written/recorded in the form

$$(\nabla\psi_2)^2 - \psi_1(\nabla\psi_1)^2 = \varepsilon,\ (\nabla\psi_1, \nabla\psi_2) = 0$$

or, if we introduce eikonals $\Psi_{1,2} = \psi_2 \mp \frac{2}{3}(-\psi_1)^{3/2}$, in the form

$$(\nabla\Psi_{1,2})^2 = \varepsilon, \tag{2}$$

moreover on caustic $\Psi_1 = \Psi_2$ and $\psi_1 = 0$.

To surfaces $\Psi_1 = \mathrm{const}$ and $\Psi_2 = \mathrm{const}$, which far from the caustic curve coincide with the constant-phase surfaces, it is possible to compare or..ogonal ones by them the family of the rays/beams which correspond to the incident and reflected waves and coincide with the rays/beams in the approximation/approach of geometric optic/optics.

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If $\mathbf{t}_{1,2} = \nabla\Psi_{1,2}/\sqrt{\varepsilon}$ —single tangential vectors to indicated beams $\mathbf{n}_{1,2}$ and $\mathbf{b}_{1,2}$ - normal and binormal to them, then in the zero approximation amplitudes are equal to

$$\mathbf{M}_0 = \Phi_1\mathbf{n}_1 + \Phi_2\mathbf{b}_1,\ \mathbf{N}_0 = F_1\mathbf{n}_2 + F_2\mathbf{b}_2,$$
$$\mathbf{P}_0 = \Phi_1\mathbf{b}_1 - \Phi_2\mathbf{n}_1,\ \mathbf{Q}_0 = F_1\mathbf{b}_2 - F_1\mathbf{n}_2,$$

where functions $\Phi_{1,2}$ and $F_{1,2}$ are found from solvability conditions

of the equations of the first approximation:

$$\operatorname{div}\left(\frac{\Phi_1^2-\Phi_2^2}{\sqrt{-\psi_1}}\sqrt{\varepsilon}\,\mathbf{t}_1\right)=0,\qquad \operatorname{div}\left(\frac{F_1^2-F_2^2}{\sqrt{-\psi_1}}\sqrt{\varepsilon}\,\mathbf{t}_2\right)=0; \tag{3}$$

$$\frac{d\chi_1}{d\sigma_1}=\frac{1}{T_1},\qquad \frac{d\chi_2}{d\sigma_2}=\frac{1}{T_2} \tag{4}$$

on the condition that the further on caustic curve $F_1=\Phi_1$, $F_2=\Phi_2$ (in the equations (4) $\chi_1=\operatorname{arctg}\frac{\Phi_1}{\Phi_2}$, $\chi_2=\operatorname{arctg}\frac{F_1}{F_2}$, T_1 and T_2 - radii of torsion, and $d\sigma_1$ and $d\sigma_2$ - elements of length of falling/incident and reflected beams). Equations (3) are analogous to the usual laws of conservation of intensity in the ray tube, however, in contrast to the approximation/approach of geometric optic/optics, on the caustic amplitudes $\Phi_{1,2}$ and $F_{1,2}$ are final, since the decrease of the section of ray tube up to zero is compensated by factor $\frac{1}{\sqrt{-\psi_1}}$. Equations (4) are the known law of the rotation of the plane of polarization of wave.

The described method of the determination of the asymptotic solution of the equations of Maxwell possesses that advantage, which gives the demonstrative picture of shaping of the field of wave near the simple caustic curve. As far as complicated caustic surfaces are concerned, comparison of the method presented with the developed at present mathematical theory of the asymptotic solutions of the quantum-mechanical problems, and also with the methods of the solution of the problems about the propagation of waves in the

plane-layered media shows that also in this case can be successfully used the ray representations; however, instead of the Airy's functions in solution (1) should be used other standard functions, which correctly reflect the character of the behavior of field.

Considerations of locality in tasks of diffraction of short waves.

V. M. Bavich.

The asymptotic behavior of the solutions of the problems of diffracting the short waves possesses the remarkable property of locality which can be formulated as follows:

1) asymptotic formula for solving of u(M, k) of diffraction task (M - observation point, k - wave number, $k \to \infty$) is divided/marked off into the sum of the asymptotic formulas

$$u = \Sigma u_i(M, k),$$

moreover components/terms/addends $u_i(M, k)$ are located in one-to-one conformity with the rays/beams, constructed according to the principle of Fermat and which arrive into point M or which pass in immediate proximity of M;

2) the phase of each component/term/addend $u_i(M, k)$ is the usual phase, computed from the rules of geometric optic/optics;

3) the asymptotic behavior of each component/term/addend

$u_i(M, k)$ when $k \to \infty$ depends only on wave field at the arbitrary point M_0 on the ray/beam, which corresponds $u_i(M, k)$, and from the character of the field of rays/beams and medium in the vicinity of the segment of ray/beam M_0M. (it is assumed that the wave goes from M_0 to M).

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Rules 1) 2) 3) are formulated sufficiently not defined, since it is unclear: a) to what tasks are applicable these constructions; b) if they are applicable, then they need the refinement of the concept of the "character of the field of rays/beams and medium in the vicinity of the segment of ray/beam M_0M". In spite of an inaccuracy in the formulations, these considerations of locality have large heuristic value. Sometimes for the derivation of asymptotic formulas it is necessary to draw on further considerations. Of course positions 1) 2) 3) are not new; however, the consecutive use/application of these positions makes it possible not only to give the new method of obtaining some already known formulas, but sometimes gives the possibility to obtain new formulas. As an example of the derivation of known formula, let us consider the task

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + k^2\right) u = -\delta(M - M_0);$$
$$u|_S = 0; \sqrt{r}\left(\frac{\partial u}{\partial r} - iku\right) \xrightarrow[r \to \infty]{} 0 \quad (1)$$

(here S convex smooth duct/contour).

Let observation point M be located on the light/world. In point M come two rays/beams: the straight line M_0M and reflected M_0s_0M $(s_0 \in S)$. Let us solve task (1) for the circle of curvature, corresponding to s_0. Let $\Phi_{\text{круг}}$ - this solution of the field of rays/beams for the solution of the problem about the circle of curvature, and in the case of task (1) they coincide in the first approximation, both for the straight lines and for the reflected beams. Therefore asymptotically

$$u(M, M_0, k) = \Phi_{\text{круг}}(M, s_0, k), \qquad (2)$$

which far from the maximum rays/beams coincides (asymptotically) with the usual geometric-optical approximation/approach, and near the maximum rays/beams - with V. S. Buslayev's formula [1].

As an example of the derivation of new formula let us consider task (1) with the replacement of condition $u|_S = 0$ to impedance $\frac{\partial u}{\partial n} + ikg(s)\,u|_S = 0$. The same considerations lead to hypothetical formula (2), where $\Phi_{\text{кр}\bar{\text{у}}\text{г}}$ - solution of problem for the impedance circle of curvature, for that case when impedance $\tilde{g}=g(s_0)$=const. Integral equation of the Ersell type makes it possible to strictly demonstrate

formula (2) in the impedance case, it is more precise the formula

$$u(M, M_0, k) = \Phi_{\text{круг}}(M, s_0, k) + O\left(k^{-\frac{1}{2}-\frac{1}{3}}\right) \quad (\operatorname{Re} g > \text{const} > 0,\ M \in S). \quad (3)$$

Thus, formula (2) is accurate in the "darkened" penumbra, as long as

$$|\Phi_{\text{круг}}(M, s_0, k)| \gg O\left(k^{-\frac{1}{2}-\frac{1}{3}}\right).$$

Substituting circle of curvature by adjacent ellipse in which at point s_0 both the curvature and derivative of it in the arc coincide with the same values for S in s_0, it is possible in formula (3) to replace $\Phi_{\text{круг}}$ by $\Phi_{\text{эллипса}}$, and $O(k^{-\frac{1}{2}-\frac{1}{3}})$ by $O(k^{-\frac{1}{2}-\frac{2}{3}})$.

The same considerations of locality and some further considerations make it possible to obtain asymptotic behavior u (M,k) in a deep shadow both near and far from the duct/contour. This gives both the known formulas of Keller and their generalization to the case of impedance cylinder.

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Change of the character of peculiarity at the front of slide after passage by the front of caustic curve.

V. S. Buldyrev.

Let in the medium extend the usual wave front and let in the vicinity of front the wave field be is assigned in the form

$$u = \frac{\pi}{\Gamma(\beta)} \gamma^{\beta-1} \varepsilon(\gamma) \{A(P) + O(\gamma)\},$$

where γ - distance from front $(\gamma = t - \tau(P)$, t - time, τ (P) - the quasi-length of ray/beam, P - the point of space), $\varepsilon(\gamma) = \begin{cases} 0, \gamma < 0 \\ 1, \gamma > 0 \end{cases}$ $A(P)$ - the intensity of wave field at the front and β - certain constant. Then after passage with the front of the wave of caustic curve the character of special feature/peculiarity at the wave front is changed. After the caustic curve at the wave front the wave field u will have a special feature/peculiarity of the form

$$\mathrm{Im}\left\{ i\pi \frac{e^{-i\pi\beta}}{\sin \pi\beta \cdot \Gamma(\beta)} (\gamma)^{\beta-1} \right\},$$
$$(\arg \gamma = \pi \text{ при}^{(1)} \ \gamma < 0).$$

Key: (1). with.

In the geometrical shadows can extend the surfaces, on which all

derivatives of wave field, although they remain continuous, nevertheless the analytical character of wave field is broken. Such surfaces were called the fronts of slide. In the vicinity of the front of slide, which extends on the undisturbed space, wave field can be represented in the form

$$u=(\gamma)^{\frac{2\beta-1}{4}}\cdot e^{-\frac{2}{3}\delta^{3/2}(3\gamma)^{-1/2}}\{A(P)+O(\gamma^{1/2})\}\,\varepsilon(\gamma),$$

where γ - distance from the front of slide, $\delta>0$ and β - some constants.

In the present work it shows that with the passage with the front of the slide of caustic curve the character of special feature/peculiarity is also changed. After the caustic curve the special feature/peculiarity will not be already so/such simple. After passage by the front of the slide of caustic curve the nonanalytic part of the wave field is described by the function

$$\operatorname{Im}\left\{ie^{-i\pi\beta}(\gamma)^{\beta-1}\int_0^\infty \zeta^{2-2\beta}\exp\left[-e^{-i\pi/3}\delta\gamma^{-1/3}\zeta-\frac{1}{3}\zeta^3\right]d\zeta\right\}.$$

This function is investigated, and for it are derived/concluded different representations.

Application of continuous integrals for the derivation of short-wave asymptotic behavior in the diffraction problems.

V. S. Buslayev.

Report is dedicated to the use/application of continuous integrals for the conclusion/output of short-wave asymptotic behavior in the two-dimensional and three-dimensional tasks of diffraction on the smooth convex objects (ducts/contours and surfaces). Almost all formulas, which thus far it was possible to obtain with the help of the continuous integrals, were derived earlier by other methods; however the method of continuous integrals leads to them, apparently, is more naturally: it is sufficiently simple and uses no a priori assumptions about the character of asymptotic behavior.

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We will for the certainty examine the two-dimensional case and the infinite smooth convex duct/contour L. The exterior of this duct/contour let us designate through D. Is examined the task

$$\left.\begin{aligned} &(-\Delta_x - k^2)\,\Gamma_{\mp}(x, x'; k) = \delta(x - x') \quad (k > 0;\ x, x' \in D), \\ &\Gamma_-|_{x\in L} = 0;\ \left.\frac{\partial \Gamma_+}{\partial n}\right|_{x\in L} = 0 \text{ и условия излучения при } |x| \to \infty. \end{aligned}\right\} \quad (1)$$

Key: (1). and radiation condition with.

Let us designate through ξ and η the orthogonal variable/alternating in $\widetilde{D}$ region, such, what $\widetilde{D}$ region is mapped onto half-plane - $\infty < \xi < \infty,\ \eta > 0.$ Let function $Q = Q(\xi, \eta; \xi', \eta'; t)$ $(-\infty < \xi, \xi'; \eta, \eta' < \infty)$ satisfy the equation

$$\frac{\partial Q}{\partial t} = \frac{1}{h_\xi h_\eta}\left[\frac{\partial}{\partial \xi}\left(\frac{h_\eta}{h_\xi}\frac{\partial}{\partial \xi}\right) + \frac{\partial}{\partial \eta}\left(\frac{h_\xi}{h_\eta}\frac{\partial}{\partial \eta}\right)\right] Q$$

and the initial condition

$$Q_{t\to+0} \to h_\xi^{-1} h_\eta^{-1} \delta(\xi - \xi')\, \delta(\eta - \eta').$$

Here h_ξ and h_η when $\eta > 0$ — Lame's coefficients system ξ, η and $h_{\xi,\eta}(\xi, \eta) \equiv h_{\xi,\eta}(\xi, -\eta)$ when $\eta \leqslant 0$. It is easy to see that

$$\Gamma_\mp (x, x'; k) = \int_c dt\, e^{k^2 t} G_\mp (x, x'; t),$$

where

$$G_\mp (x, x'; t) = Q(\xi, \eta; \xi', \eta'; t) \mp Q(\xi, \eta; \xi', -\eta'; t)$$

and c - certain duct/contour in the complex plane t.

Function Q can be represented with the help of the continuous integral which is symbolically written/recorded as follows:

$$Q(\xi, \eta; \xi', \eta'; t) =$$

$$= \int e^{-\frac{1}{4}\int\limits_0^t d\tau \left[h_\xi^2(\bar{\xi}, \bar{\eta})\left(\frac{d\bar{\xi}}{d\tau}\right)^2 + h_\eta^2(\bar{\xi}, \bar{\eta})\left(\frac{d\bar{\eta}}{d\tau}\right)^2\right]} \prod_{\tau=0}^{t} h_\xi h_\eta \, d\bar{\xi}(\tau) \, d\bar{\eta}(\tau). \quad (1)$$

Integration is conducted according to trajectories $\{\bar{\xi}(\tau), \bar{\eta}(\tau)\}\ (0 \leqslant \tau \leqslant t)$, for which $\bar{\xi}(0) = \xi$, $\bar{\eta}(0) = \eta$, $\bar{\xi}(t) = \xi'$, $\bar{\eta}(t) = \eta'$.

The initial task about asymptotic behavior Γ when $k \to +\infty$ is reduced to the task about the asymptotic behavior of function Q when $|t| \to 0$ $(\mathrm{Re}\, t > 0)$, and this task in turn, leads for integral (1), to the steepest descent method. Applying the steepest descent method, we obtain either the Gaussian continuous integrals which easily are computed by known methods or the continuous integrals, which lead to the Airy's functions.

Let us note also that continuous integrals can be used not only in the task of diffraction examined on the smooth convex objects, but also in other diffraction tasks (for the uneven bodies and in the inhomogeneous media).

Behavior as a whole of the discontinuities of hyperbolic systems and the problem of unsteady diffraction and refraction.

V. P. Maslov.

The problem of unsteady diffraction in the mathematical literature has two basic aspects. First, is studied the behavior of the interruption/discontinuity of the solution of hyperbolic equation. In the second place, is investigated the solution of hyperbolic equation with oscillating initial data.

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Connection/communication between these tasks is in detail investigated in the work of P. Lax, R. Kurant and P. Lax, which were reflected in the latter/last publication of R. Kurant's book "Mathematical physics". The most general/most common/most total result about the expansion of discontinuous solution in the series/rows is obtained in the article of Ludwig and the fundamental work of V. Babich.

In all these works is studied the behavior of the solution of

hyperbolic equation only to that moment of time when the front of interruption/discontinuity does not have special features/peculiarities, i.e., in generally speaking, for sufficiently short time t (in small).

In the physical literature is investigated the behavior of solutions in the vicinities of the special singularities of wave front. Yu. Gazarian and V. Babich investigated in the two-dimensional case the behavior of solution upon transfer through the simple caustic curve.

The author studied behavior at the arbitrary moment of the time (i.e., as a whole) of the solutions of the broad class of the hyperbolic systems, which satisfy the disruptive and oscillating initial conditions. In particular, the studied systems include wave and Maxwell equations with the boundary conditions on the smooth surfaces.

Let us illustrate basic result based on the simplest example of scalar wave equation in the inhomogeneous medium.

Let us consider solution of u(x,t) $(x = x_1, x_2, \ldots, x_n)$ of the equation

$$\frac{\partial^2 u}{\partial t^2} = c^2 (x) \Delta u; \; x = x_1, x_2, \ldots, x_n, \qquad (1)$$

satisfying the disruptive initial conditions

$$u|_{t=0} = \varphi (x) \theta \; [f (x)]; \quad u'_{t=0} = 0,$$

where

$$\theta (\xi) = \begin{cases} 0 \text{ при}^{(1)} \; \xi > 0 \\ 1 \text{ при}^{(1)} \; \xi < 0 \end{cases}; \; c (x), \; f (x) \overset{(2)}{\text{и}} \varphi (x)$$

Key: (1). with. (2). and.

sufficiently smooth functions, and, furthermore, $\varphi(x)$ is finite.

It is not difficult to see that u(x,t) can be represented in the form of the half-sum of two solutions u^+ and u^- wave equation (1), that satisfy the initial conditions

$$u^+|_{t=0} = u^-|_{t=0} = \varphi (x) \theta \; [f (x)],$$
$$(u^+)'_{t=0} = -(u^-)'_{t=0} = \varphi (x) \, c (x) \, |\text{grad} \, f (x)| \, \delta [f (x)]. \qquad (2)$$

Therefore we will pause at the solution of precisely these problems. The behavior of the interruption/discontinuity of solution with t>0 is defined, as is known, by the solution of the equations of characteristics and bicharacteristics.

Characteristic equation for the wave equation takes the form

$$\left(\frac{\partial s}{\partial t}\right)^2 = c^2(x)|\nabla s|^2,$$

i.e

$$\frac{\partial s}{\partial t} = \pm c(x)|\nabla s|.$$

$$p = p_1, \ldots, p_n;$$

Let us assume $H = (p, x) = \pm|p|c(x)$, where Sign + corresponds u^+, sign - corresponds u^-. For the certainty we will examine task for u^+, moreover sign + we will lower.

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Thus, u(x,t) - the solution of equation (1), which satisfies condition (2). Bicharacteristic system takes the form

$$\dot{X}_i = \frac{\partial H}{\partial p_i} \quad \dot{p}_i = -\frac{\partial H}{\partial x_i} \quad \frac{ds}{dt} = 0.$$

For solutions of this X(t), p(t), s(t) system are placed special form the initial conditions

$$X_i(0) = x_0; \; p(0) = \text{grad}\, f(x_0); \; s(0) = f(x_0).$$

We will examine the family of bicharacteristics X(t) and let us place X $(x_0, t) = x_0(t)$, where x_0 belongs to the carrier of function $\varphi(x)$. For sufficiently small $t < t_0$ bicharacteristic X (x_0, t) they do not intersect and the jacobian

$$\Upsilon(x_0, t) = \det \left\| \frac{\partial X_i(x_0, t)}{\partial x_{0j}} \right\|$$

is different from zero.

Let us designate through D_ε the vicinity of discontinuity surface f(x)=0.

$\varphi_1(x)$ - smooth, finite function, equal to zero outside D_ε and equal to 1 in certain ε_1 vicinity of disruptive surface.

Let us consider task as a whole. Let us designate through M (x,t) set of functions $x_0(x, t)$ - the solutions of the implicit equation $X(x_0, t)=x$.

Thus far bicharacteristics do not intersect, set M(x,t) consists of one point.

Determination. Point x_0,t (where $x_0 \in D_\varepsilon$) is called focal, if $\Upsilon(t_0, t)=0$.

By the author is introduced the determination of index $\gamma(x_0, t)$ of bicharacteristic $X(x_0, t)$. This is - integer. If with $c(x_0)\,|\operatorname{grad} f(x_0)| = \text{const}$, $\gamma(x_0, t)$ is an index according to Morse.

Theorem. c(x), f(x), $\varphi(x)$ - the sufficiently smooth functions of its arguments, grad $f(x)\neq 0$ and, furthermore, $\varphi(x)$ - finite function.

Let set M(x,t) not contain focal points, then it consists of a finite number of points x_0^k (x,t) k=1, 2, ..., K.

Under these conditions solution u(x,t) can be represented in the form

$$u(x, t) = c(x) \sum_{k=1}^{K} (-1)^{\left[\frac{\gamma(x_0^k, t)}{2}\right]} \frac{\varphi_1(x_0^k)\, P\,[f(x_0^k)]}{\sqrt{|Y(x_0^k, t)|\, c(x_0^k)}} + z(x, t), \qquad (3)$$

where $P(\xi) = \theta(\xi)$, if $Y(x_0^k, t) > 0$ and $P(\xi) = -\frac{\ln|\xi|}{\pi}$, if $Y(x_0^k, t) < 0$, and z(x,t) continuous at point x, t function.

Thus, upon transfer through the focal points interruption/discontinuity can endure "metamorphosis" being converted from $\theta(f(x_0))$ в $-\frac{1}{\pi} \ln|f(x_0)|$ and vice versa.

Let us note that formula (3) was not obtained also in the physical literature.

From the fundamental theorem proved by the author follows

considerably more general/common/total confirmation relative to the taken apart example. Specifically, efficiently is constructed this function $u_0(x,t)$, that difference $u-u_0(x,t)$ belongs C^∞.

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Behavior of the solution of the equation of Helmholtz in a shadow zone after the caustics in inhomogeneous medium.

V. P. Maslov.

Is examined solution $u(x)$ $(x = x_1, \ldots, x_k)$ of the equation

$$\Delta u + \omega^2 n(x) u = 0, \qquad (1)$$

where n(x) is analytical on the real hyperplane, moreover

$$n(x) = n_0^2 + O(|x|^{-k}) \text{ при } |x| \to \infty.$$

Key: (1). with.

Let us note that n(x) can reverse the sign.

Let us assume that from infinity along the axis x_1 falls the plane wave. The account of this fact and conditions for radiation/emission leads to the requirement

$$u(x) = e^{i\omega n_0 x_1} + |x|^{\frac{1-k}{2}} f(\varphi) e^{i\omega n_0 |x|} + o(|x|^{\frac{1-k}{2}}), \qquad (2)$$

where $\varphi = \varphi_1, \ldots, \varphi_{k-1}$ — angular coordinates in the spherical coordinates

in k-dimensional case, and $f(\varphi)$ - certain unknown function.

The asymptotic formulas, which describe the behavior of the solution of task (1)-(2) in the shadow zone, even in the two-dimensional case were not known.

It proves to be, the asymptotic behavior of solution in the shadow zone is determined by the complex solutions of the equations of geometric optic/optics. Asymptotic behavior is determined by all nonhomotopic between themselves complex rays/beams, which arrived at this point. These complex rays/beams, similar to usual real ones, form caustics and foci. The order of the smallness of solution with $\omega \to \infty$ at such focal points is lower than in nonfocal. Thus, focal points prove to be "more light".

The author gave the uniform asymptotic behavior of the solution of task (1)-(2).

Reflection of pulse signals from the Epstein layer.

L. N. Bezruchenko.

Is examined the known solution for the Epstein layer, expressed through the hypergeometric functions, and on his basis is constructed the solution of unsteady problem. With the enlistment of asymptotic methods is conducted the research of solution in the vicinity of wave fronts. For the series/row of the concrete/specific/actual forms of pulse signals in the high speed computer were performed the calculations of the fields whose results are given in the communication/report.

Unsteady propagation of waves in a heterogeneous half-space with the minimum of velocity of propagation.

I. A. Molotkov, I. V. Mukhina.

Is examined the unsteady propagation of waves in the heterogeneous stratified half-space $z\geq 0$(x, y, z - Cartesian coordinates), that is reduced to following flat/plane mixed problem for finding the function u(x, z, t):

$$\left.\begin{aligned} u_{xx}+u_{zz}-n^2(z)\,u_{tt}=0;\\ u(x,z,0)=u_t(x,z,0)=0,\ u(x,0,t)=\delta(x)\,\delta(t). \end{aligned}\right\} \quad (1)$$

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Function n(z) is considered having sole maximum with $z=z_1$ and not increasing with $z>z_1$, analytical with $0\leq z<z_2$, where $z_2>z_1$, $n(z_2)<n(0)<n(z_1)$, piecewise-continuous with $z\geq z_2$ and with $z\to\infty$ is sufficient rapidly that approaching constant value n_∞.

There is greatest interest in the propagation of waves in the layer $0\leq z\leq z_1$. Here appears shadow zone with respect to the incident wave. Both in the illuminated and in the shadow zones rays/beams turn, are formed diverse caustics. On the discontinuity surfaces or

removable discontinuities n(z) appear the leading waves. On the basis of Fermat's principle are investigated the equations of rays/beams and fronts of different waves.

The exact solution of task (1) is expressed by the repeated integral of Fourier and Mellin through the combination of independent solutions $\hat{E}^{(j)}$ (z, k, s), j=1, 2 equations

$$\frac{d^2V}{dz^2} - k^2[1 + s^2n^2(z)]V = 0, \qquad (2)$$

k and s - the variable/alternating integrations of Fourier and Mellin.

Basic content of this investigation - obtaining from exact solution (1) of asymptotic representations for u(x, z, t) near different fronts and fronts of slide. To these asymptotic representations corresponds the asymptotic behavior of Mellin's integrals in $k \to \infty$.

With $0 \leq z \leq z_1$ and $k \to \infty$ function $E^{(j)}$ (z, k, s) are expressed asymptotically through the functions of the parabolic cylinder of complicated argument and mark. In different regions of plane s this general/common/total asymptotic representation gives formulas (which it is natural to name the formulas of Debye and Foch), analogous to the appropriate formulas for the Hankel functions. Is established/installed location of zeros $\hat{E}^{(j)}$ (z, k, s) in plane s with

$k \to \infty$.

Function u(x, z, t) is divided/marked off into the components/terms/addends, that correspond to different waves, which appear in the medium in question. From each composed by different is segretated a non-analytical part of the field u(x, z, t) considered receptions/procedures contour integration as the function of the parameter γ (x, z, t), which makes sense of distance of the ray/beam of the appropriate front or front of slide. As a result in the vicinity of each special surface $\gamma=0$ for function u(x, z, t) is obtained the representation in the form of the sum of asymptotic expansion with $\gamma \to 0$, which is determining the behavior of the nonanalytic part of the field, and regular functions from γ. If $\gamma=0$ there is a surface of the first entrance for the field, then the corresponding regular function from γ is identically equal to zero.

At the front of the incident wave the field proves to be order $\gamma^{-1/2}\varepsilon(\gamma)$. where $\varepsilon(\gamma)$ — the unit function of Heaviside. At the fronts of the reflected and turned wave is a special feature/peculiarity of form $\gamma^{-2/3}$. At the primary front of slide field behaves as $\exp(-c\delta\gamma^{-1/2})\varepsilon(\gamma)$, where δ - arc length of slide, c - positive constant, which depends on values of n(0) and n'(0). More complicated proves to be special feature/peculiarity at the second front of slide, formed by the turned rays/beams. Finally, special feature/peculiarity at the front of leading wave depends on the character of special

feature/peculiarity $n(z)$ with $z=z_2$. For example, in the case gallop $n'(z)$ field at the front of leading wave proves to be order $\gamma^{-1}\varepsilon(\gamma)$.

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Application of standard equations for studying the propagation of electromagnetic waves in the smooth heterogeneous isotropic and anisotropic ionospheric layers.

G. I. Makarov, V. V. Novikov.

In the number of previous research (1)-(3) was developed the method of standard equations in connection with the solution of the problems about the propagation of electromagnetic waves in the smooth heterogeneous isotropic ionospheric layers the refractive index of which possesses simple zero. In the present work the method of standard equations is generalized in two directions. First, is examined radiowave propagation in the isotropic heterogeneous layer with the complicated turning point at which together with the refractive index they are converted into its zero firsts 2n of derivatives. Is analyzed the case both of normal and oblique incidence in plane electromagnetic wave. Is constructed the standard solution of the equations of Maxwell in entire space which is expressed either through the Hankel functions with the fractional mark, or through the Whittaker functions. The analysis of solution shows that at large distances from the mirror point, determined by

the inequality

$$k\int_0^z \sqrt{\varepsilon_m'(z)}\,dz > 1 \quad [\varepsilon_m'(0)=0],$$

the field reflected does not depend on the order of zero refractive indices. In the vicinity of mirror point is observed the "inflation" of the amplitude of electric field, moreover the "inflation" proves to be larger, the higher the order of zero of refractive index.

In the second place, the apparatus for standard equations is applied for the construction of the solution of the problem about a normal incidence in plane electromagnetic wave on the heterogeneous anisotropic ionospheric layer. The presence of magnetic field leads to the fact that the refractive index of one of the characteristic waves together with zero possesses simple pole. Is constructed the standard solution of the equations of Maxwell in entire space which is expressed as Whittaker functions. Is analyzed the effect of the pole of refractive index on the reflection coefficient.

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Ground waves in problems about the propagation of waves.

E. M. Gyunninen, G. I. Makarov.

In the work is examined the task about the radiowave propagation above flat/plane and spherical earth. As the source is selected vertical electric dipole. It is shown that for the inductive routes the field carries surface character, moreover if

$$-\frac{\pi}{2} \leqslant \arg \delta \leqslant -\frac{\pi}{4},$$

where δ - given surface impedance, in the solution is selected the term, which makes sense of the ground wave whose phase speed is lower than the speed of light (in contrast to the wave of Tsennek). In detail is examined the form of phase fronts near the earth's surface. It is possible to show that the surface character of field can be connected with energy exchange between the bottom and upper part of the wave. Further is given and is analyzed the solution of the problem of Foch for the case of the laminar underlying surface. It proves to be that in the illuminated region (case of formulas of the type Weyl-Van der Pol) of the condition of the emergence of ground wave and its form the same as for the earth/ground of flat/plane.

For studying the field in the shadow zone is examined the dynamics of eigenvalues with a change in the modulus/module and argument δ and it is shown,that the presence of this dynamics can explain some parts of the surface character of wave. As in the flat/plane case, is examined energy exchange between the bottom and upper part of the wave.

Radiowave propagation above the heterogeneous route.

Yu. M. Yanevich.

Is examined the field of the vertical electric dipole above the earth/ground for those cases when the electrical properties of the underlying surface vary along the route of propagation. The use of the approximate boundary conditions makes it possible to reduce the problem to finding of the corresponding function of weakening, in this case is accepted the model of the piecewise-uniform route. The effect of the subsequent section on previous is not considered, is excluded also from the examination certain small vicinity of interface where the properties of route vary sharply.

Initial for the calculations is the known formula of Feynberg [1] for the piecewise-discontinuous route. Is investigated the case most difficult for the calculation, when one of the sections has a small extent, and in its limits route can be simulated by plane.

Without making further limitations, is obtained expression for the function of weakening in the case two- and three-piece route (one of the sections is considered short).

The unknown expression for the two-piece route takes the form

$$W_2(D) = \sqrt{i\pi\left(\frac{ka}{2}\right)^{1/3}\frac{D}{a}}\times$$
$$\times\left\{\sum_{l=1}^{\infty}\frac{e^{\alpha_l D}}{t_l - q_{\text{д}}^2}\left[1 + \frac{\sqrt{s_{\text{к}}} - \sqrt{s_{\text{д}}}}{\alpha_l - s_{\text{к}}}\left(i\sqrt{\alpha_l}\,\Phi_{(\alpha_l r_{\text{к}})}\right) - \sqrt{s_{\text{к}}}\right] +\right.$$
$$\left. + j\,\frac{\sqrt{s_{\text{к}}} - \sqrt{s_{\text{д}}}}{\sqrt{\pi r_{\text{к}}}}\left[1 - W_1(s_{\text{к}}\, r_{\text{к}})\right]\sum_{l=1}^{\infty}\frac{e^{\alpha_l r_{\text{д}}}}{t_l - q_{\text{д}}^2}\,\frac{1}{\alpha_l + s_{\text{к}}}\right\},$$

where a - radius of the earth, k=ω/c - wave number, D - complete distance, transmitter-receiver, $r_{\text{к}}, r_{\text{д}}$ — the length of short and long sections $s_{\text{к,д}}$ — the factor of numerical distance (short and long sections respectively), W_1 - normal function of weakening Sommerfield;

$$\alpha_l = jt_l\, q_{\text{д}}\left(\frac{ka}{2}\right)^{1/3}\frac{1}{a},$$

where t_l — eigenvalues [2],

$$q_{\text{д}} = j\left(\frac{ka}{2}\right)^{1/3}\frac{1}{\sqrt{\varepsilon'_{\text{д}} - 1}}$$

and finally

$$\Phi(\sqrt{\alpha_l}\, r_{\text{к}}) = \frac{2}{\sqrt{\pi}}\int_0^{\sqrt{\alpha_l}\, r_{\text{к}}} e^{-z^2}\,dz.$$

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Analogous formula is obtained also for the three-piece route. The obtained expressions can be considered as the generalization of results [1] and [3] for the case when one of the sections of

heterogeneous route is small. Is shown that qualitative behavior of the function of weakening can be described by the formula

$$W_N(D) \approx W_{N-1}(D)\left\{1 - i\frac{2}{\sqrt{\pi}}\sqrt{s_{N-1}r_N}\right\}$$

during the intersection of boundary "dry land" - "sea" and by the formula

$$W_N(D) \approx W_{N-1}(D)\left\{i\frac{1 - W(s_k r_k)}{\sqrt{\pi s_k r_k}}\right\}$$

in the case of boundary "sea" - "dry land", where N=1.2 and $W_2(D)$ and $W_3(d)$ - the function of weakening for two- and three-piece routes respectively.

In the report are given also the results of the numerical calculations of modulus/module and phase of the function of weakening for the routes of different extent over a wide range of frequencies.

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Research of the fundamental solution of the problem of Cauchy for the system of equations of the motion of elastic anisotropic medium.

A. M. Kovalev.

1. Is constructed fundamental solution of problem of Cauchy for system of equations of motion of elastic anisotropic medium:

$$\left.\begin{array}{c} L\mathbf{U}^m = 0,\ L = \| L_{ik} \| \equiv \left\| \delta_{ik} \dfrac{\partial^2}{\partial t^2} - c_{ijkl} \dfrac{\partial^2}{\partial x_j \partial x_l} \right\|, \\ \text{при}^{(1)}\ t = 0:\ \mathbf{U}^m = 0,\ \dfrac{\partial \mathbf{U}^m}{\partial t} = \delta(x_1, x_2, x_3)\, \mathbf{e}_m, \\ \mathbf{U}^m = (U_1^m, U_2^m, U_3^m). \end{array}\right\} \quad (1)$$

Key: (1). with.

Let $L^{mn}\left(\frac{\partial}{\partial t}, \frac{\partial}{\partial x}\right)$ —the cofactor of element/cell L_{mn} of matrix/die L. Then

$$U_n^m = L^{mn} K, \quad (2)$$

where K(t, x) is solution of the problem of Cauchy for the homogeneous hyperbolic equation

$$\left.\begin{array}{l} PK = 0,\ P = \left(\dfrac{\partial}{\partial t}, \dfrac{\partial}{\partial x}\right) = \det \| L_{ik} \|, \\ \text{при}^{(1)}\ t = 0:\ \dfrac{\partial^r K}{\partial t^r} \begin{cases} 0, & r = 0, 1, 2, 3, 4 \\ \delta(x_1, x_2, x_3), & r = 5. \end{cases} \end{array}\right\} \quad (3).$$

Key: (1). with.

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Solution K(t, x) is constructed, using expansion $\delta(x)$ into the plane waves, analogous with the case of strictly hyperbolic operator P. Operator P, generally speaking, is not strictly hyperbolic, since we do not exclude the possibility of appearance of the multiple roots v of equation $P(v, \omega)=0$ at some values of the parameters ω. The proof of formally constructed solution K(t, x) is carried out by passage to the limit from the parameter ω in the functionals.

The solution of task (1) takes the form

$$U_n^m(t, x) = \frac{1}{4\pi^2} \int\limits_{P(1,\xi)=0} \frac{(-1)^{N\xi} L^{mk}(1, \xi)\, \delta'(t + x\cdot\xi)}{|\operatorname{grad} P(1, \xi)|} dS, \qquad (4)$$

where $P(1, \xi)=0$ is normal surface; $N(\xi)$ - number of points of normal surface, which lie on the same ray/beam, that also the point of integration ξ, but is nearer at the beginning of coordinates.

2. Is investigated analytical character of function $\bar{U}_n^m$ in vicinity of non-critical point x' of characteristic surface. Let us formulate result.

Let observation point be fixed/recorded, x=x'. Let ξ - plane $t'+x'\cdot\xi=0$, $t'>0$ concern the normal surface of $P(1, \xi)=0$ at point ξ', moreover it is assumed:

1) ξ' - the non-critical point of normal surface, grad P(1, ξ')≠0;

2) the Gaussian curvature of the normal surface K(ξ')≠0, i.e., point ξ' either elliptical (K>0), or hyperbolic (K<0).

Then the nonanalytic part of function $U_n^m(t, x')$ in the interval of time t, which contains t=t', is equal to

$$(U_n^m) = -\frac{(-1)^{\nu(\xi')} L^{mn}(1, \xi')}{2\pi^2 |x'| |\operatorname{grad} P(1, \xi')| |K(\xi')|^{1/2}} \times$$
$$\times \begin{cases} c\pi\delta(t + x' \cdot \xi') + \varepsilon(c(t + x' \cdot \xi'))\varphi_{mn}(t; \xi'), & \text{если}^{(1)}\ K(\xi') > 0, \\ -\frac{1}{t + x' \cdot \xi'} + (\ln|t + x' \cdot \xi'|)\psi_{mn}(t; \xi'), & \text{если}^{(1)}\ K(\xi') < 0, \end{cases} \quad (5)$$

Key: (1). if.

where c=+-1, when K(ξ')>0 and normal of surface in the vicinity of point ξ' is convex in direction +-x'; φ(t; ξ') and $\psi(t; \xi')$ – the analytic functions t; δ(x) - Dirac's function; $\varepsilon(x)$ – Heaviside's function.

Formula (5) allows/assumes following physical interpretation. If the front, which encounters to point x', has a form of convex or concave bowl, then it carries a nonanalytic contribution of the type δ(t-t'); but if the front, which encounters to point x', has a form of saddle, then it carries a nonanalytic contribution of the type $\frac{1}{t-t'}$.

3. Is examined case of hexagonal symmetry of elastic medium. As is known, in this case normal surface and characteristic surface are surface of revolution relative to axis x_3, which considerably facilitates research.

Let us assume the oval of normal surface share on region $K(\xi)>0$ and $K(\xi)<0$ circle/circumference C, along which $K=0$. It is shown that to circle/circumference C, which is the locus of the parabolic points of normal surface, corresponds the acute-angled edge of characteristic surface.

Is investigated the analytical character of field in the vicinity of the acute-angled edge of characteristic surface. It is shown that in the vicinity of edge the nonanalytic part of the wave field is described by the associated functions of Legendre.

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Diffraction of plane wave in the segment for the angles of incidence and observation, close to sliding.

V. A. Borovikov.

1. Let us consider first unsteady task of diffracting incident plane wave

$$v_{пад} = \begin{cases} \dfrac{1}{\sqrt{t - x\sin\beta - y\cos\beta}} & \text{при}^{(1)}\ t > x\sin\beta + y\cos\beta \\ 0 & \text{при}\ t < x\sin\beta + y\cos\beta \end{cases}$$

Key: (1). with.

on cutting off AB: A: (x=-δ; y=0); B: (x=δ; y=0), on which is set the boundary condition $\frac{\partial u}{\partial n} = 0$.

When the front of the incident wave reaches apex/vertex A, from this apex/vertex begins to extend cylindrical wave v_A; it is analogous, when front $v_{пад}$ reaches B, from B begins to be propagated wave v_B. Then, when front v_A reaches apex/vertex B, appears wave v_{AB} and so forth. Complete solution is the sum of all such waves and takes the form

$$v = v_{пад} + v_{отр} + v_A + v_B + v_{AB} + v_{BA} + v_{ABA} + v_{BAB} + \ldots,$$

moreover in the right side of this expression to each composed (besides $v_{пад}$ and $v_{отр}$) corresponds the broken line whose apexes/vertexes coincide with the apexes/vertexes cutting off AB (optical path).

After Fourier transform on t and multiplication of the obtained function on $\sqrt{\frac{k}{\pi i}}$ we obtain the solution of the stationary problem of diffracting on AB the incident plane wave $e^{ik(x\sin\beta+y\cos\beta)}$, which takes form $u=u_г+u_{диф}$. Here $u_г$ — solution of the geometric-optical approximation/approach, and

$$u_{диф}=u_A+u_B+u_{AB}+u_{BA}+u_{ABA}+\ldots \quad (1)$$

Let us assume $x=\rho\sin\theta$; $y=-\rho\cos\theta$, then there is a limit

$$\lim_{\rho\to\infty} e^{-ik\rho}\sqrt{\rho}\,u_{диф}=U(\beta,\theta,k),$$

which it is natural to name asymptotic behavior in the distant zone of the solution of the problem of diffracting the plane wave on AB.

Function $u_{диф}$ is the odd function y; therefore

$$U\left(\beta,\frac{\pi}{2}+\theta;\ k\right)=-U\left(\beta,\frac{\pi}{2}-\theta,k\right);\ U\left(\beta,-\frac{\pi}{2}+\theta,k\right)=$$
$$=-U\left(\beta,-\frac{\pi}{2}-\theta,k\right),$$

Since according to the reciprocity theorem $U(\beta,\theta,k)=U(\theta,\beta,k)$, we have

$$U\left(\frac{\pi}{2}+\beta,\theta,k\right)=U\left(\frac{\pi}{2}-\beta,\theta,k\right);\ U\left(-\frac{\pi}{2}+\beta,\theta,k\right)=$$
$$=-U\left(-\frac{\pi}{2}-\beta,\theta,k\right).$$

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Therefore it is possible to be bounded to the examination of values $|\beta|<\pi/2$ and $|\theta|<\pi/2$, having in this case in mind, that through the limiting values $\beta=\pm\pi/2$ and $\theta=\pm\pi/2$ $U(\beta, \theta, k)$ should be continued for odd form, becoming zero with $|\beta|=\pi/2$ or $|\theta|=\pi/2$.

Analogous with (1):

$$U(\beta,\theta,k)=U_A+U_B+U_{AB}+U_{BA}+U_{ABA}+U_{BAB}+\ldots, \quad (2)$$

where

$$U_A=\sqrt{\frac{2i}{\pi k}}\,e^{ik\delta(\sin\theta-\sin\beta)}\frac{\sin\left(\frac{\pi}{4}-\frac{\beta}{2}\right)\sin\left(\frac{\pi}{4}+\frac{\theta}{2}\right)}{\sin\beta-\sin\theta},$$
$$U_B=\sqrt{\frac{2i}{\pi k}}\,e^{ik\delta(\sin\beta-\sin\theta)}\frac{\sin\left(\frac{\pi}{4}-\frac{\theta}{2}\right)\sin\left(\frac{\pi}{4}+\frac{\beta}{2}\right)}{\sin\theta-\sin\beta},$$

but each of remaining terms in right side (2) has asymptotic expansion

$$e^{ik\delta[2(n-1)-\sin\psi-\sin\varphi]}\sum_{s=0}^{\infty}\left(\frac{i}{k}\right)^{s+\frac{n}{2}}\sum_{p,q=0}^{p+q=s}a^{(n)}_{(s,p,q)}\left(\frac{1}{\cos\left(\frac{\pi}{4}-\frac{\psi}{2}\right)}\right)^{2p+1}\times$$
$$\times\left(\frac{1}{\cos\left(\frac{\pi}{4}+\frac{\varphi}{2}\right)}\right)^{2q+1}. \quad (3)$$

Here n - number of apexes/vertexes in the optical path, which

corresponds to the wave in question; $\psi = \beta$ for the wave whose optical path begins from apex/vertex A and $\psi = -\beta$ for the wave whose optical path begins with B. Further $\varphi=\theta$ for the wave with the center into B (i.e. for the wave whose optical path it ends with apex/vertex B) and $\varphi=-\theta$ for the wave with the center into A.

Coefficients $\alpha^{(n)}_{s,p,q}$ are symmetrical relative to p and q: $\alpha^{(n)}_{s,p,q} = \alpha^{(n)}_{s,q,p}$ and they are determined according to the recursion formula

$$\alpha^{(k)}_{s,p,q} = \frac{(-1)^{s+1}\Gamma\left(s+\frac{1}{2}\right)\Gamma\left(s-p-q-\frac{1}{2}\right)}{2\sqrt{2}\,\pi^{s}\,(4\delta)^{s+1/2}\,\Gamma(s-p-q+1)};$$

with n≥2:

$$\alpha^{(n+1)}_{s,p,q} = \sum_{\varkappa=q}^{s-p}\sum_{\sigma=0}^{\varkappa-q} K^{\varkappa,\sigma}_{s,p}\,\alpha^{(n)}_{\varkappa,\sigma,q}, \qquad (4)$$

where

$$K^{\varkappa,\sigma}_{s,p} = \frac{(-1)^{s-\varkappa+1}\Gamma\left(s-\varkappa+\frac{1}{2}\right)\Gamma\left(s-\varkappa-p+\sigma+\frac{1}{2}\right)}{(4\sigma)^{s-\varkappa+1/2}\,\pi\Gamma\left(\sigma+\frac{1}{2}\right)\Gamma(s-\varkappa-p+1)}.$$

2. Formulas (3), (4) make it possible to determine asymptotic expansion U(β, θ, k) according to degrees of k^{-1} up to any fixed/recorded degree k^{-N}. However, expression obtained thus does not possess odd continuation through the values $\beta=\pm\pi/2$ and $\theta=\pm\pi/2$, i.e., it is unsuitable at the angles of incidence and observation, close to those sliding along AB.

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Actually/really, from (3) it is evident that the coefficients of

asymptotic expansion of each component/term/addend in (2) approach infinity with $\varphi \to \pi/2$ and they have even continuation through the value $\varphi = -\pi/2$ (let us recall that $\varphi = +-\theta$). Analogous situation occurs with $|\beta| \to \pi/2$. In order to obtain the correct approximation/approach $U(\beta, \theta, k)$ with $|\beta| \to \frac{\pi}{2}$ and $|\theta| \to \pi/2$, it proves to be necessary for the vicinity of each pair of limiting values $\beta = +-\pi/2$ and $\theta = +-\pi/2$ to group waves of the first part (2) and to examine together the contribution of each group. For example, $\beta \sim \pi/2$ and $\theta \sim -\pi/2$. Then

$$U(\beta, \theta, k) = U_A + [U_B + U_{BA} + U_{AB} + U_{ABA}] + \\ + [U_{BAB} + U_{BABA} + U_{ABAB} + U_{ABABA}] + \dots \quad (5)$$

it is necessary to examine together the contribution of each such tetrad to $U(\beta, \theta, k)$.

This distribution makes simple physical sense. We join those functions which with $\beta \sim \pi/2$ and $\theta \sim -\pi/2$ have the close phase factors in expansion (3), which coincide with $\beta = \pi/2$ and $\theta = -\pi/2$. For function U_A this factor with $\beta = \pi/2$ and $\theta = -\pi/2$ is equal -2δ, for the first group in (5) it is equal to 2δ, for the second 6δ and so forth.

Let us consider arbitrary group $U_{BlB} + U_{BlBA} + U_{ABlB} + U_{ABlBA}$ (where $l = 1, 2, \dots$ - a quantity of apexes/vertexes A in the optical path, which corresponds to wave U_{BlB}: $U_{B1B} = U_{BAB}$; $U_{B2B} = U_{BABAB}$ and so forth).

Let

$$F(\varphi) = \frac{2}{\sqrt{\pi i}} \int_0^{2\sqrt{k\delta}\sin\left(\frac{\pi}{4} - \frac{\varphi}{2}\right)} e^{is^2}\, ds.$$

Then

$$U_{BlB} + U_{BlBA} + U_{ABlB} + U_{ABlBA} = F(\beta)\,F(-\theta)\,U_{BlB} + F(\beta)\,U'_{BlBA} + F(-\theta)\,U''_{ABlB} + U'''_{ABlBA}, \quad (6)$$

where functions U'_{BlBA}, U''_{ABlB}, U'''_{ABlBA} have asymptotic expansion, determined by expansion (3) of function U_{BlB} (in which n=2l+1, $\varphi=\theta$ and $\psi=-\beta$) or, more precise, that assign this expansion by coefficients $\alpha^{(2l+1)}_{s,p,q}$:

$$U'_{BlBA} \approx e^{ik\delta[4l+2+\sin\theta+\sin\beta]} \sum_{s=0}^{\infty} \left(\frac{i}{k}\right)^{s+l+1} \sum_{\varkappa=0}^{s} \sum_{\sigma=0}^{\varkappa} \frac{(-1)^{s-\varkappa} A\left(s-\varkappa,\ \sigma,\ \frac{\pi}{4}+\frac{\theta}{2}\right)}{(4\delta)^{s-\varkappa+1/2}\pi} \times \sum_{p=0}^{\varkappa-\sigma} \frac{\alpha^{(2l+1)}_{\varkappa,\sigma,p}}{\left[\cos\left(\frac{\pi}{4}-\frac{\beta}{2}\right)\right]^{2p+1}}.$$

Function U''_{ABlB} is obtained from U'_{BlBA} by replacement β on $-\theta$ and θ on $-\beta$:

$$U''_{ABlB} \approx e^{ik\delta[4l+2-\sin\theta-\sin\beta]} \sum_{s=0}^{\infty} \left(\frac{i}{k}\right)^{s+l+1} \sum_{\varkappa=0}^{s} \sum_{\sigma=0}^{\varkappa} \times \frac{(-1)^{s-\varkappa} A\left(s-\varkappa,\ \sigma,\ \frac{\pi}{4}-\frac{\beta}{2}\right)}{(4\delta)^{s-\varkappa+1/2}\pi} \sum_{p=0}^{\varkappa-\sigma} \frac{\alpha^{(2l+1)}_{\varkappa,\sigma,p}}{\left[\cos\left(\frac{\pi}{4}+\frac{\theta}{2}\right)\right]^{2p+1}},$$

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Function U'''_{ABlBA} has following asymptotic expansion:

$$U^{\prime\prime}_{ABIBA} \approx e^{ik\delta[4l+4+\sin\theta-\sin\beta]} \sum_{s=0}^{\infty} \left(\frac{i}{k}\right)^{s+l+3/2} \sum_{\varkappa=0}^{s} \frac{(-1)^{s-\varkappa}}{\pi^{3}(4\delta)^{s-\varkappa+1}} \times$$

$$\times \sum_{m=\varkappa}^{s} \sum_{\sigma,p=0}^{\sigma+p=\varkappa} A\left(s-m,\, p,\, \frac{\pi}{4}+\frac{\theta}{2}\right) A\left(m-\varkappa,\, \sigma,\, \frac{\pi}{4}-\frac{\beta}{2}\right).$$

In these formulas

$$A(s, \sigma, \varphi) = \sum_{q=0}^{\infty} \frac{\Gamma\left(s+\frac{1}{2}\right)\Gamma(s+\sigma+q+3/2)}{\Gamma\left(\sigma+\frac{1}{2}\right)\Gamma(s+q+2)} \sin^{2q+1}\frac{\varphi}{2} =$$

$$= \sin\frac{\varphi}{2} \frac{\Gamma\left(s+\frac{1}{2}\right)\Gamma\left(s+\sigma+\frac{3}{2}\right)}{\Gamma(s+1)\,\Gamma(\sigma+1/2)} \int_0^1 \frac{(1-t)^{s}\,dt}{\left(1-t\sin^2\frac{\varphi}{2}\right)^{s+\sigma+\frac{3}{2}}}.$$

Since $F(\varphi)$ for odd form is continued through the value $\varphi=\pi/2$, it is easy to see that (6) also has the odd continuation through the values $\beta=\pi/2$ and $\theta=-\pi/2$.

Analogously is examined the vicinity of any pair of limiting values β and θ: $\beta=\pm\pi/2$ and $\theta=\pm\pi/2$.

Short-wave asymptotic expansions of diffraction electromagnetic fields, generated by the arbitrarily oriented dipoles, in the wedge region with ideally conducting faces.

A. A. Tuzhilin.

Electromagnetic fields E and H in the wedge region with the ideally conducting faces, generated by electrical or magnetic dipole with moment/torque P, we will determine respectively through Hertz's electrical or magnetic vector Π (r). Then the task of determining Hertz's vector Π (r) will consist in finding Green's function vector equation of Helmholtz

$$(\Delta + k^2)\,\Pi(\mathbf{r}) = -4\pi \mathbf{P}\delta(\mathbf{r} - \mathbf{r}_0), \qquad (1)$$

assigned in the wedge region D, which satisfies the generalized conditions of advancability [1] [2], and on the faces of wedge region, i.e., with $\varphi = \pm\Phi$, that satisfying edge/boundary conditions

$$\begin{gathered}[\mathbf{n}_{\pm\Phi}, \Pi(\rho, \pm\Phi, z)] = 0, \\ \partial(\mathbf{n}_{\pm\Phi}, \Pi(\rho, \pm\Phi, z))/\partial n_{\pm\Phi} = 0 \end{gathered} \qquad (2a)$$

if Π (r) - the electric vector of Hertz (r=(ρ, φ, z) $\mathbf{n}_{\pm\Phi}$ — normal to faces $\varphi = \pm\Phi$) and

$$\begin{gathered}\partial[\mathbf{n}_{\pm\Phi}, \Pi(\rho, \pm\Phi, z)]/\partial n_{\pm\Phi} = 0, \\ (\mathbf{n}_{\pm\Phi}, \Pi(\rho, \pm\Phi, z)) = 0 \end{gathered} \qquad (2b)$$

if Π (r) - Hertz's magnetic vector.

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The wedge region D in the cylindrical coordinate system (ρ, φ, z) is determined by the following inequalities: $0 < \rho < \infty, |\varphi| < \Phi, |z| < \infty$.

For so Green's specific vector functions there are representations in the form of Sommerfield's integrals with the spherical kernel:

$$\Pi(\mathbf{r}) = \frac{1}{2\pi i} \int_{\gamma} \frac{e^{ikR(\alpha)}}{R(\alpha)} \mathbf{s}(\alpha + \varphi)\, d\alpha, \qquad (3)$$

where $R(\alpha) = \sqrt{\rho^2 + \rho_0^2 + (z - z_0)^2 - 2\rho\rho_0 \cos\alpha}$, $\bar{r}_0 = (\rho_0, \varphi_0, z_0)$, and the vector function of complex variable takes the form

$$\mathbf{s}(\alpha) = \frac{\pi p}{4\Phi} \left\{ \mathbf{e}_{\alpha-\varphi_0} \operatorname{ctg} \frac{\pi}{4\Phi}(\alpha - \varphi_0) \mp \bar{e}_{\alpha+\varphi_0-2\Phi} \operatorname{ctg} \frac{\pi}{4\Phi}(\alpha + \varphi_0 - 2\Phi) \right\}. \qquad (4)$$

Into formula (4) are introduced the following designations: $p=|P|$, i.e., $P=P(\cos\theta_0, e_1+\sin\theta_0 e_2)$, the single unit vector e_1 is directed along z axis, unit vector e_2 is orthogonal e_1, moreover $\mathbf{e}_2 = \cos\psi \mathbf{i} + \sin\psi \mathbf{j}$, $\mathbf{e}_\alpha = \cos\theta_0 \mathbf{e}_1 + \sin\theta_0 (\cos\alpha \mathbf{e}_2 + \sin\alpha \mathbf{e}_3)$, $\bar{\mathbf{e}}_3 = [\mathbf{e}_1, \mathbf{e}_2]$.

In the present report will be presented the following two results.

1. On the basis of expression (3), it will be shown that Hertz's

vector Π (r) can be represented in the form

$$\Pi(\rho,\varphi,z)=\Pi_r(\rho,\varphi,z)+\Pi_g(\rho,\varphi,z); \tag{5}$$

$$\Pi_r(\rho,\varphi,z)\equiv P\left\{\sum_{n_1}\frac{e^{ikR(\varphi-\varphi_0+4n_1\Phi)}}{R(\varphi-\varphi_0-4n_1\Phi)}e_{4n\Phi}\mp\right.$$
$$\left.\mp\sum_{n_2}\frac{e^{ikR(\varphi+\varphi_0-2\Phi(2n_2+1))}}{R(\varphi+\varphi_0-2\Phi(2n_2+1))}e_{[2\Phi(2n_2+1)-2\psi]}\right\}. \tag{6}$$

Ranges of change n_1 and n_2 in formula (6) are determined from conditions $|\varphi-\varphi_0-4n_1\Phi|<\pi$ and $|\varphi+\varphi_0-2\Phi(2n_2+1)|<\pi$. the part of field Π_r easily is determined from geometric considerations and is the sum of falling/incident and reflected from the faces of waves; $\Pi_g(\rho,\varphi,z)$ — diffraction part of the field

$$\Pi_g(\rho,\varphi,z)\equiv\Pi_g^1(-\pi+\varphi-\varphi_0)-\Pi_g^1(\pi+\varphi-\varphi_0)\mp$$
$$\mp\Pi_g^2(-\pi+\varphi+\varphi_0-2\Phi)\pm\Pi_g^2(\pi+\varphi+\varphi_0-2\Phi), \tag{7}$$

where

$$\Pi_g^1(\beta)\equiv\frac{P}{4\varphi}\int_0^\infty\frac{e^{ikR(\pi+it)}}{R(\pi+it)}\,\frac{\sin\frac{\pi\beta}{2\Phi}\{e_\beta+(\operatorname{ch}t-1)[e_1,[e_\beta,e_1]]\}-\operatorname{sh}\frac{\pi}{2}\frac{t}{\Phi}\cdot\operatorname{sh}t\,[e_\beta,e_1]}{\operatorname{ch}\frac{\pi t}{2\Phi}-\cos\frac{\pi\beta}{2\Phi}}\,dt, \tag{8}$$

and $\Pi_g^2(\beta)$ differs from $\Pi_g^1(\beta)$ by the replacement of vectors e_β on vector $e_{\beta-2(\psi-\Phi)}$.

For vector functions $\Pi_g^1(\beta)$ and $\Pi_g^2(\beta)$ are derived/concluded asymptotic expansions in the series/rows in terms of asymptotic sequence $\{H_m^{(1)}(kR_0)\times(R_0/k\rho\rho_0)^m\}$ (with $k\rho\rho_0/R_0)\to\infty$), where $R_0=R(\pi)=\sqrt{(\rho+\rho_0)^2+(z-z_0)^2}$.

$$\Pi_g^{1,2}(\beta)\sim P\sum_{m=0}^{\infty}\Pi_m^{1,2}\frac{(-1)^m i\Gamma(m+1/2)}{2}\sqrt{\frac{\pi}{\rho\rho_0}}\left(\frac{R_0}{k\rho\rho_0}\right)^m H_m^{(1)}(kR_0) \tag{9}$$

if φ_0 and φ, varying in interval $(-\Phi,\Phi)$, are such, that value $\beta\neq 4n\Phi$ (n - integer).

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But if is feasible case $\beta = 4n\Phi$, then

$$\Pi_g^{1,2}(\beta) \sim P \sum_{m=0}^{\infty} \left\{ \overline{\Pi}_m^{1,2} - a_m(\beta - 4n\Phi) \begin{Bmatrix} e_{4n\Phi} \\ e_{2\Phi(2n+1)-2\psi} \end{Bmatrix} \right\} \frac{(-1)^m i\Gamma(m+1/2)}{2} \times$$
$$\times \sqrt{\frac{\pi}{\rho\rho_0}} \left(\frac{R_0}{k\rho\rho_0}\right)^m H_m^{(1)}(kR_0) - \frac{ikP}{2} \begin{Bmatrix} e_{4n\Phi} \\ e_{2\Phi(2n+1)-2\psi} \end{Bmatrix} \times$$
$$\times \int\limits_{\infty}^{\operatorname{arcsh}\left[\frac{2\sqrt{\varepsilon\rho_0}}{R(\pi-\beta+4n\Phi)}\left|\sin\frac{\beta-4n\Phi}{2}\right|\right]} H_1^{(1)}(kR(\pi-\beta+4n\Phi)\operatorname{ch}\xi)\, d\xi. \quad (10)$$

In formulas (9) and (10) are introduced the designations

$$\Pi_0^1 = A_0(\beta,\Phi)\, e_\beta;\ \Pi_m^1 = A_m(\beta,\Phi)\, e_\beta + A_{m-1}(\beta,\Phi)\,[e_1,[e_\beta,e_1]] +$$
$$+ \frac{2A^1_{m-1}(\beta,\Phi)}{2m-1}[e_\beta, e_1] \quad \overset{(1)}{\text{при}}\ m > 1, \quad (11)$$

Key: (1). with.

where

$$A_m(\beta,\Phi) = \frac{\operatorname{ctg}\frac{\pi\beta}{4\Phi}}{4\Phi} \sum_{k=0}^{m} \frac{1}{\left(\sin\frac{\pi\beta}{4\Phi}\right)^{2k}} \sum_{l=k}^{m} c_{kl}^m \left(\frac{\pi}{2\Phi}\right)^{2l},\ A_m^1(\beta,\Phi) = \partial A_m(\beta,\Phi)/\partial\beta, \quad (12)$$

but numbers c_{kl}^m are computed from the formulas

$$c_{kl}^m = \frac{(-1)^k l!}{(2l)!\,2^m} \sum_{s=l}^{m} \frac{\binom{-1/2}{m-s}\binom{s-1}{l-1}}{s!} \left(\frac{d^{2l}}{dz^{2l}}\left(\operatorname{sh}\frac{z}{2}\right)^{2k} \frac{d^{s-l}}{dz^{s-l}} \frac{z^s}{\left(\operatorname{sh}\frac{\sqrt{z}}{2}\right)^{2s}}\right)_{z=0}; \quad (13)$$

$$a_m(\beta - 4n\Phi) = \frac{(-1)^m}{2^{m+1}\pi\left(\sin\frac{\beta - 4n\Phi}{2}\right)^{2m+1}}. \quad (14)$$

Expression Π_m^1 differs from $\Pi_m^{'}$ in terms of the replacement of vectors e_3 by $e_{\beta-2(\psi-\Phi)}$.

2. Will be proved following.

Theorem. Electromagnetic fields E and H, excited by electrical or magnetic dipole in the wedge region with the ideally conducting faces, are decomposed/expanded in the asymptotic series/rows on asymptotic sequence $\{H_m^{(1)}(kR_0)(R_0/k\rho\rho_0)^m\}$ (with $k\to\infty$), these asymptotic series/rows can be obtained by the term-by-term use/application of operations grad div+k² or +-ikrot to asymptotic expansions in terms of sequence $\{H_m^{(1)}(kR_0)(R_0/k\rho\rho_0)^m\}$ of the corresponding vectors of Hertz and by the subsequent regrouping of the obtained series/row.

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Diffraction fields in the narrow wedge region with ideally soft faces.

A. A. Tuzhilin.

We will examine the task of diffracting the scalar spherical wave in the wedge region D with the ideally soft faces. This task is reduced to the determination of Green's function scalar equation of Helmholtz $(\Delta+k^2)\,\Pi(r) = -4\pi P\delta(r-r_0)$, assigned into D, satisfying condition of advanceability (or the condition of radiating/emitting Sommerfield) and by that turning into zero on the faces of wedge region, i.e., when $\varphi = \pm\Phi$ (and with $0<\rho<\infty$, $|z|<\infty$). Green's described function can be represented in the form of Sommerfield's integral with the spherical kernel

$$\Pi(\mathbf{r}) = \frac{1}{2\pi i}\int_{\gamma} \frac{e^{ikR(\alpha)}}{R(\alpha)}\, s(\alpha+\varphi)\, d\alpha, \tag{1}$$

where

$$s(\alpha) = \frac{\pi P}{4\Phi}\left\{\operatorname{ctg}\frac{\pi}{4\Phi}(\alpha-\varphi_0) - \operatorname{ctg}\frac{\pi}{4\Phi}(\alpha+\varphi_0-2\Phi)\right\}. \tag{2}$$

In the present report will be presented the following results.

1. On the basis of expression (1), Green's function $\Pi(r)$ will

be expanded into series of eigenfunctions

$$\Pi(\mathbf{r}) = \frac{2\pi P}{\Phi} \sum_{n=1}^{\infty} \sin\frac{\pi n}{2\Phi}(\varphi_0 - \Phi) \sin\frac{\pi n}{2\Phi}(\varphi - \Phi) S_{\frac{\pi n}{2\Phi}}; \quad (3)$$

$$S_{\frac{\pi n}{2\Phi}} \equiv -\frac{1}{2\pi} \int_{\gamma_1} \frac{e^{ikR(\alpha) + i\frac{\pi n}{2\Phi}\alpha}}{R(\alpha)} d\alpha, \quad (4)$$

where γ_1 - upper loop of duct/contour γ.

2. Will be proved following.

Theorem. When $\frac{\pi N}{2\Phi} < k\rho_0 < \frac{\pi(N+1)}{2\Phi}$ (N - whole non-negative number) in the region on the plane (kz, kρ), the lying/horizontal above hyperbola

$$\frac{(k\rho)^2}{\pi^2 m^2/4\Phi^2} - \frac{(kz)^2}{(k\rho_0)^2 - \pi^2 m^2/4\Phi^2} = 1 \quad (m \leqslant N), \quad (5)$$

the sequence of the functions

$$\left\{\sum_{n=1}^{m} a_n S_{\frac{\pi n}{2\Phi}}, S_{\frac{\pi(m+1)}{2\Phi}}, \ldots, S_{\frac{\pi(m+k)}{2\Phi}}, \ldots\right\} \quad (6)$$

is asymptotic with $\Phi \to 0$ and the fixed values of the parameters

$$\delta = \frac{k\sqrt{\rho^2 + \rho_0^2 + z^2}}{\pi/2\Phi}, \quad a = \frac{2\rho\rho_0}{\rho^2 + \rho_0^2 + z^2}, \quad (7)$$

and at the region, which lies is below hyperbola (5), asymptotic at $\Phi \to 0$ and those fixed/recorded δ and a, is the sequence

$$\left\{\sum_{n=1}^{m-1} a_n S_{\frac{\pi n}{2\Phi}}, S_{\frac{\pi m}{2\Phi}}, S_{\frac{\pi(m+1)}{2\Phi}}, \ldots, S_{\frac{\pi(m+k)}{2\Phi}}, \ldots\right\} \quad (8)$$

(a_n — arbitrary functions from Φ, oscillating with $\Phi \to 0$).

This theorem allows with the studies of field in the narrow wedge region, i.e., when $\Phi \to 0$, to be bounded in series/row (3) to several first terms. Thus, when $0 < k\rho_0 < \pi/\Phi$ the first term of series/row (3) is the asymptotic representation of field when $\Phi \to 0$ on the entire plane (kz, kρ).

3. For functions S_p will be derived following asymptotic representations with $p \to \infty$ and fixed/recorded δ and a.

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4. Case kρ_0<p. In the region on the plane (kz, kρ) out of the ellipse

$$\frac{(k\rho)^2}{p^2} + \frac{(kz)^2}{p^2 - (k\rho_0)^2} = 1, \qquad (9)$$

of the certain not containing vicinity of ellipse, it is correct

$$S_p \sim \frac{k e^{-p\left(\gamma - \frac{a\delta^2}{2}\cos\beta\,\mathrm{sh}\,\gamma\right)}}{\sqrt{2\pi}\,p^{3/2}\sqrt{\delta^2 - 2 - 2i\sqrt{\delta^2 - \frac{a^2\delta^4}{4} - 1}}\,\sqrt[4]{\delta^2 - \frac{a^2\delta^4}{4} - 1}}\, e^{ip\left(\frac{a\delta^2}{2}\sin\beta\,\mathrm{ch}\,\gamma - \beta\right)} \qquad (10)$$

where $\mathrm{ch}\,\gamma = \frac{\sqrt{2(1+\sqrt{1-a^2})}}{a\delta}$, $(\gamma > 0)$; $\cos\beta = \frac{\sqrt{2(1-\sqrt{1-a^2})}}{a\delta}$, $(0 < \beta < \pi/2)$.

In certain vicinity of ellipse (9) it is correct

$$S_p \sim \frac{k e^{-p\left(\alpha_0 - \delta\sqrt[4]{1-a^2}\right)}}{\sqrt{2\pi}\,(p\delta)^{1/3}\sqrt[6]{1+\sqrt{1-a^2}}}\, w\left[\frac{\sqrt{2}\,p^{2/3}\left(1 - \frac{\delta}{\sqrt{2}}\sqrt{1+\sqrt{1-a^2}}\right)}{\sqrt[3]{\delta\sqrt{1+\sqrt{1-a^2}}}}\right] \qquad (11)$$

where $\mathrm{ch}\,\alpha_0 = \frac{1+\sqrt{1-a^2}}{a}$ $(\alpha_0 > 0)$, a $w(z) = u(z) + iv(z)$ — Airy's function.

Within ellipse (9) the asymptotic behavior of function S_p has aperiodic character, we do not write it out.

Case $k\rho_0>p$. At the region, the lying/horizontal below hyperbola

$$\frac{(k\rho)^2}{p^2}-\frac{(kz)^2}{(k\rho_0)^2-p^2}=1 \qquad (12)$$

and certain not containing vicinity of hyperbola, is correct relationship/ratio (10).

In certain vicinity of hyperbola (12) it is correct

$$S_p\sim\sqrt{\frac{2}{\pi}}\,\frac{ke^{ip(\delta^4\sqrt{1-a^2}-\alpha^0)}}{p^{1/3}\sqrt[6]{1-\sqrt{1-a^2}}}\,v\left[-\frac{\sqrt{2}p^{2/3}\left(1-\frac{\delta}{\sqrt{2}}\sqrt{1-\sqrt{1-a^2}}\right)}{\sqrt[3]{\delta\sqrt{1-\sqrt{1-a^2}}}}\right], \qquad (13)$$

where $\cos\alpha^0=\frac{1-\sqrt{1-a^2}}{a}$, $(0<\alpha^0<\pi/2)$, $v(z)$ — alleged part of the Airy's function.

At the region, lying/horizontal above hyperbola (9) and certain not containing vicinity of hyperbola, it is correct

$$S_p\sim\frac{k}{\sqrt{\pi}p^{1/2}\sqrt[4]{1-\delta^2+\frac{a^2\delta^4}{4}}}\left\{\frac{e^{i\frac{3\pi}{4}}}{\sqrt{a\delta^2\sin\beta^1}}e^{ip\left(\frac{a\delta^2}{2}\sin\beta^1-\beta^1\right)}+\right.$$
$$\left.+\frac{e^{-i\frac{3\pi}{4}}}{\sqrt{a\delta^2\sin\beta^2}}e^{ip\left(\frac{a\delta^2}{2}\sin\beta^2-\beta^2\right)}\right\}, \qquad (14)$$

where $\cos\beta^1=\frac{2}{a\delta^2}\left(1-\sqrt{1-\delta^2+\frac{a^2\delta^4}{4}}\right)$, $(0<\beta^1<\pi)$; $\cos\beta^2=\frac{2}{a\delta^2}\times$
$\times\left(1+\sqrt{1-\delta^2+\frac{a^2\delta^4}{4}}\right)$, $(0<\beta^2<\pi/2)$.

Expressions (10), (11), (13) and (14) make it possible to in detail investigate the structure (in particular, to determine paths of constant phase and rays/beams along which they extend are wave) the waves in question.

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Research of Green's function in problems of diffraction on transparent sphere and circular cylinder.

V. S. Buldyrev.

The mathematical formulation of the problem of diffraction on the transparent uniform body, limited by the smooth locked surface Ω, consists of the following. It is necessary to find function U(P) (P - the point of space), which satisfies:

1) the equation

$$\Delta u(P) + k^2(P)\, u(P) = -\delta(P - Q),$$

in which k(P) the piecewise constant function

$$k(P) = \begin{cases} k_1, & P \ \text{вне}^{(1)}\ \Omega \\ k_2, & P \ \text{внутри}^{(2)}\ \Omega; \end{cases}$$

Key: (1). outside. (2). inside.

$\delta(P-Q)$ - delta-function and point Q is arranged/located outside Ω;

2) by some conditions for coupling for u and $\frac{\partial u}{\partial n}$ on the surface Ω;

3) to radiation condition at infinity.

Function u(P) is Green's function problem 1-3. In the present report is investigated Green's function u(P) in the problems of diffraction on the transparent ones circular cylinder and the sphere, for which relative refractive index $n=k_2/k_1$ is lower than unity. In the case of cylinder the problem is considered as the flat/plane ($P(r, \varphi)$ and $Q(r_0, 0)$ - the point of plane, Ω - circle/circumference $r=a$). In the case of sphere the problem is axially symmetric ($P(r, \theta, \varphi)$, $Q(r_0, 0, 0)$ - point of three-dimensional space, Ω - sphere $r=a$).

With $n<1$ is observed the phenomenon of total reflection. The rays/beams, which fall to the surface of cylinder or sphere at the angles, close to the angle of total reflection ψ_0 $(\sin\psi_0 = n)$, in the internal medium extend along the chords with those disappearing by a small length. Secondary refraction of these rays/beams generates in the environment of the waves, constant-phase surface of which they approach with an increase of the number of reflections in the internal medium certain boundary surface. In the vicinity of this

boundary surface occurs the interference of the waves, which tested different number of reflections within the cylinder, as a result of which is formed/shaped interference type leading wave. With the decrease of angle of incidence the chord length over which the rays/beams extend in the internal medium, grows. This leads to the fact that the waves, which tested different number of reflections within the cylinder, leave from the process of interference in the vicinity of leading wave and acquire their individuality.

The basic goal of the present investigation consists of the isolation/liberation from Green's function, in the first place, the expressions, which are the waves, which tested a small number of reflections within the cylinder, and, in the second place, the expressions which describe the total effect of those waves, for which a number of reflections is great. These latter/last expressions will be the field of interference type leading wave.

For the implementation of the planned program Green's function, which has the form of infinite series on cos $n\varphi$ in the case of cylinder and on $P_n(\cos\theta)$ in the case of sphere, is converted according to Watson's method into the contour integral on the plane by complex variable n. After the strain of the duct/contour of integration and expansion of integrand in the infinite series of the type of geometric progression for Green's function is obtained the

representation of the form

$$u = S + S_T + \sum_{l=0}^{\infty}\left\{\sum_{m=0}^{M-1}(s_{lm}^{+} + s_{lm}^{-}) + S_{lM}^{+} + S_{lM}^{-}\right\}.$$

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Each component in the right side of this formula - contour integral, which makes specific physical sense.

Integral S (straight line and reflected of wave) and integrals $s_{lm}^{\pm}$ (waves, which bridged around the origin of coordinates 1 of times against (-) and on (+) the hour hand and those tested in this case 2l+m+1 and 2l+m reflections) can be calculated with $|k_2 a|>>1$ according to the steepest descent method.

Integral S_T (diffraction waves or the wave of slide) is computed with the help of the remainder theorems.

Integrals S_{lM}^{+} and S_{lM}^{-} describe interference type leading wave, completed 1 of revolutions around the origin of coordinates. Mach number, which indicates a number of waves which left the process of interference included within the limits $0.3\gamma-1<M\leq 0.3\gamma$, where $\gamma=(\theta-\theta_0)(k_2 a/2)^{1/3}$ and $\theta-\theta_0$ - angular distance between observation point and ray/beam, reflected at angle ψ_0.

Integrals $S^{\pm}_{lm}$ are reduced to the integrals of the type

$$\int\limits_{\infty e^{-\frac{2\pi}{3}i}}^{\infty e^{i\pi/3}} \left[\frac{w_1(t)}{w_2(t)}\right]^M \cdot \frac{e^{i\gamma t}}{w_2(t)\, w_3(t)}\, dt, \qquad (1)$$

in which $w_j(t)$ (j=1, 2, 3) - Airy's function. These integrals are some special functions of the parameter γ and are subject to tabulation. For their calculation were comprised the programs for the high speed machine.

It should be noted that functions of type (1) have the universal character: through them will be expressed the field of interference type leading wave in all those problems where this wave appears.

The problem of diffraction on flattened transparent bodies.

B. Z. Katsenelenbaum, V. V. Malin.

1. Interest in this problem is determined first of all by appearance of quasi-optical lens lines for transmission of millimeter waves. The basic elements of line are thin dielectric lenses. In the existing theory each lens is treated as phase corrector. If any component of field on the plane before lens (z=0) is u(0, x, y) (z - axis of lens), then at output (z=d)u(d, x, y)=u(0, x, y) $\exp[-i\psi(x, y)]$. Here $\psi(x, y)$ - optical length of straight/direct parallel axis and combining point (0, x, y) and (d, x, y). This geometric solution does not consider diffraction that for the millimeter waves it is possible to lead to appreciable errors, it, actually, does not consider also refractions of rays/beams. Are indicated below some receptions/procedures, which, probably, it will be possible to form the basis of of a stricter theory.

Let us take dielectric constant ε as the continuous function of coordinates; fields in this case also must be continuous. Transition to disruptive ones ε must be produced during the concrete/specific/actual calculations. We will be bounded to

two-dimensional scalar problem, i.e., we will seek function u(x, z) from the equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2} + k^2 \varepsilon u = 0$$
$$\varepsilon = \varepsilon(x, z). \qquad (1)$$

Apparatus easily can be generalized to the equations of Maxwell.

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Function ε varies with coordinate x slower than with z, so that the order of relation $\partial\varepsilon/\partial x : \partial\varepsilon/\partial z$ is a low value $\alpha(\alpha<<1)$ - this equivalent to the condition that the face of body, on which occurs the diffraction (lenses, prisms), it is almost perpendicular to z axis, normal to them forms with z axis the angle of order α. Derivative $\frac{\partial u}{\partial x}$ also is approximately/exemplarily α times less than $\frac{\partial u}{\partial z}$. The geometric-optical solution is obtained from (1), if we reject/throw $\frac{\partial^2 u}{\partial x^2}$. Then is obtained the ordinary differential equation

$$\frac{d^2 u}{dz^2} + k^2 \varepsilon u = 0, \qquad (2)$$

in which x it is the parameter; it actually/really has, as is known, the approximate (if we possible disregard/neglect reflection) solution

$$u(x, z) = u(x, 0)\, \varepsilon^{-1/4} \exp\left[-ik\int_0^z \sqrt{\varepsilon}\, dz\right]. \qquad (3)$$

2. Taking into account relative smallness of derivatives on x, it is possible to preserve simple form of equation (2) and to seek

u(x, z) as limit (if it exists), which are approached with $n \to \infty$ functions $u_n(x, z)$, determined from iterative process

$$\frac{d^2 u_n}{dz^2} + k^2 \varepsilon u_n = -\frac{\partial^2 u_{n-1}}{\partial x^2}. \qquad (4a)$$

In each stage will have to solve one and the same ordinary differential equation or, which is the same, to compute the same-type integrals

$$u_n(x, z) = -\int g(x, z, z') \frac{\partial^2 u_{n-1}(x, z')}{\partial x^2} dz', \qquad (5)$$

where g - Green's function equation (2) with the appropriate boundary conditions. Process (4a) coincides with the iterative process of solving the equation

$$u(x, z) = -\int g(x, z, z') \frac{\partial^2 u}{\partial x^2} dz', \qquad (6)$$

being obtained from (1).

Another process, which also brings, if it descends, to the solution of equation (1), is described by the equations

$$\frac{d^2 u_n}{dz^2} + \left(k^2 \varepsilon + \frac{1}{u_{n-1}} \frac{d^2 u_{n-1}}{\partial x^2}\right) u_n = 0. \qquad (4б)$$

In each stage it is necessary to solve although different, homogeneous equations. The general view of the iterative process, which reduces the solution of equation in the partial derivatives to the solution of the sequence of ordinary differential equations, is given by the formula

$$\frac{d^2 u_n}{dz^2} + f_n \frac{du_n}{dz} + (k^2 \varepsilon + \varphi_n) u_n = -\frac{\partial^2 u_{n-1}}{\partial x^2} + F_{n-1} \frac{\partial u_{n-1}}{\partial z} + \Phi_{n-1} u_{n-1}, \qquad (4)$$

where function f_n, F_n, φ_n and Φ_n are selected with arbitrary form $f_n, F_n \to f;\ \varphi_n, \Phi_n \to \varphi$. As $u_0(x, z)$ it is possible to take the function

of type (3) or the more complicated geometric-optical solution (containing eikonal), which considers refraction of rays/beams.

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3. Relative slowness of change u(x, z)c x makes it possible to use also so-called parabolic equation technique. In application to (1) should be instead of u(x, z) introduced the new unknown function A(x, z), after defining it as factor in the formula

$$u(x, z) = A(x, z)\, v(x, z), \qquad (7)$$

where v(x, z) - the solution of equation (2), and in the obtained equation for A(x, z) to reject/throw term $\partial^2 A/\partial z^2$. Although the Cartesian coordinates are not ray coordinates and $\partial v/\partial x \sim k\alpha v$, i.e. increases with k, the parabolic equation

$$\frac{\partial A}{\partial z} + \frac{1}{2\frac{\partial v}{\partial z}} \frac{\partial^2 (Av)}{\partial x^2} = 0 \qquad (8)$$

remains valid and with $k \to \infty$, since rejected/thrown component/term/addend is α^2 times less than preserved. Furthermore, if d (thickness of lens) is not very great ($d \ll L$, $L = 1/k\alpha^2$ - scale of change A with z), then in second term in (8) it is possible to replace A(x, z) by A(x, 0), and to explicitly find correction to the geometric-optical solution

$$A(x, z) - A(x, 0) = -\frac{1}{2}\int_0^z \frac{1}{\partial v/\partial z} \frac{\partial^2}{\partial x^2}[A(x, 0)\, v]\, dz. \qquad (9)$$

Asymptotic representation of the solution of the problem about the diffraction of plane electromagnetic wave on the ideally conducting sphere, valid at arbitrary height of observation point.

O. I. Fal'kovskiy, A. Z. Fradin.

On the basis of the series/rows according to the spherical wave functions of a strict solution of problem, which are determining all six components of complete field in the spherical coordinates, is obtained the asymptotic representation of solution, valid at arbitrary spot height of observation and value $ka \gg 1$ ($k=2\pi/\lambda$ - wave number, a - radius of sphere). The derived formulas are suitable for the numerical calculations and make it possible to give the demonstrative interpretation of field.

For obtaining the asymptotic representation the series/rows of a strict solution at first are converted according to Watson into the contour integrals in the plane of complex variable. In the case of the location of observation point in the shadow zone and penumbra these integrals are computed through the series/rows of the deductions of integrands. Because of the fact that $kr \geq ka \gg 1$ (r - distance from the center of sphere to observation point), in these

series/rows instead of the functions of Riccati-Bessel and their derivatives of the argument are used the asymptotic representations of G. D. Malyuzhints [1], the valid at arbitrary spot height of observation. As a result this complete field it is expressed as the series/rows into members of which enter the Airy's functions of the complex argument and their derivatives. These series/rows descend sufficiently rapidly, and in the region of a deep shadow in them it is to possible retain only first term.

The obtained formulas make it possible in the first approximation, to consider complete field in the shadow as the result of the superposition only of two fields. One of them is formed by the "straight/direct" diffracted ray/beam, which arrived into observation point by the shortest path from the boundary of geometric shadow, and another - by the "polar" diffracted ray/beam, which arrived into observation point by the shortest path of the diametrically opposite point of the boundary of shadow and crossed the "polar axis of shadow".

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In proportion to approximation/approach to an "axis of shadow" the contribution to the overall field of "straight/direct" ray/beam is reduced, and the contribution of "polar" ray/beam increases. On the

"axis of shadow" both these of contribution become equal to each other.

In the case of the location of observation point in the illuminated region initial contour integrals are divided/marked off into the series/row of new integrals (as a result of using the asymptotic expressions for the derivatives of Legendre's functions). Computing the part of the newly obtained integrals by the method of steady state, we come to field expressions which coincide with the expressions of geometric optic/optics - by the so-called reflecting formulas. Remaining integrals are computed through the series/rows of deductions in the same way as this was done for the contour integrals in the shadow zone, and give diffraction corrections to the reflecting formulas.

The formulas obtained for the illuminated region make it possible in the first approximation, to consider complete field as the result of the superposition of two fields one of which is determined by the reflecting formulas, which consider the expansion of the pencil of rays after reflection from the surface of sphere, and another is formed by "polar" diffracted ray/beam.

The field, which corresponds in the shadow zone to "polar" diffracted ray/beam, is identical to corrections to the reflecting

formulas in the illuminated region. Hence it follows that the special features/peculiarities of the structure of field in the region of penumbra must be determined by the formulas, which correspond to "straight/direct" ray/beam, and that these formulas in the illuminated region must pass into the expressions of geometric optic/optics. Research in the region of the penumbra of the formulas, which correspond to "straight/direct" ray/beam, showed that they actually/really give continuous transition from the expressions, valid in the region of a deep shadow, to the reflecting formulas in the illuminated region.

In this case in the region, which directly adjoins the boundary of geometric shadow, the caused by "straight/direct" ray/beam field at the sufficiently large distances on the sphere with an accuracy to almost constant component/term/addend (giving diffraction background) will be changed just as with the diffraction of plane electromagnetic wave on the half-plane. This result coincides with result, previously obtained V. A. Fok. However, earlier it was obtained for the case when distance from observation point to the surface of sphere is small in comparison with its radius. Here we are free from this limitation. It is important only so that would be small the angle of diffraction, which corresponds to observation point.

In special cases of the location of observation point on the

surface of sphere and in the wave zone asymptotic representations pass into the expressions, obtained earlier for these limiting cases by other authors.

The limits of the applicability of asymptotic representations were evaluated via the comparison of the numerical results, obtained with the help of these representations and with the help of the numerical addition of the series/rows of a strict solution.

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Wave diffraction in a heterogeneous half-space with the refractive index, which depends on two coordinates.

I. A. Molotkov.

For half-space $z>0$, $-\infty<x$, $y<\infty$ (x, y, z - Cartesian coordinates) is examined two-dimensional problem

$$\left.\begin{array}{c} v_{xx}+v_{zz}+\omega^2 n^2(x, z)\, v=0, \\ v|_{z=0}=\delta(x)\ \text{(I)}\ \text{или}\ v_z|_{z=0}=\delta(x)\ \text{(II)}, \end{array}\right\} \quad (1)$$

Key: (1). or.

radiation condition with $x^2+z^2\to\infty$, about finding of function $v(x, z, \omega)$. Refractive index $n(x, z)$ is counted to the satisfying following conditions:

1) $n(x, z)$ positively is analytical with $0\leqslant z<\infty$, $|x|<\infty$;

2) grow with the depth in the vicinity of boundary $n_z(x, z)>0$ when $0<z<\varepsilon$;

3) for all rays/beams $z=f(x)$ the extremal of line integral $\int n(x, z)\,ds$ is correct inequality $n_z-n_x f'(x)>0$, indicating the

constant sign of the curvature of rays/beams and the absence of turning points; rays/beams are assumed to be those not intersecting (absence of caustic curves);

4) with $x^2+z^2\to\infty$ and $z\geq0$ $n(x, z)$ approach constant.

It is necessary to find asymptotic behavior $v(x, z, \omega)$ in shadow zone with $\omega\to\infty$.

Asymptotic behavior $v(x, z, \omega)$ is located by the method of parabolic equation. First function $v(x, z, \omega)$ is sought in the thin layer, which adjoins boundary of $z=0$. In this layer is examined the function of the weakening

$$V(x, z, \omega) = v(x, z, \omega)\exp\left[-i\omega\int_0^x n(\alpha, 0)\,d\alpha\right].$$

In the differential equation for V are introduced new - across and along the rays/beams - the scales, which contain the different degrees of frequency ω, and such, that with respect to the new variable/alternating the function of weakening and its derivatives prove to be the values of one order. As a result of neglect in the equation by small terms, which are of the order $O(\omega^{-2/3})$ in comparison with the main things, appears the problem about the integration of parabolic equation for the function of weakening.

In the problem in question parabolic equation proves to be very complicated. However, using the possibility to supplement to it the terms of order $\omega^{-2/3}$, in it after some substitutions it is possible to divide variable/alternating. After satisfying other conditions, including condition in the source, we obtain asymptotic behavior v(x, z, ω) in the boundary layer with the thickness of the order $\omega^{-1/3}$.

Following stage - obtaining asymptotic behavior v(x, z, ω) in the depth of half-space in shadow zone. This transition can be completed with the help of Green's formula or by the ray continuation of field into the depth of half-space, after taking as the initial data of value in the boundary layer.

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Let us give final result for case I:

$$v(x, z, \omega) = c\omega^{1/6} e^{-\frac{i\pi}{12}} \exp\left(i\omega T + \omega^{1/3}\Delta e^{\frac{5\pi i}{6}}\right)\left[1 + O(\omega^{-1/3})\right], \qquad (2)$$

$$c = -\frac{n_z(0, 0)\, n_z^{-1/6}(\delta, 0)\, n^{1/3}(\delta, 0)}{2^{7/6} v'(-2^{1/3}\varkappa_1)\, R^{1/2}(\delta, 0; x, z)},$$

$$\Delta = \varkappa_1 \int_0^{\delta} \frac{n_z^{2/3}(x, 0)}{n^{1/3}(x, 0)}\, dx, \qquad (3)$$

where δ - arc length of slide, T - the minimum propagation time from source to the observer, v(t) - the integral of Airy, R - geometric disagreement of ray tube. Calculation R is reduced to the integration

of the Cauchy problem for the ordinary differential equation of the second order.

The knowledge of the high-frequency asymptotic behavior of the solution of stationary problem makes it possible to find the nonanalytic part of field u(x, z, t) in the vicinity of fronts or fronts of the slide of the corresponding unsteady problem. Here t - time, calculated off the moment/torque of including the source. Functions u(x, z, t) and v(x, z, ω) are connected with Fourier transform. For studying the behavior u(x, z, t) in the vicinity of the front of slide $\gamma \equiv n^{-1}(0, 0)(t-T)=0$ above the Fourier integral of v(x, z, ω) are produced the conversions, analogous to those described in [3]. As a result is established asymptotic with $\gamma \rightarrow 0$ the formula

$$u(x, z, t) = A(\gamma) \exp\left(-\frac{2}{3} \frac{\Lambda^{3/2}}{\sqrt{3\gamma}}\right) \varepsilon(\gamma) [1 + O(\sqrt{\gamma})], \qquad (4)$$

in which $A(\gamma)$ - the exponential function, which depends on the law of the inclusion/connection of source, $\varepsilon(\gamma)$ - the Heaviside unit function.

The formulas, analogous to (2) and (4), are obtained also in the case of the location of source within the half-space.

Finally, if we from function n(x, z) in the depth of half-space instead of the analyticity require only piecewise continuity, then in the half-space will arise the further waves, connected with the

reflections from the discontinuity surfaces n(x, z) and its derivatives. This leads to further components/terms/addends (with other phases) in (2), but it is not reflected in (4), since the fronts of slide remain the surfaces of the first entrance.

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Estimations of field in a shadow zone with the diffraction of cylindrical wave on the limited convex cylinder.

V. M. Babich, I. V. Olympius.

Is examined the following task. Is given the equation

$$(\Delta + k^2)\, u_{Q_0}(Q) = \delta(Q - Q_0) \qquad (1)$$

in exterior s_E locked convex to the curve s. We will assume that at all points curved s the curvature is different from 0 and ∞. Then $Q_0 \in s_E$ does not belong to s.

Are assigned the boundary condition

$$\left.\frac{\partial u}{\partial n}\right|_s = 0 \qquad (2)$$

and the condition for the radiation/emission

$$r^{1/2}\left(\frac{\partial u}{\partial r} - iku\right) \underset{r\to\infty}{\to} 0 \qquad (3)$$

Is sought asymptotic behavior $u_{Q_0}(Q)$ with $k\to\infty$ in the shadow zone for source Q_0.

In the work it is established/installed.

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Theorem 1. With $k \geqslant k_0$ for solving task (1)-(3) in the shadow zone occurs the estimation

$$|u_{Q_0}(Q)| \leqslant c_1 k^{-1/3} e^{-c_2 k^{1/3} f(Q_0, Q)}, \qquad (4)$$

where $f(Q_0, Q) \geqslant \mathrm{const}(D) > 0$ for any locked limited region D, wholly lying/horizontal within the shadow zone; $f(Q_0, Q)$ does not depend on k; with the approach to the boundary of light/world and shadow $f(Q_0, Q) \to 0$.

Through $c_{1,2}$ are designated the constants, which depend only on sizes/dimensions of s whose numerical values do not interest us.

In the proof of theorem we first obtain estimation $u_{Q_0}(Q)$, if Q belongs to arc s, which is located in the shadow, or, which is equivalent, if source is located on the boundary, and observation point in the shadow. This means that is solved the problem

$$(\Delta + k^2) G_{P_0}(Q) = 0; \qquad (5)$$

$$\left.\frac{\partial G}{\partial n}\right|_s = 2i\delta(P - P_0); \qquad (6)$$

$$r^{1/2}\left(\frac{\partial G}{\partial r} - ikG\right) \underset{r\to\infty}{\to} 0. \qquad (7)$$

Occurs theorem 2.

In the shadow zone for source P_0 when $\rho(Q; S) \geqslant \delta > 0$ is valid the estimation

$$|G_{P_0}(Q)| \leqslant c_1 k^{-1/3} e^{-c_2 k^{1/3} f(P_0, Q)}, \qquad (8)$$

where $l(P_0, Q)$ the length of smaller arc curved s from point P_0 to the point of contact of the tangency of ray/beam, carried out from Q to s.

Estimation (8) is obtained, if we reduce the problem (5)-(7) to the integral equation of Fredholm of the second order. For the construction of the kernel of this equation serves as special form the corrected solution of task (5)-(7) for the circle.

The task of diffraction on the elliptical cylinder and some estimations of Green's function of Helmholtz's operator in the case of diffraction on the arbitrary convex cylinder.

V. D. Andronov.

1. In works, dedicated to tasks of diffraction on elliptical cylinder, they are studied diffraction and scattering plane wave or is examined case when source and observation point are located in immediate proximity of elliptical cylinder.

In the present work is examined the case of the arbitrary location of point source and is indicated the method of the determination of the asymptotic expression of Green's function, when observation point is located on the cylinder. Is examined also the task of the approximation of Green's function for the arbitrary convex cylinder by Green's function for the elliptical cylinder, which has the contact of the third order with the data by cylinder.

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2. In elliptical system of coordinates (ξ, η)

$$x = c \operatorname{ch} \xi \cos \eta,$$
$$y = c \operatorname{sh} \xi \sin \eta$$

out of ellipse E: $\xi=\xi_0$ (c - distance between foci) is examined task

$$(\Delta+k^2)\,\Phi(M_0,M)=-\delta(M-M_0); \qquad (1)$$

$$\frac{d\Phi}{dn}\Big|_E=0,\ \sqrt{r}\left(\frac{\partial\Phi}{\partial r}-ik\Phi\right)_{r\to\infty}\to 0, \qquad (2)$$

where Δ - operator of Laplace, k - wave number, δ - Dirac's function, $M=M(\xi,\eta)$, $M_0=M_0(\xi',\eta')$. The divided equations take the form

$$\left[\frac{d^2}{d\xi^2}+(h^2\,\mathrm{sh}^2\,\xi+\lambda)\right]H(\xi)=0; \qquad (3)$$

$$\left[\frac{d^2}{d\eta^2}+(h^2\sin^2\eta-\lambda)\right]Y(\eta)=0, \qquad (4)$$

($h^2=c^2k^2$); the first of conditions (2) takes the form

$$\frac{\partial\Phi}{\partial\xi}\Big|_{\xi=\xi_0}=0. \qquad (5)$$

$g_1(\xi,\xi';\lambda)$ and $g_2(\eta,\eta';-\lambda)$ Green's function of operators (3), (4) respectively. Then, as shown in [2]

$$g_1(\xi\xi';\lambda)=\frac{1}{2iH_1'(\xi_0,\lambda)}\,[H_1'(\xi_0,\lambda)\,H_2(\xi,\lambda)-H_2'(\xi_0,\lambda)\,H_1(\xi,\lambda)]\,H_1(\xi',\lambda),$$

$$\xi'>\xi; \qquad (6)$$

$$g_2(\eta,\eta';-\lambda)=\frac{1}{2}\left[\frac{Y_1(\eta)\,Y_1(\pi-\eta')}{Y_1'(\pi)}+\frac{Y_2(\eta)\,Y_2(\pi-\eta')}{Y_2(\pi)}\right],\ \eta'>\eta. \qquad (7)$$

Here $H_1(\xi,\lambda)$, $H_2(\xi,\lambda)$ - two solutions of equations (3)whose Wronskian determinant is equal to 2i, moreover $H_1(\xi,\lambda)\in L_2(\xi_0,+\infty)$; Y_1, Y_2 solutions (4), determined by conditions $Y_1(0)=1$, $Y'_1(0)=0$, $Y_2(0)=0$, $Y'_2(0)=1$. For determining of $g_1(\xi,\xi';\lambda)$ with $\xi>\xi'$ and $g_2(\eta,\eta';-\lambda)$ when $\eta>\eta'$ is necessary in (6) and (7) to interchange

the position $\xi \to \xi'$, $\eta \to \eta'$. Occurs the condition

$$g_2(\eta+2\pi, \eta'; -\lambda) = g_2(\eta, \eta'; -\lambda).$$

Using the formula (for example, see [2])

$$\Phi(\xi', \eta'; \xi, \eta) = \frac{1}{2\pi i}\int_{\gamma} g_1(\xi, \xi'; \lambda)\cdot g_2(\eta, \eta'; -\lambda)\, d\lambda, \qquad (8)$$

where the duct/contour γ contains zero $H'_1(\xi_0, \lambda)$, we will obtain on the cylinder $\xi=\xi_0$

$$\Phi(\xi', \eta'; \xi_0, \eta) = -\frac{1}{2\pi i}\int_{\gamma} \frac{H_1(\xi', \lambda)}{H'_1(\xi_0, \lambda)} g_2(\eta, \eta'; -\lambda)\, d\lambda. \qquad (9)$$

3. In equations (3), (4) it is assumed/set $\lambda=-h^2 sh^2 \xi_1$; it is located asymptotic behavior of their solutions (for (3) different in regions of light/world and shadow) and roots λ_s of equation $H'_1(\xi_0, \lambda)=0$.

In the case of the location of point (ξ_0, η_0), in the light/world for $H_1(\xi, \lambda)$ is used asymptotic behavior ($h \to +\infty$)

$$H_1(\xi, \lambda) = \frac{e^{-i\frac{\pi}{4}}}{h^{1/2}\sqrt[4]{sh^2\xi - sh^2\xi_1}} e^{ih\int_{\xi_1}^{\xi}(sh^2\xi - sh^2\xi_1)^{1/2} d\xi}. \qquad (10)$$

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For function $g_2(\eta', \eta_0; -\lambda)$ is obtained the asymptotic equality

$$g_2(\eta', \eta_0; -\lambda) = \frac{i\exp\left\{ih\int_{\eta_0}^{\eta'} K(\eta)\, d\eta\right\}}{2h\,[K(\eta')\,K(\eta_0)]^{1/2}}, \qquad (11)$$

where $K(\eta) = (\sin^2\eta + sh^2\xi_1)^{1/2}$. From (9), (10), (11) follows

$$\Phi(\xi', \eta'; \xi_0, \eta_0) = \frac{1}{2\pi i}\int \frac{\tau \exp\left\{ih\left[\int\limits_{\xi_0}^{\xi'} (\mathrm{sh}^2\xi - \tau^2)^{1/2} d\xi + \int\limits_{\eta_0}^{\eta'} (\sin^2\eta + \tau^2)^{1/2} d\eta\right]\right\}}{\sqrt[4]{\sin^2\eta' + \tau^2}\,\sqrt[4]{\sin^2\eta_0 + \tau^2}\cdot\sqrt[4]{\mathrm{sh}^2\xi' - \tau^2}\cdot\sqrt[4]{\mathrm{sh}^2\xi_0 - \tau^2}}\, d\tau, \qquad (12)$$

($\tau = \mathrm{sh}\ \xi_1$, $\eta' > \eta_0$). Integral (12) is computed from the method of steady state. Equation $\omega'(\tau)=0$ for determining saddle points where

$$\omega(\tau) = \int\limits_{\xi_0}^{\xi} (\mathrm{sh}^2\xi - \tau^2)^{1/2} d\xi + \int\limits_{\eta_0}^{\eta'} (\sin^2\eta + \tau^2)^{1/2} d\eta, \qquad (13)$$

after bringing of integrals to the elliptical ones of the first kind and use of a theorem of addition for them accepts the form

$$\frac{\mathrm{ch}\,\xi_0\,\mathrm{sh}\,\xi'\cdot\sqrt{\mathrm{sh}^2\xi' - \tau^2} - \mathrm{ch}\,\xi'\,\mathrm{sh}\,\xi_0\sqrt{\mathrm{sh}^2\xi_0 - \tau^2}}{\tau^2 + 1 - \mathrm{ch}^2\xi'\,\mathrm{ch}^2\xi_0} =$$
$$= \frac{\cos\eta'\sin\eta_0\sqrt{\sin^2\eta_0 + \tau^2} - \sin\eta'\cos\eta_0\sqrt{\sin^2\eta' + \tau^2}}{\tau^2 + 1 - \cos^2\eta'\cos^2\eta_0}. \qquad (14)$$

The required solution of this equation takes the form

$$\tau_{\mathrm{cr}}^2 + 1 = \qquad (15)$$
$$= \frac{(\mathrm{ch}\,\xi'\cos\eta' - \mathrm{ch}\,\xi_0\cos\eta_0)^2 + (\mathrm{sh}\,\xi'\cdot\sin\eta'\,\mathrm{ch}\,\xi_0\cos\eta_0 - \mathrm{ch}\,\xi'\cos\eta'\,\mathrm{sh}\,\xi_0\sin\eta_0)^2}{(\mathrm{ch}\,\xi'\cos\eta' - \mathrm{ch}\,\xi_0\cos\eta_0)^2 + (\mathrm{sh}\,\xi'\sin\eta' - \mathrm{sh}\,\xi_0\sin\eta_0)^2}.$$

By the triple use of a theorem of addition for the elliptical second-order integrals (13) it is established that $\omega(\tau_{\mathrm{cr}}) = \frac{1}{c}|MM_0|$. Solution (15) has the following geometric interpretation. For different positions of point M on the ellipse it can seem that the continuation of ray/beam M_0M inside the ellipse $\xi=\xi_0$:

a) intersects its large axis out of the segment (-c, +c);

b) falls into the focus of ellipse;

c) intersects segment (-c, +c);

d) is tangent to the ellipse.

In the case of a) value $c\cdot(\tau_{cT}^2+1)^{1/2}$ is a semimajor axis of the ellipse, confocal to given, for which ray/beam M_0M is tangent. In particular, if this ray/beam falls into the focus (case b), then this ellipse is degenerated in the segment (-c, +c) and $\tau_{cT}=0$. In the case of c) value $c(\tau_{cT}^2+1)^{1/2}$ is a distance from the center of ellipse $\xi=\xi_0$ for the apex/vertex of the hyperbola, confocal with the given ellipse, for which ray/beam M_0M is tangent. In the case of d) $c\cdot(\tau_{cT}^2+1)^{1/2}$ is a semimajor axis of this ellipse, i.e., $\tau_{cT}=\operatorname{sh}\xi_0$. It is established that $\omega''(\tau_{cT})<0$.

4. Is examined task (1)-(2) (relative to function $u(M_0, M)$) for arbitrary duct/contour S of positive curvature.

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It shows that for this duct/contour there is always an ellipse, which has at point s of duct/contour S the contact of the third order and wholly lying within S (in the vicinity of point of contact of

tangency).

At each point s of duct/contour S is constructed this ellipse and for it is solved problem (1)-(2); by the series/row of estimations for Φ and $\frac{d\Phi}{dn}$ is proven.

Theorem 1. For values of u on duct/contour S occurs asymptotic formula ($k\to\infty$)

$$u(s)=\Phi(M_0, s)+O(k^{-1/3})\max_S|\Phi|, \qquad (16)$$

where $u(s)=u|M\in S$, $\Phi(M_0, s)$ - solution of the corresponding problem for the ellipse, carried out at point s, as noted above. Hence, because of estimation $\Phi(M_0^-, s)=O(\exp(-ck^{1/3}))$ for shadow zone, it is obtained.

Theorem 2. If point s on duct/contour S lies/rests at shadow zone, then occurs the estimation

$$u(s)=O(\exp(-ck^{1/3})), \ (c=\mathrm{const}>0). \qquad (17)$$

Formula (16) is an improvement in the analogous formula, obtained by V. M. Babich for the case of circle of curvature.

Interference wave field near the surface of elastic heterogeneous sphere.

A. I. Lanin.

Is investigated wave field in the elastic sphere $r \leq a$ with the elastic constants λ, μ and ρ, excited by the rotary source, applied to its surface.

The wave field of source at observation point can be represented by the sum of the waves, which tested 0, 1, 2, ... reflections from the surface of sphere. Waves with a small number of reflections by 0, 1, ..., M-1 can be examined separately (wave of geometric optic/optics). Waves, a number of reflections for which is great M, M+1 ... (Mach number depends on the wavelength λ and distance from the source to observation point), they will interfere with each other, forming complicated wave field. This field can be named interference type ground wave.

The method of isolation/liberation from the exact solutions of the tasks of expressions, which describe the waves of a similar nature, is proposed by V. S. Buldyrev (see V. S. Buldyrev's report).

For the uniform elastic sphere the exact solution of task Q(r, $\vartheta) = Q \cdot e_\varphi$ (e_φ - the unit vector of spherical coordinates) takes the form

$$Q(a, \vartheta) = \frac{1}{8\pi a^2 \mu k} \lim_{r \to a} \sqrt{\frac{a}{r}} \sum_{n=1}^{\infty} (2n+1) \frac{I_{n+1/2}(kr)}{I'_{n+1/2}(ka) - \beta I_{n+1/2}(ka)} \cdot P'_n(\cos\vartheta), \quad (1)$$

where β=3/2ka, k=ω/b, b - velocity of propagation of transverse waves, $I_{n+1/2}(x)$ - Bessel function, $P_n^1(\cos\vartheta)$ - the first associated function of Legendre.

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Applying the conversions of Watson, let us represent infinite sum (1) by the contour integral

$$Q(a, \vartheta) = \frac{-i}{8\pi a^2 \mu k} \lim_{r \to a} \sqrt{\frac{a}{r}} \int_c f(r, a, \nu) \frac{\nu P^1_{\nu-\frac{1}{2}}[\cos(\pi - \vartheta)]}{\cos \nu\pi} d\nu, \quad (2)$$

where

$$f(r, a, \nu) = \frac{I_\nu(kr)}{I'_\nu(ka) - \beta I_\nu(ka)}.$$

Integration occurs on duct/contour c, which contains the positive part of the real axis.

As a result of different conversions of integral (2) it is

possible to isolate the integrals, which describe separate waves and interference-surface wave.

For the separate waves with ka>>1 is obtained the following formula of geometric approximation/approach:

$$S^{+}_{0m} = -i\frac{ka}{2\pi\mu a^{1/2}}\frac{\sin^{3/2}\varphi_m}{\sqrt{2a(m+1)\cos\varphi_m\cdot\cos\vartheta}}\, e^{i\left[k2a(m+1)\cos\varphi_m-\frac{\pi}{2}m\right]}_{m\leqslant M-1,\ 0<\vartheta<\pi}\left\{1+O\left(\frac{1}{30\gamma}\right)\right\}.$$

In given formula φ_m - angle in the source between a radius of sphere and the ray/beam, which arrives after m reflections into observation point; $\sin^{3/2}\varphi_m$ - coefficient of the directionality of source; $2a(m+1)\cos\varphi_m$ - length of the path passed by wave; $\pi/2m$ - change in the phase of wave after m-fold passage of caustic curve; $\gamma=\vartheta\cdot(ka/2)^{1/3}$ - given angular distance.

The integrals, which describe interfering wave, it is possible to reduce to the form

$$S^{+}_{0M} = \frac{i\cdot(-1)^M}{4\pi^{3/2}a^2\mu}\left(\frac{ka}{2}\right)^{1/6}\cdot\frac{e^{i\left[ka\vartheta-\frac{\pi}{4}\right]}}{\sqrt{\sin\vartheta}}\cdot\Gamma_M(\gamma)\times$$
$$\times\left\{1+O\left[(M+1)\left(\frac{2}{ka}\right)^{2/3}\right]+O\left(\frac{e^{-A\gamma}}{\gamma}\right)\right\}, \qquad (3)$$

where

$$\Gamma_M(\gamma)=\int\limits_{Ae^{i\frac{4\pi}{3}}}^{Ae^{i\frac{\pi}{3}}}\frac{1}{W_2'(t)\,[W_1'(t)+W_2'(t)]}\left[\frac{W_1'(t)}{W_2'(t)}\right]^M e^{i\gamma t}\,dt,\ W_j(t),\ j=1,\,2-$$

Airy's function, $A\approx3$-4.

Function $\Gamma_M(\gamma)$ can be easily tabulated. For its calculation was comprised the program for the high speed electronic computer BESM-2.

For the heterogeneous sphere with the wave propagation velocity b=b(r), which increases with the depth, the process of the formation of interference-surface wave and waves of geometric optic/optics remains the same. Difference consists only of the fact that the rays/beams will come into observation point after m-fold reflection, extending not along the chords, but on the arcs of different curvature.

Conversion and research of solution for the heterogeneous sphere are carried out by the methods, analogous to the methods, used with b=const. The formulas, obtained with b=const, can be generalized in case of b=b(r).

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5. Method of the generalized seriess of Fourier.

Approximate solution of the maximum tasks of mathematical physics.

V. D. Kupradze.

The rapid progress of computer technology, characteristic for our time, puts forth new tasks in the region of the research of the "universal" methods of approximate solution of the tasks of mathematical natural science. In the present report will described one new "version" of method Fourier, which makes it possible construct approximate solutions almost for all linear maximum tasks of mathematical physics and continuum mechanics.

Equations and system of equations of elliptical type. The simplest, but completely typical example to the illustration of method gives Dirichlet problem for the harmonic function: to find function u(x), harmonic in region B_i, by Lyapunov's limited locked surface S, knowing that limit u(x) with the tendency of point x from within toward point x_0 on the boundary is equal to the assigned continuous function $f(x_0)$. The regular solution of this problem

satisfies the relationships/ratios of Green

$$u(x)=\frac{1}{4\pi}\int_S f(y)\frac{\partial}{\partial n_y}\frac{1}{r(x,y)}dS-\frac{1}{4\pi}\int_S\frac{\varphi(y)}{r(x,y)}dS,\ x\in B_i; \quad (1)$$

$$0=\int_S f(y)\frac{\partial}{\partial n_y}\frac{1}{r(x,y)}dS-\int_S\frac{\varphi(y)}{r(x,y)}dS,\ x\in B_e, \quad (2)$$

where by B_e is understood the external infinite region, which supplements B_i to the complete space, and φ(y) is a value normal derivative unknown function. It is proven, that if φ(y) is determined from functional equation (2), then (1) gives solution of problem. It is proven also, that (2) actually/really has, and besides only, the solution and that it is possible to clearly compute its Fourier coefficients. For this is introduced the set of functions

$$[r(x_k,y)]^{-1}\equiv\omega_k(y),\ y\in S;\ k=1,2,3,4\ldots,$$

where x_k is a calculating point set, everywhere tightly arranged/located on the arbitrary locked surface of S_1, situated in B_e, clasping S and not having with it common points. Is proven the fundamental theorem: set $\{\omega_k(y)\}$ is linearly independent and complete in space L_2 (S).

Introducing the orthonormalized system

$$\varphi_i(y)=\sum_{k=1}^{i}A_{ik}\omega_k(y),$$

from (2), in an obvious manner we obtain

$$\Phi_i\equiv\int_S\varphi(y)\varphi_i(y)\,dS=\sum_{k=1}^{i}A_{ik}\int_S f(y)\frac{\partial}{\partial n_y}\frac{1}{r(x_k,y)}dS.$$

Hence, accorc ıg to known facts about the properties of Fourier series in the complete Hilbert spaces, we have for N→∞

$$\left\| \varphi (y) - \sum_{i=1}^{N} \Phi_i \varphi_i \right\|_{L_2} \to 0.$$

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It is now easy to ascertain that the function

$$u_N (x) = \frac{1}{4\pi} \int_S f (y) \frac{\partial}{\partial n_y} \frac{1}{r (x, y)} dS - \frac{1}{4\pi} \int_S \frac{1}{r (x, y)} \sum_{i=1}^{N} \Phi_i \varphi_i (y) dS \qquad (4)$$

is unknown approximate value of function u(x). For this it suffices to compose difference (1) and (4) and to consider its modulus/module with the help of Schwarz-Buniakowski's inequality

$$| u (x) - u_N (x) | \leqslant \frac{1}{2\pi} \left\{ \int_S \left(\frac{1}{r (x, y)} \right)^2 dS \int_S \left[\left(\varphi (y) - \sum_{i=1}^{N} \Phi_i \varphi_i (y) \right)^2 \right] dS \right\}^{1/2}.$$

Hence by virtue of (3) we have

$$u (x) = \lim_{N \to \infty} u_N (x).$$

This methodology almost without any changes passes to use/application to all linear "elliptical" tasks for the homogeneous and piecewise-discontinuous media, for equations and systems of equations, in particular for the boundary-value problems of the theory of diffraction, theories of elasticity and hydrodynamics. Details see in [1]; see also the report to D. Z. Avazashvili.

Equations and system of equations of parabolic type. As the simplest typical example can serve mixed problem of the theory of the thermal conductivity:

$$\Delta u(x, t) - \frac{\partial u}{\partial t} = 0, \quad x \in B_i, \quad t \in [0, T], \quad (T > 0)$$
$$\lim_{x \to x_0 \in S} u(x, t) = f_1(x_0, t),$$
$$\lim_{t \to 0} u(x, t) = f_2(x).$$

It is proven, that with some, sufficiently general/common/total properties of functions f_1 and f_2 and surface S the task has a solution for which are valid Green's formulas:

$$u(x, t) = \int_0^t d\tau \int_S \rho(x, t; y, \tau)\, \varphi(y, \tau)\, dS - F(x, t), \; x \in B_i; \qquad (5)$$
$$0 = \int_0^t d\tau \int_S \rho(x, t; y, \tau)\, \varphi(y, \tau)\, dS - F(x, t), \; x \in B_e. \qquad (6)$$

Here F(x, t) - assigned function, which depends on f_1 and f_2, $\varphi(y, \tau) = \frac{\partial u(y, \tau)}{\partial n_y}$ and $\rho(x, t; y, \tau) = \frac{1}{8\pi^{3/2}(t-\tau)^{3/2}} e^{-\frac{r^2(x, y)}{4(t-\tau)}}, \; y \in S, \; \tau < t$ is a basic solution of the equation of thermal conductivity.

Together with points x_k, by those introduced above, now we introduce other points $t_i, i = 1, 2 \ldots \; |t_i| < |t_{i+1}|$, which "tightly" cover/coat entire segment [0, T], where T - arbitrarily fixed/recorded positive number, and we construct the set

$$\sigma(x_k, t_i; y, \tau) = \begin{cases} \rho(x_k, t_i; y, \tau) & \tau < t_i \\ 0 & t_i \leqslant \tau \leqslant T. \end{cases} \qquad (7)$$

It is not difficult to check that the elements/cells of this set for $y \in S, \tau \in [0, T]$ are continuous together with all derivatives.

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After placing the somehow introduced set, we can write:

$$\omega_j(y, \tau) = \sigma(x_{k_j}, t_{i_j}; y, \tau), \tag{8}$$

and in space L_2 (Sx [0, T]) with the norm for the element/cell $\varphi(y, \tau)$,

$$\sqrt{\int_0^T d\tau \int_S \varphi^2(y, \tau)\, ds}$$

we introduce the orthonormalized set

$$\varphi_n(y, \tau) = \sum_{j=1}^{n} A_{n_j} \omega_j(y, \tau). \tag{9}$$

Occurs the following fundamental theorem:

Set $\{\varphi_n(y, \tau)\}$ is linearly independent and it is full in space L_2 (Sx [0, T]).

From (6) it follows that

$$\int_0^{t_{i_j}} d\tau \int_S \varphi(y, \tau)\, \rho(x_{k_j}, t_{i_j}; y, \tau)\, dS = F(x_{k_j}; t_{i_j})$$

or by virtue of (7) and (8)

$$\int_0^T d\tau \int_S \varphi(y, \tau)\, \omega_j(y, \tau)\, dS = F(x_{k_j}, t_{i_j}),$$

and finally hence by virtue of (9) we obtain

$$\Phi_n = \int_0^T d\tau \int_S \varphi(y, \tau)\, \varphi_n(y, \tau)\, dS = \sum_{j=1}^{n} A_{n_j} F(x_{k_j}, t_{l_j}).$$

After constructing the function

$$u_N(x, t) = \int_0^t d\tau \int_S \rho(x, t;\, y, \tau) \sum_{n=1}^{N} \Phi_n \varphi_n(y, \tau)\, dS - F(x, t) \qquad (10)$$

and, relying on (5), (10), in view of fundamental theorem and inequality of a Schwarz-Buniakowski we obtain

$$u(x, t) = \lim_{N \to \infty} u_N(x, t).$$

More general maximum problems are examined in the works of O. Napetvaridze and A. Akizhanov. M. Grigoliya showed the use/application of a method for approximate solution of the basic mixed unsteady task for the linearized system of equations of Navier-Stokes.

Uses/applications to hyperbolic type tasks. L. Paatashvili showed the use/application of a method for approximate solution of mixed problem of wave equation. First with the help of the Laplace transform task is reduced to "elliptical" type boundary-value problem; to approximate solution of this latter, constructed according to p. 1, it proves to be applicable inverse transformation of Laplace that also it leads to approximate solution of initial task.

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Uses/applications to the mixed boundary-value problems. Let S' and S" be two pieces of surface S and S'+S" =S; let for S' be is assigned value $u=f_1$, and on S" - value $\frac{\partial u}{\partial n}=f_2$ of harmonic in B_i function u(x). This is - basic mixed problem of the theory of potential. Its solution is expressed by the formula

$$u(x)=\frac{1}{4\pi}\lim_{N\to\infty}\int_{S''}\frac{\partial G(x,y)}{\partial n_y}\sum_{i=1}^{N}\Phi_i\varphi_i(y)\,dS+F(x),\ x\in B_i, \qquad (11)$$

where G(x, y) is Green's function for certain region. When this function is located clearly and this proves to be possible for the sufficiently wide circle of tasks, formula (11) gives the explicit solution of mixed problem. In [1] are described the uses/applications to mixed problems of the theory of elasticity.

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6. Integral and operational identities.

One theorem for the analytic functions and its generalizations for wave potentials.

G. D. Malyuzhinets

Let us assume on the complex plane z are arranged/located three smooth closed curves l_1, l, l_2 so that l contains l_1, l_2 in turn, contains l.

Let us designate through D_1 the infinite region, external with respect to l_1, and through D_2 - final, internal with respect to l_2. The curve l belongs simultaneously to both regions D_1, D_2 and it is wholly arranged/located in their intersection $D_1 \cap D_2$, being vicinity by the curve l. Then has place.

Theorem 1. Let the analytic function $w(z)$, regular in D region and continuous in $D+l_1+l_2$, accept on duct/contour l of value $w(z_l)$. Then function $w(z_l)$ can be represented in the form of the sum

$$w(z_l)=w_1(z_l)+w_2(z_l), \tag{1}$$

where $w_1(z_l)$ and $w_2(z_l)$ - value for l of the analytic functions $w_1(z)$ and $w_2(z)$, from which $w_1(z)$ is regular in D_1 and $w_2(z)$ is regular in D_2, moreover

$$w_{1,2}(z_l)=\frac{1}{2}w(z_l)\mp\frac{1}{2\pi i}\int_l\frac{w(\xi)}{\xi-z_l}\,d\xi. \tag{2}$$

Proof. Applying the Cauchy theorem to the region between two closed curves l'_1 and l'_2 located respectively in the regions between the curves l_1l and ll_2, we have

$$w(z)=-\frac{1}{2\pi i}\int_{l_1}\frac{w(\xi)}{\xi-z}\,d\xi+\frac{1}{2\pi i}\int_{l_2}\frac{w(\xi)}{\xi-z}\,d\xi, \tag{3}$$

where the integration for both ducts/contours l'_1, l'_2 is produced in the positive direction. The confronting in right side (3) integrals

$$w_{1,2}(z)=\mp\frac{1}{2\pi i}\int_{l'_2}\frac{w(\xi)}{\xi-z}\,d\xi \tag{4}$$

represent the unknown functions $w_1(z)$, $w_2(z)$.

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Their regularity with respect in D_1 and to D_2 follows from the possibility to combine ducts/contours l'_1, l'_2 in accordance with l_1 and l_2. Counting further point z of that arranged/located between l and l'_2 in the case of $w_1(z)$ or between l'_1 and l in the case of

$w_2(z)$ and combining l'_1 or respectively l'_2 with l, and then directing z to certain point z_l on l, we obtain expressions (2) for $w_1(z_l)$ and $w_2(z_l)$, containing Cauchy integral.

Proved theorem 1 is the obvious modification of the known theorem, utilized during the factorization in the method of Wiener - Hopf - Foch.

Theorem 1 can be reformulated for the case of the harmonic functions u. For this purpose let us decompose function w(z) into the real and alleged parts of w(x+iy)=u(p)+iv(p), where through p is designated point (x, y), and let us retain in the real plane x, y previous designations for regions D_1, D, D_2 and closed curves l_1, l, l_2, after introducing additionally standards/normals n with corresponding indexes, directed to the exterior of these curves. In the general case function u(p) satisfies the equation

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=F_1(p)+F_2(p), \qquad (5)$$

where the function (perhaps, generalized) $F_1(p)\equiv 0$ when $p\in D_1$ and respectively $F_2\equiv 0$ when $p\in D_2$ so that in D region functions u, v are harmonics (Δu=Δv=0) and satisfy the conditions of Cauchy - Riemann. Then it is valid.

Theorem 2. Let function u(p), harmonic in D and continuous together with first-order derivatives in $D+l_1+l_2$, accept on the

closed curve l of value $u(p_l)$, and its normal derivative - value $\frac{\partial u}{\partial n}(p_l)$. Then occur the representations

$$u(p_l) = u_1(p_l) + u_2(p_l),$$
$$\frac{\partial u}{\partial n}(p_l) = \frac{\partial u_1}{\partial n}(p_l) + \frac{\partial u_2}{\partial n}(p_l),$$

where $u_1(p_l)$ and $u_2(p_l)$ - value for l of functions $u_1(p)$ and $u_2(p)$, harmonics with respect in D_1 and to D_2. In this case

$$\left.\begin{aligned} u_{1,2}(p_l) &= \frac{1}{2}u(p_l) \mp \frac{1}{2\pi}\int_l \left[\frac{\partial u}{\partial n'}(p')\ln\frac{1}{r} - u(p')\frac{\partial}{\partial n'}\ln\frac{1}{r}\right]dl', \\ \frac{\partial u_{1,2}}{\partial n}(p_l) &= \frac{1}{2}\frac{\partial u}{\partial n}(p_l) \mp \int_l \left[\frac{\partial u}{\partial n'}(p')\frac{\partial}{\partial n}\ln\frac{1}{r} - u(p')\frac{\partial^2}{\partial n\partial n'}\ln\frac{1}{r}\right]dl' \end{aligned}\right\} \quad (6)$$

For the proof let us assume in (4) z=x+iy; $\zeta = \xi + i\eta$; $w_{1,2}(z) = u_{1,2}(p)+iv_{1,2}(p)$; $w_{1,2}(\zeta)=u_{1,2}(p')+iv_{1,2}(p')$ and with the help of Cauchy-Rieman's conditions let us represent

$$\frac{1}{\zeta - z} = \frac{\partial \ln r}{\partial \xi} - i\frac{\partial \ln r}{\partial \eta} \quad (r = |\zeta - z|).$$

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Then real part (4) is converted into the line integral

$$u_{1,2}(p) = \mp\frac{1}{2\pi}\int_{l_{1,2}} \left\{\left[v(p')\frac{\partial \ln r}{\partial \eta} + u(p')\frac{\partial \ln r}{\partial \xi}\right]d\eta + \left[v(p')\frac{\partial \ln r}{\partial \xi} - u(p')\frac{\partial \ln r}{\partial \eta}\right]d\xi\right\},$$

which after integration in parts with the use of conditions of Cauchy-Rieman and identity $d\eta\frac{\partial}{\partial \xi} - d\xi\frac{\partial}{\partial \eta} = dl'\frac{\partial}{\partial n'}$ obtains the form

$$u_{1,2}(p) = \mp\frac{1}{2\pi}\int_{l_{1,2}} \left[\frac{\partial u}{\partial n'}(p')\ln\frac{1}{r} - u(p')\frac{\partial}{\partial n'}\ln\frac{1}{r}\right]dl',$$

and after differentiation along the normal n to curve l - the form

$$\frac{\partial u_{1,2}}{\partial n}(p) = \mp\frac{1}{2\pi}\int\limits_{l'_{1,2}}\left[\frac{\partial u}{\partial n'}(p')\ln\frac{1}{r} - u(p')\frac{\partial^2}{\partial n\partial n'}\ln\frac{1}{r}\right]dl'.$$

Applying then with respect to of ducts/contours l'_1, l'_2 and point p the same procedure, as in theorem 1, and using the properties of the potentials (and their normal derivatives) of simple and double layer when $p \to p_l$, we come to expressions (6). Harmony $u_{1,2}(p)$ in $D_{1,2}$ is established/installed also by previous method. Let us note that if function u(p) everywhere satisfies equation (5), then functions $u_{1,2}(p)$ everywhere satisfy equations $\Delta u_{1,2}=F_{1,2}(p)$.

Theorem 2, besides her conclusion/output from theorem 1, can be proved directly with the help of Green's formula and on this path allows/assumes generalization for the wave potentials. Let us show this at the case of the three-dimensional space where they occur the analogous to previous, but already three-dimensional regions D_1, D, D_2 and the locked ducts/contours l_1, l, l_2 are substituted by locked smooth surfaces of S_1, S, S_2, moreover normal n to S is directed to the outer side from S.

Theorem 3. Let function u(p) satisfy everywhere equation $\Delta u+k^2u=F_1(p)+F_2(p)$ (dependence on time e^{-ikct}), where $F_{1,2}(p)\equiv 0$ when $p\in D_{1,2}$. Are assigned values $u(p_S)$ and $\frac{\partial u}{\partial n}(p_S)$ on surface of S. Then occur the representations

$$u(p_S) = u_1(p_S) + u_2(p_S);\quad \frac{\partial u}{\partial n}(p_S) = \frac{\partial u_1}{\partial n}(p_S) + \frac{\partial u_2}{\partial n}(p_S), \qquad (7)$$

where $u_{1,2}(p_S)$ - value for S of solutions $u_{1,2}(p)$ equations $\Delta u_{1,2}+k^2u_{1,2}=F_{1,2}(p)$, $u_1(p)$ satisfying radiation condition at infinity. Functions $u_{1,2}(p_S)$ and $\frac{\partial u_{1,2}(P_S)}{\partial n}$ are determined by the formulas

$$\left.\begin{aligned} u_{1,2}(p_S) &= \frac{1}{2}u(p_S) \mp \frac{1}{4\pi}\iint_S \left[\frac{\partial u}{\partial n'}(p')\frac{e^{ikR}}{R} - u(p')\frac{\partial}{\partial n'}\frac{e^{ikR}}{R}\right]dS, \\ \frac{\partial u_{1,2}}{\partial n}(p_S) &= \frac{1}{2}\frac{\partial u}{\partial n}(p_S) \mp \frac{1}{4\pi}\iint_S \left[\frac{\partial u}{\partial n'}(p')\frac{\partial}{\partial n}\frac{e^{ikR}}{R} - u(p')\frac{\partial^2}{\partial n\partial n'}\frac{e^{ikR}}{R}\right]dS. \end{aligned}\right\} \quad (8)$$

Proof. We will be bounded here for the brevity to the derivation of formulas (8). Let us use Green-Helmholtz's formula to functions $u_1(p)$ and $u_2(p)$ respectively in the regions out of and within surface of S. Then, assuming/setting $p = p_S$, we have

$$u_{1,2}(p_S) = \mp \frac{1}{2\pi}\iint_S \left[\frac{\partial u_{1,2}}{\partial n'}(p')\frac{e^{ikR}}{R} - u_{1,2}(p')\frac{\partial}{\partial n'}\frac{e^{ikR}}{R}\right]dS.$$

Now, subtracting one equality of another and using (7), we find

$$u_1(p_S) - u_2(p_S) = -\frac{1}{2\pi}\iint_S \left[\frac{\partial u}{\partial n'}(p')\frac{e^{ikR}}{R} - u(p')\frac{\partial}{\partial n'}\frac{e^{ikR}}{R}\right]dS. \quad (9)$$

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Analogously let us find

$$\frac{\partial u_1}{\partial n}(p_S) - \frac{\partial u_2}{\partial n}(p_S) = -\frac{1}{2\pi}\iint_S \left[\frac{\partial u}{\partial n'}(p')\frac{\partial}{\partial n}\frac{e^{ikR}}{R} - u(p')\frac{\partial^2}{\partial n\partial n'}\frac{e^{ikR}}{R}\right]dS. \quad (10)$$

After solving the system of four equations (7), (9), (10) relatively $u_{1,2}(p_S)$ and $\frac{\partial u_{1,2}}{\partial n}(p_S)$, we obtain formulas (8).

For the case of unsteady wave fields is valid completely analogous theorem 4 about the representation of solution of U(p, t) of equation $\Delta U - \frac{1}{c^2}\frac{\partial^2 U}{\partial t^2} = F_1(p, t) + F_2(p, t)$ by the sum of solutions $U_{1,2}(p, t)$ equations $\Delta U_{1,2} - \frac{1}{c^2}\frac{\partial^2 U_{1,2}}{\partial t^2} = F_{1,2}(p, t)$. In this case the formula for

$U_{1,2}(p_S, t)$, obtained by Fourier transform from (8) or it is direct, with the help of the theory of the delaying potentials, they take the form

$$U_{1,2}(p_S, t) = \frac{1}{2}U(p_S, t) \mp \frac{1}{4\pi}\iint_S \Big[\frac{1}{R}\frac{\partial U}{\partial n'}(p', t') + \frac{1}{cR}\frac{\partial R}{\partial n'}(p', t')\frac{\partial U}{\partial t'}(p', t') - \frac{\partial}{\partial n'}\Big(\frac{1}{R}\Big)U(p', t')\Big]dS,$$

$$\frac{\partial U_{1,2}}{\partial n}(p_S, t) = \frac{1}{2}\frac{dU}{\partial n}(p_S, t) \mp \frac{1}{4\pi}\iint_S \Big\{\frac{\partial}{\partial n}\Big(\frac{1}{R}\Big)\frac{\partial U}{\partial n'}(p', t') - \frac{\partial^2}{\partial n \partial n'}\Big(\frac{1}{R}\Big)U(p', t') + \frac{1}{cR}\Big[\Big(\frac{\partial^2 R}{\partial n \partial n'} - \frac{2}{R}\frac{\partial R}{\partial n}\frac{\partial R}{\partial n'}\Big)\frac{\partial U}{\partial t'}(p', t') - \frac{\partial R}{\partial n}\frac{\partial^2 U}{\partial t' \partial n'}(p', t') - \frac{1}{c}\frac{\partial R}{\partial n}\frac{\partial R}{\partial n'}\frac{\partial^2 U}{\partial t'^2}(p', t')\Big]\Big\}dS,$$

where after differentiation under the integrals is placed t'=t-R/c.

Application of Lorentz's lemma to the calculation of the radiation fields of the assigned sources in different media.

I. G. Kondratyev, V. I. Talanov.

Is proposed the general method of calculating the radiation fields the assigned source distribution in different linear homogeneous media: isotropic and anisotropic, with the spatial dispersion and without it, motionless and moving, etc. Sources are assumed to be stationary ones in the space and those harmonically changing in the time. The method, based on the use/application of Lorentz's lemma, generalizes the known theory of the excitation of waveguides, worked out by Weinstein, to the case of normal plane waves in the unbounded media.

In general form, suitable in the examination of anisotropic media with the weak spatial dispersion, Lorentz's lemma is formulated as follows:

$$\frac{4\pi}{c}\cdot\int_v(\mathbf{j}_1^e\cdot\mathbf{E}_2-\mathbf{j}_2^e\cdot\mathbf{E}_1-\mathbf{j}_1^m\cdot\mathbf{H}_2+\mathbf{j}_2^m\cdot\mathbf{H}_1)\cdot dv=\oint_S\Big\{([\mathbf{E}_1\cdot\mathbf{H}_2]-[\mathbf{E}_2\cdot\mathbf{H}_1])\cdot\mathbf{n}$$
$$-ik_0\cdot\left[\gamma_{ikl}\cdot E_{1k}\cdot E_{2i}+\delta_{iklm}\cdot\left(\frac{\partial E_{1k}}{\partial x_m}\cdot E_{2i}-\frac{\partial E_{2k}}{\partial x_m}\cdot E_{1i}\right)\right]\cdot n_l\Big\}\cdot dS. \quad (1)$$

where coefficients $\gamma_{ikl}, \delta_{iklm}$ are determined from the expansion [1]

$$D_i=\varepsilon_{ik}\cdot E_k+\gamma_{ikl}\cdot\frac{\partial E_k}{\partial x_l}+\delta_{iklm}\cdot\frac{\partial^2 E_k}{\partial x_l\cdot\partial x_m}.$$

FOOTNOTE [1]. It is assumed that B=H, although this limitation is not in principle. ENDFOOTNOTE.

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Field E_2, H_2 in (1) in the general case relates to the "transposed" medium, in which both the direction of external magnetostatic field and direction the motions of medium are substituted to the reverse/inverse.

Let E_1, H_1 - unknown field, created by assigned sources $j_1^e \equiv j^e$, $j_1^m \equiv j^m$. As E_2, H_2 let us take the field of the plane wave

$$\tilde{\mathbf{E}}_2(\mathbf{k}_\gamma) = \tilde{\mathbf{E}}_\gamma^0(\mathbf{k}_\gamma)\cdot e^{-ik_\gamma r}, \quad \tilde{\mathbf{H}}_2(\mathbf{k}_\gamma) = \tilde{\mathbf{H}}_\gamma^0(\mathbf{k}_\gamma)\cdot e^{-ik_\gamma r}, \tag{3}$$

being normal wave of γ type in the "transposed" medium. When one and the same value $|\mathbf{k}|$ they answer different normal waves (as in the case of polarizational degeneration in the isotropic medium), we will consider it their orthogonalized in the sense of executing of the relationship/ratio

$$\begin{gathered} I_{\nu\gamma} = \{[\mathbf{E}_\nu^0(\mathbf{k})\cdot\tilde{\mathbf{H}}_\gamma^0(-\mathbf{k})] - [\tilde{\mathbf{E}}_\gamma^0(-\mathbf{k})\cdot\mathbf{H}_\nu^0(\mathbf{k})]\}\cdot\mathbf{n} - \\ - ik_0\cdot\left\{\gamma_{ikl}\cdot E_{\nu k}^0(\mathbf{k})\cdot\tilde{E}_{\gamma i}^0(-\mathbf{k}) + \delta_{iklm}\left[\frac{\partial}{\partial x_m}E_{\nu k}(\mathbf{k})\cdot\tilde{E}_{\gamma i}(-\mathbf{k}) - \right.\right. \\ \left.\left. - \frac{\partial}{\partial x_m}\tilde{E}_{\gamma k}(-\mathbf{k})\cdot E_{\nu i}(\mathbf{k})\right]\right\}\cdot n_l = 0, \end{gathered} \tag{4}$$

where n - unit vector in the direction of the group velocity of wave $\mathbf{E}_\nu$, $\mathbf{H}_\nu$. Value $I_{\nu\nu} = N_\nu$ let us name the norm of a plane wave of the type ν.

The field of the assigned sources at certain point of the sufficiently distant sphere in the direction of unit vector $\overline{r}_0(\vartheta, \varphi)$ we will seek in the form of the superposition of the normal plane waves

$$\mathbf{E}(\mathbf{r}_0, r) = \sum_{\nu} a_{\nu}(\mathbf{r}_0, r)\cdot\mathbf{E}_{\nu}^{0}(\mathbf{k}_{\nu})\cdot e^{-i\mathbf{k}_{\nu}\mathbf{r}},$$
$$\mathbf{H}(\mathbf{r}_0, r) = \sum_{\nu} a_{\nu}(\mathbf{r}_0, r)\cdot\mathbf{H}_{\nu}^{0}(\mathbf{k}_{\nu})\cdot e^{-i\mathbf{k}_{\nu}\mathbf{r}} \quad (5)$$

with slowly varying amplitudes of $a_{\nu}(\mathbf{r}_0, \mathbf{r})$. In view of the uniformity of space natural to assume that in distant zone ($r \to \infty$) the group velocity of each of the normal plane waves and which locally is divided/marked off radiation field, is directed along radius (r_0). By this consideration is determined the form of dependence $\mathbf{k}_{\nu}(\mathbf{r}_0)$ in (3). The corresponding surface integral, which is obtained as a result of substitution (3) and (5) in (1), with $r \to \infty$ is computed by the method of steady state, moreover under the done assumption about dependence $\mathbf{k}_{\nu}(\mathbf{r}_0)$ of stationary proves to be the point at which $\mathbf{k}_{\nu}(\mathbf{r}_0) = -\mathbf{k}_{\gamma}$, i.e. the point of contact of the tangency of the fronts of the emitted and test waves. The resultant expression for amplitudes $a_{\nu}(\vartheta, \varphi, r)$ takes the form

$$a_\nu(\vartheta, \varphi, r) = \frac{1}{r} \cdot \left\{ \frac{2 \cdot \frac{\partial \mathbf{k}_\nu}{\partial \varphi} \cdot \frac{\partial \mathbf{r}_0}{\partial \varphi} + \frac{\partial^2 \mathbf{k}_\nu}{\partial \varphi^2} \cdot \mathbf{r}_0}{2 \cdot \frac{\partial \mathbf{k}_\nu}{\partial \vartheta} \cdot \frac{\partial \mathbf{r}_0}{\partial \vartheta} + \frac{\partial^2 \mathbf{k}_\nu}{\partial \vartheta^2} \cdot \mathbf{r}_0} \cdot \left[\left(2 \cdot \frac{\partial \mathbf{k}_\nu}{\partial \vartheta} \cdot \frac{\partial \mathbf{r}_0}{\partial \vartheta} + \frac{\partial^2 \mathbf{k}_\nu}{\partial \vartheta^2} \cdot \mathbf{r}_0 \right)^2 - \right.\right.$$
$$\left.\left. - \left(\frac{\partial \mathbf{k}_\nu}{\partial \vartheta} \cdot \frac{\partial \mathbf{r}_0}{\partial \varphi} + \frac{\partial \mathbf{k}_\nu}{\partial \varphi} \cdot \frac{\partial \mathbf{r}_0}{\partial \vartheta} + \frac{\partial^2 \mathbf{k}_\nu}{\partial \vartheta \cdot \partial \varphi} \cdot \mathbf{r}_0 \right)^2 \right] \right\}^{1/2} \times$$
$$\times \frac{1}{\sin \vartheta} \cdot \frac{2}{N_\nu} \cdot \frac{i}{c} \cdot \int_V [\mathbf{j}^e \cdot \widetilde{\mathbf{E}}_\nu(-\mathbf{k}_\nu) - \mathbf{j}^m \cdot \widetilde{\mathbf{H}}_\nu(-\mathbf{k}_\nu)] \cdot e^{i\mathbf{k}_\nu \mathbf{r}} \cdot dv. \qquad (6)$$

For ν the type in question factors confronting in relationship/ratio (6) before the integral are the universal function of the parameters of medium. In terms of this it differ significantly from the expressions, which escape/ensue from the usual reciprocity theorem whose use/application requires the knowledge of the fields of elementary sources.

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Relationship/ratio (6) is illustrated by a number of examples. In the special cases, which were being examined earlier in the literature, it leads to the already known results.

The region of the applicability of the method proposed is not contained by the tasks about the radiation/emission of the assigned source distribution in the unbounded homogeneous medium; it extends also to the piecewise-uniform media, if is known the solution of the problem about the diffraction of normal plane waves with this configuration of system.

Equivalent transformations of quasi-optical systems.

V. I. Talanov.

In the practice of the use of quasi-optical waveguides and resonators increasingly more frequently appears the need in the construction of the combined systems, which consist of different amplitude-phase converters of the wave bundle: lenses, mirrors, prisms, plates, etc. With the use/application of such systems are connected, in particular, attempts at the suppression of the parasitic types of oscillations in the resonators of lasers and the quasi-optical plumbing.

The calculation of the complicated combined quasi-optical systems substantially is simplified during the use/application of an operational method of describing the wave bundles. The basic operators of the transformation of wave bundle are: the operator of Green, who mutually connects field distributions in two different beam sections in the free space; the operators of the aberrations of the first and second orders; the operator of the Fourier transform of wave bundle; the operator of the transverse displacement of bundle to the constant vector, the operator of the scale transformation of bundle; the operator of the irising of bundle ideally absorbing

barrier with the opening/aperture. For the operators indicated is established/installed the series/row of the operational identities which permit implemention of the equivalent transformations of the quasi-optical systems of amplitude-phase transformers, analogous to the transformations of optical systems, produced on the basis of ray theory. By equivalent transformations of quasi-optical systems are understood the transformations of the transformers of the fields which either leave without the change the operator, which connects field distributions in two fixed/recorded sections of system or are converted it in accordance with the replacement of the function, which describes wave bundle.

As the examples, which illustrate the use/application of equivalent transformations, are examined the following tasks: about the passage of wave beam on the quasi-optical waveguide with the arbitrarily arranged/located periodically repeating diaphragms; about the types of the oscillations of spherical resonator with the intermediate lens; about the construction of the short combined resonator, equivalent according to its characteristics to longer confocal resonator; about the reflection of wave beam from the quadratic converter with the flat/plane mirror shield, arranged/located after it, inclined toward the optical axis of system.

Interesting transformations of the quasi-optical systems are connected with the use/application of identities, which contain the transformation of beam into its angular spectrum. Applying Fourier transform to the beam in the real quasi-optical system, is possible to pass to the examination of certain interconnected circuit, the role of "beam" in which will play the angular spectrum of wave beam in the reference system. To flat/plane uniform layer in the real system corresponds quadratic converter in that conjugated/combined and, on the contrary, to quadratic converter in the reference system corresponds the layer of uniform space in that conjugated/combined.

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Analogous mutual relationships/ratios have between the operator of first-order aberrations and the operator of beam displacement. As an example of interconnected circuits can serve the ray waveguide, comprised of the dissimilar quadratic converters, and the waveguide, which contains the identical, but nonequidistantly arranged/located converters.

Transition to the examination of equivalent interconnected circuit proves to be especially useful when the transformation of the angular spectrum of wave beam by the separate elements of system is described more simply than the coordinate transformation of beam

itself. As an example of this task is given the calculation of the types of the oscillations of spherical resonator with that introduced to it inclined toward the optical axis of system flat/plane Fabri-Perot interferometer.

One modification of diffraction integral in the theory of volumetric scattering.

M. L. Levin.

The task about the diffraction on the uniform transparent body in the approximation/approach of the theory of weak scattering is reduced, as is known, to the determination of the integral

$$I=\int \frac{e^{-ik(R_1+R_2)}}{R_1R_2}\,dV, \qquad (1)$$

where $\mathbf{R}_1=R_1\mathbf{r}_1$, $\mathbf{R}_2=R_2\mathbf{r}_2$ - radius-vectors of the element of volume dV of the scattering body, carried out from source and observation point.

Integrand in (1) is divergence of vectorial field

$$\mathbf{F}=\frac{i}{k}\frac{e^{-ik(R_1+R_2)}}{(R_1+R_2)^2-D^2}(\mathbf{r}_1+\mathbf{r}_2)=\frac{i}{2k}\frac{e^{-ik(R_1+R_2)}}{R_1R_2}\cdot\frac{\mathbf{r}_1+\mathbf{r}_2}{1+\cos\alpha}.$$

Here D - distance from the source to observation point, α - angle between unit vectors $\mathbf{r}_1$ and $\mathbf{r}_2$. On the ray/beam, which combines source with observation point, field F has a special feature/peculiarity. Using the gauss divergence theorem and taking into account this special feature/peculiarity, it is possible to reduce (1) to the form

$$I=\oint F_n\,dS+\frac{2\pi}{ik}\,l\,\frac{e^{-ikD}}{D}, \qquad (2)$$

where l - section of ray/beam D, which is located within the body.

Transition from volumetric integral (1) to surface integral (2) is analogous in a certain sense to the replacement of the picture of Fresnel by Young's picture in the classical theory of diffraction. During the solution of the specific problems (in particular, when the incident wave it is flat/plane) calculations according to formula (2) prove to be substantially more simply than according to formula (1).

7. Infinite systems of equations.

Mathematical questions of the theory of diffraction on a flat periodic lattice.

Ye. N. Podolskiy.

1. As is known, task about diffraction of plane electromagnetic wave, which falls at arbitrary angle to flat/plane periodic lattice, formed by parallel infinitely thin and ideally conducting metallic strips/films, is reduced to two scalar tasks A and B which are formulated below in parallel.

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On axis oy of plane yOz is arranged/located the lattice, formed by periodically arranged/located axis intercepts. The period of lattice is equal to d, the width of slots - Δ. It is necessary to find function v(y, z), which satisfies everywhere in the plane, with exception of the points of lattice, the equation of Helmholtz $\Delta v+k^2v=0$, continuous up to the boundary, that satisfies the boundary condition

$$v|_{\text{на реш}} = 0 \quad \text{в задаче А} \tag{1}$$

Key: (1). in problem.

(respectively

$$\left.\frac{\partial v}{\partial z}\right|_{\text{на реш}} = 0 \text{ в задаче}^{(1)} \text{ B})$$

Key: (1). in problem.

and represented in the form

$$v(y, z) = v_{\text{пад}}(y, z) + v_1(y, z),$$

where

$$v_{\text{пад}} = e^{i\mu y + i\nu z};\ v_1(y, z) = \varkappa e^{i\mu y} \sum_{n=-\infty}^{\infty} a_n e^{inpy} e^{i\nu_n |z|}\ (z \neq 0)$$

(factor α is equal to one in task A ; $\frac{|z|}{z}$ in task B).

Here

$$\mu^2 + \nu^2 = k^2,\ \operatorname{Im}\mu = 0,\ p = \frac{2\pi}{d},$$
$$\nu_n = \sqrt{k^2 - (\mu + np)^2},$$

moreover radical sign is chosen from condition $\operatorname{Im}\nu_n > 0$, and if $\operatorname{Im}\nu_n = 0$, then $\operatorname{Re}\nu_n > 0$.

So that the solution v(y, z) would satisfy condition of the finiteness of energy in any limited part of the space, it suffices to require satisfaction of the following condition: there is periodic with period d a summarized in the period function F(y), such, that

$$\left|\frac{\partial v(y, z)}{\partial z}\right| < F(y)$$

in certain band $|z| < \sigma$ $(\sigma > 0)$.

Under these conditions are proven the existence and the uniqueness of the solution of tasks A and B.

2. Task B is reduced to task A; therefore it is possible to be bounded to examination of task A.

Task A is reduced to the solution of the infinite system of linear algebraic equations. It is proved that this system satisfies Koch's conditions and for its solutions in the space of sequences l^2 are valid Fredholm theorems. It is shown that any solution of system from l^2 gives the solution of stated problem A. Therefore it singularly and in view of Fredholm's alternative exists.

Since the mentioned system of linear algebraic equations satisfies Koch's condition, its solution can be obtained in the form of the relations of the convergent infinite determinants of Cramer. Thus, by truncation of infinite system there can be obtained approximate solution of the system (but it means, and the boundary-value problem A) presented in the principle to any degree of accuracy.

Let us introduce dimensionless parameter $\varkappa = \frac{k}{p} = \frac{d}{\lambda}$, equal to

the ratio of the period of lattice to the length of the incident wave. Are of interest two limiting cases: small $\varkappa$ and large $\varkappa$.

3. In long-wave approximation/approach we will count k, μ and ν fixed/recorded, and d vanishing. First let us assume in this case is retained the constant ratio of the width of slot Δ to period d. Then the solution of task A can be represented in the form

$$v(y, z) = v^{(1)}(y, z) + O(\varkappa^3),$$

where the estimation is uniform relative to y and z.

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For $v^{(1)}(y, z)$ is given explicit expression. From it it is possible to obtain with $z > \sigma > 0$ the approximation/approach of Lamba-Rayleigh which, as it is proven, has an error in order $O(\varkappa^3)$. Is of interest another version of the long-wave approximation/approach when simultaneously with tendency d toward zero approaches one relation Δ to d. Namely, let $p \to \infty$, moreover

$$\lim_{p\to\infty} \frac{1}{p} \ln \frac{1 + \cos\frac{\pi\Delta}{d}}{2} = -D.$$

In this case the solution of task A with $p \to \infty$ is even in any closed domain, which does not contain the points of axis oy, it approaches limit function

$$v_0(y, z) = e^{i\mu y + i\nu z} - \frac{1}{1 - i|\nu| D} e^{i\mu y + i|\nu z|}.$$

4. In examination of short-wave approximation/approach we consider lattice of that fixed/recorded ($d=2\pi$), and k as that approaching infinity (in this case direction of propagation of incident wave is kept constant).

The obtained result consists of the following. The solution of task A can be represented in the form

$$v(y, z) = e^{i\mu y}\left(e^{i\nu z} - e^{i|\nu z|}\right) + e^{i\mu y} \sum_{n=-\infty}^{\infty} \sin n \frac{\Delta}{2} \cdot \frac{1}{\pi n} e^{iny} e^{i\nu_n |z|} + \varepsilon(y, z).$$

Function $\varepsilon(y, z)$ satisfies the inequality

$$\iint_G |\varepsilon(y, z)|^2\, dy\, dz < \frac{C(G)}{\sqrt{k}}.$$

In this case flow $w_\zeta[\varepsilon]$ of the energy through the segment with the length of 2π of straight line $z=\zeta\neq 0$, which falls to function $\varepsilon(y, z)$, is considered as follows:

$$|w_\zeta[\varepsilon]| < \frac{C}{k}.$$

If we formally decompose ν_n according to degrees 1/k and to be bounded in the expansion two first terms, then it is possible to obtain the representation of solution in the form $v^\Gamma(y, z) + \varepsilon^\Gamma(y, z)$;

$$v^\Gamma(y, z) = e^{i\mu y}\left(e^{i\nu z} - e^{i|\nu z|}\right) + e^{i\mu y} \sum_{n=-\infty}^{\infty} \sin n \frac{\Delta}{2} \cdot \frac{1}{\pi n} e^{in\left(y - \frac{\mu}{|\nu|}|z|\right)}$$

is the approximation/approach of geometric optic/optics, and $\varepsilon^\Gamma(y, z)$ satisfies the following inequalities:

$$\iint_G |\varepsilon^\Gamma(y, z)|^2\, dy\, dz < \frac{C(G)}{\sqrt{k}}; \quad |w_\zeta[\varepsilon^\Gamma]| < C\sqrt{\frac{|\zeta| + 1}{k}}.$$

Thus, in this task is obtained a strict proof of geometric optic/optics.

5. In intermediate range x is checked possibility of numerical calculation with a high degree of accuracy in electronic digital computers with operating speed of order "Arrow/pointer".

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Diffraction and propagation of electromagnetic waves in flat and cylindrical periodic structures of special geometric form.

G. N. Gestrin, L. N. Litvinenko, K. V. Maslov, V. P. Shestopalov.

It is well known, as it is difficult to obtain the efficient calculation methods in the diffraction tasks when the wavelength of the incident field of the same order as as the sizes/dimensions of obstructions. In the work of Z. S. Agranovich, V. A. Marchenko and V. P. Shestopalov (ZhTF, XXXII, iss. 4, 381, 1962) was proposed the method of the solution of the problem of diffraction on the correct foil lattice, which makes it possible represent solution in the form convenient for the qualitative and quantitative research. Following the work indicated appeared the whole series of other works in which this method applied to more complicated type periodic systems. One of the in practice interesting tasks, for which the use/application of a method of the work indicated proves to be possible, is the task of diffraction on the flat/plane periodic lattice, which consists of the parallel metallic bands of different width d and d_1. Diffraction on this lattice possesses the specific special features/peculiarities, which make the use/application of such lattices more advantageous in

a sense, than simple lattices. Furthermore, the solution of this problem for the case of the arbitrarily incident plane wave is allowed then, using the averaged boundary conditions, to obtain the results, which relate to the propagation of waves in the circular and spiral waveguides of analogous structure (i.e., consisting of the strips/films of different width).

Let us consider briefly the simplest task of diffraction.

Let in plane xOy be arranged/located the flat/plane infinitely thin and ideally conducting metallic lattice with period l, which consists of alternating bands with a width of d_1 and d. Axis Ox is parallel to the bands of lattice. From half-space z>0 normal to this lattice falls plane electromagnetic wave $E_x = e^{-i(kz+\omega t)}$, $E_y = E_z = 0$. To component E_x of the unknown field it is possible to present in the form of Fourier series in half-spaces z>0 and z<0. Using boundary conditions in plane z=0, we will easily obtain infinite system of equations for determining the unknown Fourier coefficients. However, this system little is convenient for research and obtaining approximate solutions. Via reducing of initial task to Riemann-Hilbert's task it is possible to obtain the equivalent system of equations for the Fourier coefficients, which proves to be extremely convenient for the solution of problem. The small number of equations of this system gives the possibility to perform with the

high degree of accuracy the numerical calculations in the range of values $\varkappa\left(\varkappa=\frac{kl}{2\pi}\right)$ from 0 to the values of the order of several ones when has already been made by possible the use/application of short-wave approximation/approach.

During the research of solution special interest presents the case when width d_1 of narrower band is small in comparison with the period of lattice. If $\varkappa \ll 1$ (long-wave approximation/approach), then from the system indicated for the coefficient of reflection b_0 it is possible to find

$$b_0=\frac{2i\varkappa\,(R_{[\sigma]}\tilde{R}^1_{[\sigma]}-\tilde{R}_{[\sigma]}R^1_{[\sigma]})}{2i\varkappa\,(R_{[\sigma]}\tilde{R}^1_{[\sigma]}-\tilde{R}_{[\sigma]}R^1_{[\sigma]})+R^1_{[\sigma]}-\tilde{R}^1_{[\sigma]}}, \qquad (1)$$

where values $R_{[\sigma]}$, $R^1_{[\sigma]}$, $\tilde{R}_{[\sigma]}$ and $\tilde{R}^1_{[\sigma]}$ asymptotically (when $\cos\frac{\pi d_1}{l}=v\to 1$) are equal to

$$\tilde{R}^1_{[\sigma]}=-\frac{1}{2\sqrt{2(1-u)}}\ln\frac{1-v}{32}\cdot\frac{(1-u+\sqrt{2(1-u)})^2}{(1-u)^2};$$

$$R^1_{[\sigma]}=\frac{1}{\sqrt{2(1-u)}}\ln\frac{\sqrt{1-u^2}}{1-u+\sqrt{2(1-u)}};$$

$$\tilde{R}_{[\sigma]}=\tilde{R}^1_{[\sigma]}-\ln\left(1+\sqrt{\frac{1-u}{2}}\right);$$

$$R_{[\sigma]}=R^1_{[\sigma]}-\frac{1}{2}\ln\frac{1+u}{2};$$

$$u=\cos\frac{\pi(l-d)}{l}.$$

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If we in (1) pass to the limit with $v\to 1$, then we will obtain the known Lamb approximation/approach for the usual lattice. However, because of the fact that the tendency toward zero appropriate terms

in (1) very slow (as $\frac{1}{\ln(1-v)}$), presence v even little different from 1 noticeably is reflected to value b_0.

It proves to be possible to obtain by the same means the solution for H-polarized wave and further for the case of oblique incidence in the wave. This in turn, makes it possible to establish/install equivalent boundary conditions for these lattices and to consider then a question about the propagation of the damped and undamped electromagnetic waves in the circular and spiral waveguides of the corresponding form. In this case, if we during the establishment of equivalent boundary conditions are bounded by Lamb approximation/approach, then these conditions will coincide with those obtained with the solution of quasi-static problem. But if we use more precise values of the coefficients of reflection (passage), then boundary conditions can be used also for the not very short periods of structure.

Strict solution of the problem about the propagation of electromagnetic waves in special circular waveguides.

S. S. Tret'yakova, V. P. Shestopalov.

Circular waveguide is the periodic structure, which consists of the infinitely thin, isolated/insulated from each other ideally conducting rings, with period 1 and width of the slots between rings d. Such structures find wide application as the waveguide filters in the transmission lines, they are applied in the linear accelerators, it is possible to use them for obtaining the radiation/emission, caused by the motion of electron beam along the axis of circular waveguide.

In the real devices/equipment circular waveguide is placed into the dielectric tube, surrounded by metal shield. The presence of dielectric and shielding waveguide, obviously, must affect the characteristics of circular waveguide. The purpose of this work is obtaining a strict solution of the problem about the propagation of electromagnetic waves in the circular waveguide, placed into the dielectric medium with arbitrary value ε and shielded ideally conducting cylindrical waveguide.

Is examined the case of the symmetrical E- and H-waves. Because of the periodicity of structure electromagnetic field is represented in the form of Fourier series. Unknown Fourier coefficients are found from the subordination of fields to precise boundary conditions on the surface of the shielding and circular waveguides: equality to zero of tangential field component on the slot. By different transformations boundary-value electrodynamic problem is reduced to the systems of equations which are already well studied and are solved via their reducing to Riemann-Hilbert's task for the analytic function.

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The solution of the latter is obtained in the form of the uniform infinite system of linear algebraic equations relative to the unknown Fourier coefficients of electromagnetic field, for which is suitable the method of reduction. Equality to zero determinants of this system is dispersion equation for each transmission mode; dispersion equations are obtained in the form, convenient for the calculations on computer(s).

In the simplest special case of zero approximation dispersion equations acquire the sufficiently simple form:

the E-waves

$$J_0(p_0a) = \frac{1}{2i}\left[J_0'(p_0a) - \frac{p_0\varepsilon}{p_0'} J_0(p_0a)\frac{J_0'(p_0'a)H_0(p_0'b) - J_0(p_0'b)H_0'(p_0'a)}{J_0(p_0'a)H_0(p_0'b) - J_0(p_0'b)H_0(p_0'a)}\right] \times$$
$$\times \frac{P_\nu(-u) + P_{\nu-1}(-u)}{P_\nu(-u) - P_{\nu-1}(-u)},$$

H- wave

$$J_1(p_0a) = -\frac{1}{2i}\left[J_0(p_0a) - \frac{p_0}{p_0'} J_0'(p_0a)\frac{J_0'(p_0'a)H_0'(p_0'b) - J_0'(p_0'b)H_0(p_0'a)}{J_0(p_0'a)H_0'(p_0'b) - J_0'(p_0'b)H_0'(p_0'a)}\right] \times$$
$$\times \frac{P'_\nu(u) - P_{\nu-1}(u)}{P'_\nu(u) + P_{\nu-1}(u)},$$

where

$$p_0^2 = k^2 - h_0^2, \quad p_0'^2 = k^2\varepsilon - h_0^2; \quad k = \frac{2\pi}{\lambda_0}; \quad h_0 = \frac{2\pi}{\lambda_b}; \quad \nu = \frac{h_0 l}{2\pi}$$

$P_\nu(u)$, $P_{\nu-1}(u)$ - Legendre's function from argument $u=\cos\frac{\pi d}{r}$; $J_0(p_0a)$, $H_0(p_0a)$, ... - the Bessel function and Hankel of the first order of arguments p_0a, p_0b, p'_0a, p'_0b, $J'_0(p_0a)$, $H'_0(p_0a)$, ... - derivatives of the argument; a - radius of circular waveguide; b - radius of the shielding waveguide.

Diffraction of electromagnetic waves on the dielectric lattices, comprised of beams of rectangular cross section.

Yu. T. Repa.

Is examined the task about a normal incidence in plane electromagnetic wave on the lattice, comprised of parallel dielectric beams of rectangular cross section. Bars are assumed to be those prepared of the uniform isotropic dielectric with complex dielectric constant $\varepsilon_1=\varepsilon'_1+i\varepsilon''_1$. Space between them is filled with dielectric with $\varepsilon_2=\varepsilon'_2+i\varepsilon''_2$. Is examined the case of E-polarization (vector of electric field is oriented along the bars of lattice) for the lattices with different h (thickness of lattice in the direction of propagation of wave) and θ (coefficient, analogous to duty factor). Period is undertaken by equal 2π.

Task is reduced to the numerical finding of the coefficients of reflection and passage and the analysis of their dependence on the ratio of period to the wavelength. Interest in it is caused by the possibility of practical use/application and by the fact that it possesses the greatest generality in comparison with the analogous metallic structures. The examination of the latter can be represented as a special case of this task.

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The periodicity of structure in the direction, perpendicular to incident direction in the wave, makes it possible to present the fields above, under, and within the structure in the form of Fourier series. Subsequently on the fields are superimposed the requirements of satisfaction to the equations of Maxwell in the appropriate regions, to propagation condition at infinity and to boundary conditions on the structure. Obtaining the infinite systems of linear algebraic equations relative to the coefficients of reflection and passage is realized by reexpanding the sets of functions of complete in one interval of the sets of functions complete ones in other interval with the subsequent use of a property of orthogonality.

The difficulty of using the obtained infinite systems consists in the fact that the complexity of structure leads to the dependence of equations from three independent indices, i.e., to peculiar "it is three-dimensional/space" system of equations.

The solution of systems was produced by the method of reduction. For this purpose was comprised the program for ETsVM [' ЭЦВМ - digital computer] "Ural-4". The evaluation of the convergence of the

solutions of the truncated systems to the solution of infinite system of equations was produced numerically via calculation of the truncated systems of different orders and comparison of results. It is possible to draw the conclusion that the solution of similar type problems exhausts the possibilities of machines of the type "Strela, "Ural", "BESM" almost completely since the order of the systems which must be solved for the comparison, grows according to the law of nl^2, where n - number of infinite systems of equations which are required to solve together, and l=1, 2,3.... Thus, in this task with l=3 it is necessary to calculate integrated systems 2nd, eighth and of the 18th or the real systems of the 4th, 16th and 36th orders.

By passage to the limit from the obtained expressions it is possible to obtain formulas for the coefficients of reflection and passage from the layer with the uniform and isotropic dielectric. These formulas coincide precisely with the known results.

Finally, the dependences, obtained at specific limiting values ε, coincide sufficiently well with the appropriate dependences for the metallic lattices.

Diffraction of H-polarized electromagnetic waves on the foil lattice, comprised of the bars of rectangular cross section.

S. A. Masalov.

Is examined the task about the diffraction of inclined incident H-polarized plane electromagnetic wave on the lattice from the parallel ideally conducting bars of rectangular cross section, between which is arranged/located uniform and isotropic dielectric. To lattice parameters and wavelength in comparison with the period of the lattice no limitations are superimposed. The diffracted field is sought in the form of Fourier series respectively above and under the lattice and within the slots. Boundary conditions and conditions of join reduce to two infinite systems of the linear algebraic equations in which as the unknowns enter the half-sums and the half-differences of the wave amplitudes of the diffraction spectrum.

In certain cases it is possible to show that these infinite systems have solutions, moreover the sum of the squares of unknowns descends. In [1] for the mixed boundary-value problem found the solution of the equation of Laplace within the circle in the form of infinite series is made an evaluation of the decrease of Fourier coefficients at $n \to \infty$. Analogously for the task examined/considered by us in certain cases it is possible to produce the estimation of tendency toward zero Fourier coefficients with $n \to \infty$.

At some values of the parameters is produced the solution of systems on computer(s) by reduction method. Was discovered a good convergence of solutions of the truncated systems.

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The obtained results clarify the character of transition from the lattice of final thickness to the infinitely thin lattice, on one hand, and to the system of waveguides - on the other hand.

If we consider the task about the propagation of H-polarized waves along the lattice, assuming/setting fields by the identical ones above and under the lattice, then is obtained the infinite system of linear homogeneous algebraic equations. From the condition for inversion into zero determinations of this system, using Newton's method the determination of the roots of equations, numerically on computer(s) are determined propagation constant of field along the lattice. The task of propagation along the lattice proves to be in view of the symmetry of field to the equivalent task of propagation along the rack with the rectangular teeth.

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Diffraction of plane electromagnetic wave on the lattice from parallel coaxial cylinders.

Ye. A. Ivanov, S. F. Il'yukevich.

Is given a strict solution of the problem about the diffraction of plane electromagnetic wave $\mathbf{H}^0 = \{0, 0, H_z^0\}$, $\mathbf{E}^0 = \{E_x^0, E_y^0, 0\}$, $H_z^0 = He^{ik\mathbf{n}\mathbf{R} - i\omega t}$ on the system from an arbitrary number of parallel coaxial lines of the circular infinite cylinders, which form in the space foil lattice, on the assumption that the physical properties of each of the cylinders of system remain constant/invariable along the axis of cylinder (physical properties of all internal cylinders they are considered identical, they are determined by values $\varepsilon_3, \mu_3, \sigma_3$. The physical properties of jackets are also considered identical, they are determined by values $\varepsilon_2, \mu_2, \sigma_2$) and by the fact that the wave is propagated in the sense of the vector n, which forms arbitrary angle α with the line of centers of cylinders and angle $\pi/2$ with their axes. The coordinate system Oxyz is introduced so that axis Oz coincides with axis of one of the pairs of rollers, and axis Ox is directed along the center line. Cylinders are numbered: s=0, O+-1, ...+-N, moreover index s=0 is ascribed to the pair of rollers with which is connected the coordinate system Oxyz. Furthermore, are introduced local coordinate systems $O_s x_s y_s z_s$, $s = 0, \pm 1, \ldots \pm N$. connected

with the appropriate pair of coaxial rollers. Distance between centers of pairs of rollers is equal to $l_{ss'}$, $s \neq s'$; $s, s' = 0, \pm 1, \ldots$ +-N, radii of jackets are designated through a_s, and radii of the internal cylinders through b_s, $s = 0, \pm 1, \ldots \pm N$. If we designate now the field, external with respect to the cylinders of lattice, through (I), the field, included between the surfaces of each pair of coaxial rollers, through (II), and the field, determined by all internal cylinders of lattice, through (III), then for finding the component H_z of vector H of secondary field in each of the fields indicated we will arrive at two-dimensional problem of the mathematical theory of diffraction, which consists of the determination of the solution of equation $\Delta H_z^{(i)} + k_i^2 H_z^{(i)} = 0$, $i =$ I, II, III, where

$$k_i^2 = \begin{cases} \frac{\varepsilon_1 \mu_1 \omega^2}{c^2}, \ \varepsilon_1 = \mu_1 = 1, \ \text{область I} \\ \frac{\varepsilon_2 \mu_2 \omega^2 + 4\pi\sigma_2 \mu_2 \omega i}{c^2}, \ \text{область II} \\ \frac{\varepsilon_3 \mu_3 \omega^2 + 4\pi\sigma_3 \mu_3 \omega i}{c^2}, \ \text{область III}, \end{cases}$$

Key: (1). region. satisfying boundary conditions:

$$H_z^{(1)} = H_z^{(0)} + H_z^{(1)} = H_{z,s}^{(2)}, \ \frac{\partial H_{z,s}^{(1)}}{\partial \rho_s} = \frac{\varepsilon_1 (4\pi\sigma_1 - i\omega)}{\varepsilon_2 (4\pi\sigma_2 - i\omega)} \frac{\partial H_{z,s}^{(2)}}{\partial \rho_s}$$

with

$$\rho_s = a_s, \quad s = 0, \pm 1, \ldots \pm N$$

and

$$H_{z,s}^{(2)} = H_{z,s}^{(3)}, \quad \frac{\partial H_{z,s}^{(2)}}{\partial \rho_s} = \frac{\varepsilon_2 (4\pi\sigma^2 - i\omega)}{\varepsilon_3 (4\pi\sigma_3 - i\omega)} \frac{\partial H_{z,s}^{(3)}}{\partial \rho_s}$$

with

$$\rho_s = b_s, \ s = 0, \pm 1, \ldots \pm N.$$

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Here, furthermore, function $H_z^{(1)}$ must satisfy radiation conditions at infinity. This problem is solved in the polar coordinates, in which the unknown functions are represented in the form of infinite series according to their own wave functions of circular cylinder, registered in the local coordinates:

$$H_z^{(1)} = \sum_{s=-N}^{N} \sum_{n=-\infty}^{\infty} x_n^{(s)} H_n^{(1)}(k_1\rho_s)\, e^{in\varphi_s} + H_z^0,$$

$$H_{z,s}^{(2)} = \sum_{n=-\infty}^{\infty} y_n^{(s)} J_n(k_2\rho_s)\, e^{in\varphi_s} + \sum_{n=-\infty}^{\infty} z_n^{(s)} H_n^{(1)}(k_2\rho_s)\, e^{in\varphi_s},$$

$$H_{z,s}^{(3)} = \sum_{n=-\infty}^{\infty} u_n^{(s)} J_n(k_3\rho_s)\, e^{in\varphi_s}, \quad s = 0, \pm 1, \ldots \pm N.$$

Unknown coefficients $x_n^{(s)}, y_n^{(s)}, z_n^{(s)}$ and $u_n^{(s)}$ are found from the boundary conditions, which bring as a result to by the infinite systems of linear equations for them. To these systems it is not possible to use soundly any of the known methods of their solution. However, after replacement

$x_n^{(s)} = J_n(k_1 a_s) X_n^{(s)}$, $y_n^{(s)} = H_n^{(1)}(k_2 a_s) Y_n^{(s)}$, $z_n^{(s)} = J_n(k_2 b) Z_n^{(s)}$, $u_n^{(s)} = H_n^{(1)}(k_3 b_s) U_n^{(s)}$ for new unknowns $X_n^{(s)}, Y_n^{(s)}, Z_n^{(s)}, U_n^{(s)}$ are obtained the infinite systems, solved with the method of truncation when $l_{ss'} > a_s + a_{s'}, s \neq s'$; s, s'=0, +-1, ...+-N, |s-s'|=1. In this case it proves to be that

$$\sum_{n=-\infty}^{\infty} |X_n^{(s)}|^2 < \infty, \qquad \sum_{n=-\infty}^{\infty} |Y_n^{(s)}|^2 < \infty, \qquad \sum_{n=-\infty}^{\infty} |Z_n^{(s)}|^2 < \infty,$$

$\sum_{n=-\infty}^{\infty} |U_n^{(s)}|^2 < \infty$. Similarly is solved problem, also, for the case when $\mathbf{E}^0 = \{0, 0, E_z^0\}$, $\mathbf{H}_0 = \{H_x^0, H_y^0, 0\}$, $E_z^0 = Ee^{iknR - i\omega t}$.

Is examined the case of two pairs of coaxial rollers, for which are given examples of the numerical solution of task by strict formulas for some particular values of parameters ($a_s=a$, $b_s=b$, $s=\pm 1$; $k_2b=1{,}25$, $k_2a=2.5$; 3, 4, $k_2=1.6\ k_1$). For the case of 2N+1 pairs of coaxial rollers from strict formulas are derived the approximate formulas, which fall the possibility to investigate solution with some assumptions ($\min k_1 l_{ss'} \gg 1$, $a_s = a$, $b_s = b$, $s, s' = 0, \pm 1, \ldots \pm N$, $s \neq s'$; $k_2 a \ll 1$ etc.).

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Diffraction of electromagnetic waves on two coaxial disks.

Ye. A. Ivanov.

Report is devoted to a strict solution of the problem about the diffraction of the field of magnetic dipole on two coaxial infinitely thin and ideally conducting disks, arranged/located in a vacuum at a distance of l from each other, on the assumption that in the general case radii of disks are different and that the direction of moment/torque **m** of the dipole, arranged/located at any point on the axis of disks (including in center of one of them), forms arbitrary angle α with the axis of disks. In view of the linearity of task after resolving of vector **m** into two components of vector $\mathbf{m}=\mathbf{m}_r+\mathbf{m}_z$, where $\mathbf{m}_r=\mathbf{m}\sin\alpha$, and $\mathbf{m}_z=\mathbf{m}\cos\alpha$, general problem is reduced two quotients: a) to the task about diffraction of the field of vertical magnetic dipole on two coaxial disks and b) to the task about the diffraction of horizontal magnetic dipole on two coaxial disks. Problem a) is solved in the coordinates of flattened spheroid ξ, η, φ,

in which $\mathbf{E}=(0, 0, E_\varphi)$, $\mathbf{H}=(H_\xi, H_\eta, 0)$, where H_ξ and H_η are expressed as E_φ from equation rot $\mathbf{E}$=ik$\mathbf{H}$ and where $E_\varphi=E_\varphi^{(1)}+E_\varphi^{(2)}$. Here $E_\varphi^{(1)}$ - known component of the primary field of dipole, and $E_\varphi^{(2)}$ - sought component of vector $\mathbf{E}$ of secondary field. $E_\varphi^{(2)}$ is sought as the solution of the wave equation of Helmholtz, which satisfies condition $E_\varphi^{(1)}+E_\varphi^{(2)}=0$ on the surface of each disk and condition for radiation/emission at infinity, moreover it is assumed that $E_\varphi^{(2)}=\sum_{s=\pm 1} E_s$, where E_s is represented by infinite series according to its own wave spheroidal functions, registered in the local coordinates of the s spheroid, s=+-1. For the unknown coefficients of expansions from the boundary conditions is obtained the infinite system of linear equations, which after the replacement of previous unknown coefficients to the new ones in some formulas becomes quasi-regular, solved the method of truncation. Task b) is more complicated. Its solution is found with the help of Hertz's magnetic vector $\mathbf{\Pi}$ by formulas $\mathbf{E}=ik\,\mathrm{rot}\,\mathbf{\Pi}$, $\mathbf{H}=\mathrm{grad}\,\mathrm{div}\mathbf{\Pi}+k^2\mathbf{\Pi}$ (temporary/time dependence is considered assigned factor $e^{-i\omega t}$) by generalizing the method, used by M. G. Belkina in the task about the diffraction of the field of horizontal magnetic dipole on one disk (Coll. Diffraction of electromagnetic waves on some bodies of revolution. M., Sov. radio, 1957) and in the solved by us earlier task about the diffraction of the field of horizontal magnetic dipole on two disks of equal radii (Zh. of the comp. math. and math. physics, 3, No 2, 1963; Coll. "Numerical methods of solving the differential and integral equations and

quadrature formulas". M., publishing house "Science", 1964) in the case of two coaxial disks of different radii. As in the recently indicated works, problem is solved first in the cylindrical coordinates in which it is possible to divide boundary conditions for unknown potential functions Φ and Ψ on the surface of each disk (here $\Phi=\Phi_1+\Phi_2=\Pi_x^{(1)}+\Pi_x^{(2)}, \Pi_z=\Pi_z^{(2)}=\Psi\cos\varphi, \Pi_y=0$, where $\Pi^{(1)}=\{\Phi_1=|\mathbf{m}|\cos\alpha e^{ikR}/R, 0, 0\}$ - vector of Hertz of primary field, dipole, and $\Pi^{(2)}=\{\Pi_x^{(2)}, 0, \Pi_z^{(2)}\}$ - vector of Hertz of secondary field), and then in the coordinates of flattened spheroid, in which Φ_2 and Ψ are sought as the solutions of the equations of Helmholtz, which satisfy on the surface of disks conditions $\partial\Phi/\partial\xi_s=C_s a_s \eta_s$, $\Psi=C_s a_s\sqrt{1-\eta_s^2}$, $s=\pm1$ and condition for radiation/emission at infinity. In the case when dipole does not lie/rest on surface of one of the disks, for determining the unknown integration constants C_s is used only the condition of Mayksner, which is reduced to the requirement of turning into zero radial component of the vector of the density of induced currents on the edges/fins of disks.

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In the case when dipole proves to be arranged/located in center of one of the disks, for determination C_s it is used, furthermore, reciprocity theorem for two magnetic dipoles. Functions Φ_2 and Ψ are sought in the form of infinite series according to their own

spheroidal wave functions, registered in the local coordinates of the s disk, for unknown coefficients of which from the boundary conditions are obtained the infinite systems of linear equations. To these systems it is not possible to use the soundly known methods of solving the infinite systems; however, after the replacement of previous unknowns to the new ones according to some formulas for the latter are obtained the systems, solved with truncation.

Is investigated a question about the possibility of using the obtained formulas for the numerical calculation on them. In particular, it shows that for the sufficiently wide range of parameter $C_s = ka_s$, a_s - a radius of the s disk, s=+-1, the numerical results always can be obtained when $l > a_{+1} + a_{-1}$. As the illustration of method are given examples of the numerical solution of task for c=1; 2 and different kl. For kl>>1 (wavelength is considerably lower than the distance between the disks) are obtained from strict solutions the approximation formulas, convenient for the research and the numerical calculation.

Diffraction of plane wave on double lattice, comprised of thin bands (oblique incidence).

G. Sh. Kevanishvili.

Let us assume that plane electromagnetic wave

$$E_1 = e^{ikx\cos\theta + iky\sin\theta}$$

falls on the periodic structure, formed of two foil lattices each of which is comprised of the infinitely long ideally conducting thin bands. Let us assume also that the lattices occupy in the space such position, that the bands of one lattice are arranged/located accurately against the openings/apertures between the bands of another lattice. The distance between the lattices let us designate through h and let d - period of lattice, a - bandwidth. Vector E is parallel to the edge of band.

The electric intensity of the scattered from the double lattice indicated wave takes the following form:

$$E = -\frac{iAkd}{2\pi^2} \sum_{m=-\infty}^{\infty} F(m)\, e^{ik_1(m+D\sin\theta)y} e^{-k_1 x\sqrt{(m-D\sin\theta)^2-D^2}}, \qquad (1)$$

where

$$F(m) = \frac{\sin(\pi a d^{-1} m)}{(m+D\sin\theta)\sqrt{(m-D\sin\theta)^2-D^2}}\left[1+(-1)^m e^{-ikh\cos\theta - i\pi D\sin\theta}\right].$$

$$(k_1 = 2\pi/d,\ D = d/\lambda).$$

In formula (1) $k=2\pi/\lambda$, λ - wavelength; A is determined from the following relationship/ratio:

$$A = \frac{4\pi^3 \sin\left[\frac{ka}{2}\sin\theta\right] / \sin\theta}{ik^2d^2\left[\sum_{m=-\infty}^{\infty} F_1(m) + e^{-ikh\cos\theta - i\pi D\sin\theta}\sum_{m=-\infty}^{\infty}(-1)^m F_2(m)\right]}. \qquad (2)$$

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Here

$$F_1(m) = \frac{\sin^2(\pi a d^{-1}(m + D\sin\theta))}{(m + D\sin\theta)^2\sqrt{(m - D\sin\theta)^2 - D^2}},$$

$$F_2(m) = \frac{\sin^2(\pi a d^{-1}(m + D\sin\theta))\, e^{-2\pi h d^{-1}\sqrt{(m-D\sin\theta)^2-D^2}}}{(m + D\sin\theta)^2\sqrt{(m - D\sin\theta)^2 - D^2}}.$$

If in (1) we complete passage to the limit with $x\to\infty$, we will obtain the value of stray field in the far zone. Assuming that $1 - D\sin\theta > 1$, i.e., $D<1/1+\sin\theta$, we will have

$$E(\infty) = -\frac{iAkd}{2\pi^2}\frac{\sin\left(\frac{ka}{2}\sin\theta\right)}{iD^2\sin\theta\cos\theta}\left(1 + e^{-2ikh\cos\theta - i\pi D\sin\theta}\right)e^{-ikx\cos\theta + iky\sin\theta}.$$

The expression, which stands in the curly brace, is the coefficient of reflection of electromagnetic waves from the double lattice. Let us designate it through R, then, taking into account (2), after simple transformations we obtain

$$R = -\frac{1}{1 + \frac{D^2\cos\theta\sin^2\theta}{2\sin^2\left(\frac{ka}{2}\sin\theta\right)}\left[i - \mathrm{tg}\left(kh\cos\theta + \frac{\pi D}{2}\sin\theta\right)\right]\sum_{\substack{m=-\infty \\ m\neq 0}}^{\infty}\Phi(m)}, \qquad (3)$$

where

$$\Phi(m) = F_1(m) + (-1)^m F_2(m).$$

In the case of normal incidence and when $2\pi h d^{-1}\sqrt{1-D^2} > 2{,}5$ formula (3) takes simpler form; in this case for the modulus/module of reflection

coefficient it is obtained

$$|R|=\frac{1}{\sqrt{(1-\Delta\,\mathrm{tg}\,kh)^2+\Delta^2}}, \qquad (4)$$

where

$$\Delta=\frac{D}{\pi^2}\left(\frac{d}{a}\right)^2\sum_{m=1}^{\infty}\frac{\sin^2(\pi m a d^{-1})}{m^2\sqrt{m^2-D^2}}.$$

The analysis of formula (4) with D^2=0.5 and a/d=0.25 shows that the maximum value of reflection coefficient is realized at the following two values of the distance between the lattices: h=1.15λ and h=1.6λ. The reflection of the wave incident to the double lattice is absent with h=1.25λ and h=1.72λ.

Diffraction of plane wave, which passed through the heterogeneous layer, on a periodically uneven surface of arbitrary form.

Yu. P. Lysanov.

Is examined the task about the diffraction of the plane harmonic wave, which falls at arbitrary angle from the uniform half-space z>H to the heterogeneous layer, limited by surfaces of z=H and $z=\zeta(x, y)$, where $\zeta(x, y)$ - periodic function x and y ($\zeta(x, y)\leqslant 0$). No limitations on the parameters of uneven surface (height/altitude, three-dimensional/space periods) are superimposed. Sole assumption lies in the fact that the function $\zeta(x, y)$ is considered piecewise-differentiated on x and y. The heterogeneous properties of medium in the layer $\zeta(x, y)<z\leqslant H$ are characterized by refractive index $\mu(z)$, that are the function only of vertical coordinate.

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Complete field in region z>H is sought in the form

$$\psi(x, y, z)=e^{i(\alpha_0 x+\beta_0 y-\gamma_{00} z)}+\sum_{n,m=-\infty}^{+\infty} A_{nm} e^{i(\alpha_n x+\beta_m y+\gamma_{nm} z)}, \qquad (1)$$

where the first term presents the incident wave, rest - diffracted field, which consists of mirror reflected wave (n=m=0) and diffracted

waves of the highest numbers. Here $(\alpha_0, \beta_0, \gamma_0)$ - the components of the wave vector of the incident wave along the coordinate axes, that satisfy condition $\alpha^2_0+\beta^2_0+\gamma^2_{00}=k^2_0$, where $k_0=\omega/C_0$ - wave number in the homogeneous medium;

$\alpha_n = \alpha_0 + nq_1;\ \beta_m = \beta_0 + mq_2;\ \gamma_{nm} = \sqrt{k_0^2 - (\alpha_n^2 + \beta_m^2)};$

$\mathrm{Im}\gamma_{nm} > 0$ with $\mathrm{Im}k_0>0$; $q_1=2\pi/L_1$; $q_2=2\pi/L_2$; L_1 and L_2 - three-dimensional/space periods of uneven surface along the axes x and y respectively.

The unknown amplitudes of diffracted waves A_{nm} are represented in the form of definite integrals of complete field $\tilde{\varphi}$ and its normal derivative $\frac{\partial\tilde{\varphi}}{\partial\nu}$ (ν - internal normal to the uneven surface) on the uneven surface.

With the help of Gauss-Ostrogradskiy's formula is obtained integral equation for complete field $\tilde{\varphi}$ and his normal derivative $\frac{\partial\tilde{\varphi}}{\partial\nu}$ on the uneven surface:

$$\int_0^{\frac{2\pi}{q_1}}\int_0^{\frac{2\pi}{q_2}} \left\{\left[\tilde{\varphi}\left(Q_{nm}^{(1)}\frac{\partial\varphi_{nm}^{(2)}}{\partial\nu} - Q_{nm}^{(2)}\frac{\partial\varphi_{nm}^{(1)}}{\partial\nu}\right) - \right.\right.$$
$$\left.\left. - \frac{\partial\tilde{\varphi}}{\partial\nu}\left(Q_{nm}^{(1)}\varphi_{nm}^{(2)} - Q_{nm}^{(2)}\varphi_{nm}^{(1)}\right)\right]\frac{1}{\nu_z}\right\}_{z=\zeta(x,y)} dxdy =$$
$$= \begin{cases} \dfrac{8\pi^2 W_{00}}{q_1 q_2} e^{-i\gamma_{00}H} & n = m = 0, \\ 0 & n \neq 0,\ m \neq 0 \end{cases} \qquad (2)$$

where

$$Q_{nm}^{(1,2)} \equiv \left(F_{nm}^{(1,2)} - \frac{1}{i\gamma_{nm}}\frac{dF_{nm}^{(1,2)}}{dz}\right)_{z=H};\ \varphi_{nm}^{(1,2)}(x, y, z) = F_{nm}^{(1,2)}(z)\, e^{-i(\alpha_n x+\beta_m y)};$$

$$W_{00} \equiv \left(F_{00}^{(1)}\frac{dF_{00}^{(2)}}{dz} - F_{00}^{(2)}\frac{dF_{00}^{(1)}}{dz}\right)_{z=0}$$

$F^{(1)}_{nm}(z)$ and $F^{(2)}_{nm}(z)$ - two linearly independent solutions of the equation

$$\frac{d^2F_{nm}}{dz^2}+[k_0^2\mu^2(z)-(\alpha_n^2+\beta_m^2)]F_{nm}=0. \qquad (3)$$

Integral equation (2) is the generalization of the equation, obtained by R. G. Barantsev, to the case when uneven surface limits heterogeneous layer.

The structure of integrand in (2) is such, that is possible the examination of the case when boundary condition is assigned in the form of linear combination $\tilde{\psi}$ and $\frac{\partial\tilde{\psi}}{\partial v}$.

In the particular case of the absolutely pliable surface when $\tilde{\psi}(x, y, \zeta(x, y))=0$, the first term under the integral in (2) vanishes.

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Transforming the function

$$\left(\frac{\partial\tilde{\psi}}{\partial v}\frac{1}{v_z}\right)_{z=\zeta(x,y)}=\sum_{k,l=-\infty}^{+\infty}B_{kl}e^{i(\alpha_k x+\beta_l y)}, \qquad (4)$$

it is obtained for determining the coefficients B_{kl} the following infinite system of the linear algebraic equations

$$\sum_{k,\,l=-\infty}^{+\infty} B_{kl} C_{nmkl} = \begin{cases} -\dfrac{8\pi^2 i V_{00}}{q_1 q_2} e^{-i\gamma_{00} H} & n = m = 0; \\ 0 & n \neq 0,\ m \neq 0, \end{cases} \tag{5}$$

where

$$C_{nmkl} \equiv \int_0^{\frac{2\pi}{q_1}} \int_0^{\frac{2\pi}{q_2}} \left(Q_{nm}^{(1)} F_{nm}^{(2)} - Q_{nm}^{(2)} F_{nm}^{(1)} \right)_{z=\zeta(x,\,y)} e^{i(k-n)x + i(l-m)y}\, dx\, dy. \tag{6}$$

Diffraction of electromagnetic wave on a lattice, comprised of thin rectangular plates (oblique incidence).

G. Sh. Kevanishvili.

Let the flat/plane periodic structure be comprised of the thin ideally conducting right-angled plates with the transverse sizes/dimensions 2a and 2b; let us allow also that the period of structure in both directions is identical and equal to d.

If to this system at angle θ falls the plane wave

$$E_1 = e^{ikx\cos\theta + iky\sin\theta} \quad (k = 2\pi/\lambda),$$

then diffraction field will take the following form:

$$E = \frac{k_1^3 j_0 d}{i4\pi^3 \omega\varepsilon} \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \Phi(n, m, \theta)\, e^{ik_1[mz + (n+D\sin\theta)y]}\, e^{-k_1 x\sqrt{m^2+n_1^2-D^2}}, \quad (1)$$

where

$$\Phi(n, m, \theta) = \frac{\sin[k_1 a(n + D\sin\theta)]}{n + D\sin\theta}\left\{\frac{\sin[k_1 a(m-D)]}{2(m-D)} + \frac{\sin[k_1 a(m+D)]}{2(m+D)} - \right.$$

$$\left. - \frac{\cos kb \sin(k_1 bm)}{m}\right\} \frac{(D^2 - m^2)}{\sqrt{m^2 + (n - D\sin\theta)^2 - D^2}};$$

$$n_1 = n - D\sin\theta, \; k_1 = 2\pi/d, \; D = d/\lambda.$$

Value j_0, which is the amplitude of the density of surface current on the plate, takes this form:

$$j_0 = \frac{i4\pi^3 b \sin(ka\sin\theta)/\eta_0 d \sin\theta}{\sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \Phi(n, m, \theta) F(n, m, \theta)}, \quad (2)$$

where

$$F(n, m, \theta) = \frac{\sin(k_1 am)}{m} \cdot \frac{\sin[2\pi bd^{-1}(n + D\sin\theta)]}{(n + D\sin\theta)};$$

$$\eta_0 = 120\pi \text{ ом}.$$

Key: (1). ohm

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Formula (1) makes it possible to determine the value of stray field in the distant zone (i.e., with $x \to \infty$), thus, for instance, with D<1 easily we find

$$E(\infty) = \frac{k^3 j_0 d}{i4\pi^2 \omega\varepsilon}[\Phi(0, 0, \theta)/iD\cos\theta]\, e^{-ikx\cos\theta + iky\sin\theta}.$$

Whence taking into account (2) for the reflection coefficient we obtain

$$R = -\frac{kb\Phi(0, 0, 0)\sin(ka\sin\theta)/\sin\theta}{iD\cos\theta \sum\limits_{n=-\infty}^{\infty}\sum\limits_{m=-\infty}^{\infty}\Phi(n, m, \theta)\, F(n, m, \theta)}. \qquad (3)$$

Let us consider the special case: $a/d \approx 1/2$, then formula (3) substantially is simplified and takes this form:

$$R = -(2b/d)\frac{\sin(\pi D\sin\theta)}{\sin(k_1 bD\sin\theta)} \quad (b \leqslant 0{,}5d).$$

Now let us assume that in (1) $d/\lambda = D > 1$, then it is possible to find such numbers N and M, that only for them $M^2 + (N - D\sin\theta)^2 - D^2 > 0$ and with $x \to \infty$ the value of field takes this form:

$$E(\infty) = -\frac{j\omega d}{4\pi^2\omega\varepsilon}\sum_{n=-N}^{N}\sum_{m=-N}^{M}\Phi_1(m, n)\, e^{i2\pi d^{-1}mz}\, e^{i2\pi md^{-1}(n + D\sin\theta)\, v},$$

where

$$\Phi_1(n, m) = \Phi(n, m, \theta)\, e^{-i2\pi d^{-1}x\sqrt{D^2 - (n - D\sin\theta)^2 - m^2}}.$$

The terms of double series are the diffraction fields of higher orders, which extend at the angle

$$\alpha_{n,m} = \arccos \sqrt{1-(n/D-\sin\theta)^2-(m/D)^2}$$

to external normal to the surface of plate.

If we observe the strong inequality $D^2\cos^2\theta \gg 1$, then after the first interference correction the value of reflection coefficient will take this form:

$$R = -\frac{i\omega\eta_0 kd\Phi(0, 0, \theta)}{4\pi^3}(1+\Delta), \tag{4}$$

where $\Delta = \Phi(0, 1, \theta)/\Phi(0, 0, \theta)$.

In the particular case when $a/d \sim {}^1/_2$, $D=4$, $b/d={}^1/_4$, formula (4) substantially is simplified and takes the form

$$R = -\frac{\cos(2\pi\sin\theta)}{1-15\frac{\sin^2\theta\cos(2\pi\sin\theta)}{(1+4\sin\theta)^2\sin(2\pi\sin\theta)}}.$$

The analysis of this formula shows that at $\theta=14°30'$ and $\theta=30°$ reflection coefficient is converted in these zero, therefore, angles correspond to the angles of complete refraction at which the lattice becomes transparent for the electromagnetic waves.

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Diffraction of electromagnetic waves on space lattice.

G. Sh. Kevanishvili.

If electromagnetic wave $E = e^{ikx}$ normally falls on the space lattice in nodes/units of which are arranged/located passive electrical radiators/resonators/elements with the final conductivity σ, then the electric intensity of the wave scattered from the lattice will take this form:

$$E = \frac{k^3 J_0}{i8\pi^3\omega\varepsilon_0} \sum_{\nu=0}^{N-1} \sum_{p=-\infty}^{\infty} \sum_{q=-\infty}^{\infty} F_1(q, l, D)\, F_2(\nu, p, q, x)\, e^{i2\pi p d^{-1} y + i2\pi q d^{-1} z}, \quad (1)$$

where N - number of lattices in the direction of axis x; I_0 - amplitude of the density of surface current along the radiator/resonator/element, moreover vector I_0 is directed in parallel to z axis;

$$F_1(q, l, D) = \frac{\sin\left[2\pi \frac{l}{d}(q - D)\right]}{q - D} + \frac{\sin\left[2\pi \frac{l}{d}(q + D)\right]}{q + D} - 2\,\frac{\cos kl \sin(2\pi q l d^{-1})}{q};$$

$$F_2(\nu, p, q, x) = \left(1 - \frac{q^2}{D^2}\right) e^{-i2\pi\lambda^{-1}\nu h - 2\pi\nu h d^{-1}\sqrt{p^2+q^2-D^2}}\, \frac{e^{-2\pi x d^{-1}\sqrt{p^2+q^2-D^2}}}{\sqrt{p^2 + q^2 - D^2}};$$

l - halflength of radiator/resonator/element; h - distance between the lattices along x axis; d - the same in directions y and z; D=d/λ; λ - wavelength; k=2π/λ.

The value of current density I_0 is given by the formula

$$I_0 = -\frac{2lE_0}{A\sum_{\nu=0}^{N-1}\sum_{p=-\infty}^{\infty}\sum_{q=-\infty}^{\infty}F_1(q,l,D)F_2(\nu,q,p,a+h)\frac{\sin(2\pi qld^{-1})}{q}+WF(kl)}. \quad (2)$$

Here $F(kl) = \frac{2}{k}(\sin kl - kl\cos kl)$, $A = k\eta_0 d/i8\pi^3$, $\eta_0 = 120\pi$ (ohm), W= [1+i) /2πa] $\sqrt{\mu\omega/2\sigma}$, moreover a and μ a radius and magnetic permeability of radiator/resonator/element, ω - angular field frequency.

If we in expression (1) complete passage to the limit, we will obtain the value of field in the distant zone

$$E(\infty) = \lim_{x\to\infty} E = -\frac{k\eta_0 I_0 F_1(0,l,D)}{8\pi^2 D}\left(\sum_{\nu=0}^{N-1} e^{-i\xi\pi\lambda-i\nu h}\right)e^{-ikx}.$$
$$(D<1)$$

The value of reflection coefficient can be determined from the relationship/ratio

$$R = \lim_{x\to\infty} E/E_1,$$

consequently, taking into account (2) we obtain

$$R = \frac{(k\eta_0 l/i8\pi^2 D)\,F(0,l,D)\sum_{\nu=0}^{N-1} e^{-i\xi\pi\nu\lambda-ih}}{A\sum_{\nu=0}^{N-1}\sum_{p=-\infty}^{\infty}\sum_{q=-\infty}^{\infty}F_1(q,l,D)F_2(\nu,p,q,a+h)\frac{\sin(2\pi qld^{-1})}{q}+WF(kl)}.$$

In special cases, namely, with D<<1, h/d>1 and l/d∼1/2, latter/last formula substantially is simplified and takes the form

$$R = \frac{2l/d}{1 - i2Df(h,N)\ln\gamma - \frac{8\pi^2 DWF(kl)f(h,N)}{k\eta_0 F_1(0,l,D)d}}.$$

Here $f(h,N) = e^{ikh(N-1)}\frac{\sin kh}{\sin khN}$, $\gamma = 1 - e^{-2\pi ad^{-1}}$.

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We give below the values of reflection coefficients under the varied conditions:

1. N=1, W=0 (radiators/resonators/elements possess ideal conductivity).

$$|R| = \frac{1}{\sqrt{1 + 4D^2 \ln^2 (1 - e^{-2\pi a d^{-1}})}};$$

2. $N = 2,\ W = 0,$

$$|R| = \frac{1}{\sqrt{(1 + D \ln \gamma \cdot \mathrm{tg}\, kh)^2 + D^2 \ln^2 \gamma}};$$

3. $N = 3,\ W = 0,$

$$|R| = \frac{1}{\sqrt{\left(1 - D_1 \frac{\sin kh \cos kh}{3 - 4\sin^2 kh}\right)^2 + D_1^2 \left(\frac{\sin^2 kh - \cos^2 kh}{3 - 4\sin^2 kh}\right)^2}},$$

where $D_1 = D \ln\gamma$;

4. $N = 4,\ W = 0,$

$$|R| = \frac{1}{\sqrt{\left[1 + \frac{D_1}{2} \frac{\sin kh\,(3 - 4\sin^2 kh)}{\cos kh\,(\cos^2 kh - \sin^2 kh)}\right]^2 + \left[\frac{D_1}{2} \frac{3 - 4\cos^2 kh}{\cos^2 kh - \sin^2 kh}\right]^2}};$$

5. $N = 1,\ W \neq 0;$

$$|R| = \frac{1}{\sqrt{2B^2 - 2B + 5}},$$

$$B = (1/{}_2 4\pi^2)\sqrt{\mu\omega/2\sigma};$$

6. $N = 2,\ W \neq 0,$

$$|R| = \frac{1}{\sqrt{\left[\left(1 + \frac{B}{2}\right) + \left(D \ln\gamma - \frac{B}{2}\right) \mathrm{tg}\, kh\right]^2 + \left[\left(D \ln\gamma - \frac{B}{2}\right) + \frac{B}{2} \mathrm{tg}\, kh\right]^2}}.$$

The formulas given above for the modulus/module of reflection coefficient easily undergo detailed analysis.

Wave diffraction on discrete periodic lattice.

I. A. Urusovskiy.

Is examined two-dimensional problem of diffraction of harmonic acoustic and electromagnetic waves on the arbitrary periodic lattice, which consists of the separate elements/cells. Cascade profile Γ is described by the equations

$$x = \xi(l), \quad z = \zeta(l), \qquad (1)$$

moreover $\xi(l+D)=d+\xi(l)$, $\zeta(l+D)=\zeta(l)$. Here x, z - Cartesian coordinates, and l - arc length, calculated lengthwise Γ to current point, d - the period of translation along the axis x, D - the complete arc length of the duct/contour of one element/cell. With $-D/2 \leqslant l-sD < D/2$ equations (1) describe the duct/contour of the s element/cell of lattice. It is assumed that the ducts/contours of adjacent elements/cells do not intersect.

In particular, for the case of the acoustically absolutely soft or absolutely conducting lattice the task about the determination of respectively acoustic or components along y axis of electric field is placed thus.

With assigned function $p_i(x, z)$ (field in the incident wave) to find continuous and limited in region $G+\Gamma$ (G - the external with respect to the elements/cells of lattice region of space) function $p_r(x, z)$ (diffracted wave), which satisfies in G region to the equation of Helmholtz $(\Delta+k^2)p_r(x, z)=0$, where k - wave number, Imk²>0, to boundary condition $p_r+p_i=0$ on Γ, on also to the normal conditions, which require the absence of sources of field in the salient points Γ, if the same are, and at infinity.

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The method of solution is based on what the kernel of integral equation to solution of which is reduced the task, that is the function of two variable/alternating, with the fixed/recorded difference in these variable/alternating is periodical in one of them. This makes it possible to reduce the solution of equation to the solution of the system of algebraic equations, which can be found with the method of reduction. Let us give here the diagram of the determination of the solution of problem.

The diffracted field is represented as expansion in terms of the quasi-plane waves

$$p_r(X, Z)=\frac{1}{4\pi}\int_{-\infty}^{\infty}\Big\{e^{i(\varkappa_x X+\varkappa_z Z)}\sum_n E_n^{+}(Z, -\varkappa_x)\Phi(nq-\varkappa_x\mu)+$$
$$+e^{i(\varkappa_x X-\varkappa_z Z)}\sum_n[D_n^{-}(-\varkappa_x)-E_n^{-}(Z, -\varkappa_x)]\Phi(nq-\varkappa_x\mu)\Big\}\frac{k}{\varkappa_z}d\varkappa_x. \quad (2)$$

Here $q = 2\pi/D,\ \mu = d/D,\ \Phi(u) = -2\sum_n \Pi(u+nq)\, X_n(u+nq)/a_0(u+nq),$

$$\Pi(u) = \int_{-\infty}^{\infty} p_i[\xi(l),\ \zeta(l)]\, e^{iul}\, dl,$$

function $X_n(u)$ - the solution of the algebraic system

$$\sum_m b_{n-m}(u-nq)\, X_m(u) = \delta_n,$$

where $\delta_0 = 1,\ \delta_{n\neq 0} = 0,\ b_n(u) = a_n(u)/a_0(u),$

$$a_n(u) = \frac{k}{2D}\int_{-D/2}^{D/2} e^{-inql}\, dl \int_{-\infty}^{\infty} H_0^{(1)}(kr)\, e^{iu\tau}\, d\tau,$$

$$r = \sqrt{[\xi(\tau+l)-\xi(l)]^2 + [\zeta(\tau+l)-\zeta(l)]^2},$$

$$E_n^{\pm}(Z, \varkappa_x) = \frac{1}{D}\int_{-D/2}^{D/2} \frac{1+\operatorname{sign}[Z-\zeta(l)]}{2}\, e^{i\{\varkappa_x[\xi(l)-\mu l] \mp \varkappa_z\zeta(l) - nql\}}\, dl,$$

$$D_n^{\pm}(\varkappa_x) = \frac{1}{D}\int_{-D/2}^{D/2} e^{i\{\varkappa_x[\xi(l)-\mu l] \mp \varkappa_z\zeta(l) - nql\}}\, dl,$$

$$\varkappa_z = \sqrt{k_2 - \varkappa_x^2};\ \mathrm{Re},\ \mathrm{Im}\,\varkappa_z > 0.$$

With Z≥max ζ(λ) and with Z≤min ζ(1) (2) it is reduced respectively to the expressions

$$p_r(X, Z) = \frac{1}{4\pi}\int_{-\infty}^{\infty} e^{i(\varkappa_x X + \varkappa_z Z)} \sum_n D_n^{+}(-\varkappa_x)\, \Phi(nq-\varkappa_x\mu)\, \frac{k}{\varkappa_z}\, d\varkappa_x$$

and

$$p_r(X, Z) = \frac{1}{4\pi}\int_{-\infty}^{\infty} e^{i(\varkappa_x X - \varkappa_z Z)} \sum_n D_n^{-}(-\varkappa_x)\, \Phi(nq-\varkappa_x\mu)\, \frac{k}{\varkappa_z}\, d\varkappa_x,$$

by that presenting the superposition of the plane waves, which exit from the lattice or which slide along it.

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In the case of the incident plane wave of form $p_i = \exp\,[i\,(k_x^0 x - k_z^0 z)]$,

$$p_r(X, Z) = \sum_m [A_m^+(Z) e^{i\left(k_x^m x + k_z^m z\right)} + A_m^-(Z) e^{i\left(k_x^m x - k_z^m z\right)}],$$

where $k_x^m = k_x^0 + m\,(2\pi/d),\ k_z^m = \sqrt{k^2 - (k_x^m)^2},$

$$A_m^+(Z) = \frac{k}{2\mu k_z^m} \sum_n v_{m-n}(k_x^0)\, E_n^+(Z, -k_x^m),$$

$$A_m^-(Z) = \frac{k}{2\mu k_z^m} \sum_n v_{m-n}(k_x^0)\, [D_n^-(-k_x^m) - E_n^-(Z, -k_x^m)],$$

but functions $v_n(k_x^0)$ are the solution of the algebraic system

$$\sum_m a_{n-m}(-k_x^n \mu)\, v_m(k_x^0) \doteq -2D_n^+(k_x^0).$$

With $Z \geqslant \max \zeta(l)$ and $Z < \min \zeta(l)$ $A_m^\pm$ do not depend on Z:

$$A_m^+ = \frac{k}{2\mu k_z^m} \sum_n v_{m-n}(k_x^0)\, D_n^+(-k_x^m),\ A_m^- = 0 \overset{(1)}{\text{при}}\ Z > \max \zeta(l),$$

$$A_m^+ = 0,\ A_m^- = \frac{k}{2\mu k_z^m} \sum_n v_{m-n}(k_x^0)\, D_n^-(-k_x^m) \overset{(1)}{\text{при}}\ Z < \min \zeta(l).$$

Key: (1). when.

The proposed theory is valid in the case when the elements/cells of lattice close border on each other, forming the uneven surface whose profile/airfoil here is conveniently taken as Γ.

Are investigated the properties of solution and are examined

some special cases.

The obtained results directly are transferred to the case of layered inhomogeneous medium with the refractive index, which depends on z. For this function $H^{(1)}_0(kr)$ should be replaced Green's function of the corresponding inhomogeneous medium.

8. Inverse problems.

Synthesis of antenna with flat aperture.

Ye. G. Zelkin.

The task of the synthesis of antenna with the flat/plane aperture can be formulated thus: it is necessary to determine the law of field distribution according to antenna aperture and the form of aperture, which will ensure obtaining the assigned field **E** in the distant zone, polarized according to any assigned law.

Recently both in the Soviet and in the foreign press appeared a large quantity of articles, dedicated to the solution of this problem. Let us pause at the general/common/total results, obtained during the development of the theory of the synthesis of antenna with the flat/plane aperture.

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As is known, field in distant zone $E(\theta, \psi)$, created by antenna with the flat/plane aperture, it is possible to represent in the form

$$\mathbf{E} = -\frac{k}{4\pi i} \cdot \frac{e^{-ikR}}{R} [\mathbf{i}_R [\mathbf{p}\mathbf{N}]], \tag{1}$$

where

$$\mathbf{N} = \iint_S \mathbf{F} e^{ik(x \sin\theta \cos\psi + y \sin\theta \sin\psi)} dS, \quad \mathbf{p} = \mathbf{n} - \mathbf{i}_R \tag{2}$$

- tangential field component in antenna aperture, S - the surface of aperture, $\mathbf{i}_R$, θ, ψ - unit vector and the quardants of direction to observation point, $\mathbf{n}$ - the unit vector of normal to the aperture, x, y - the coordinate of the point of aperture. (Expression (1) it is obtained from the formula of Huygens-Kirchhoff's type for the field in the distant zone. In this case the antenna was certain ideally conducting surface with flat/plane radiating aperture. Formula approximated, since during her conclusion/output it was assumed that the tangential components of both electrical and magnetic of vectors on the metallic surface were equal to zero. However, it gives sufficiently good approximation/approach for large-size apertures, i.e., for comparatively pencil-beam antennas.).

Expression in the brackets is the radiation pattern of flat/plane aperture. Let us designate it through **D**.

The vector function **D** is assigned, the unknown function is **F**.

By introduction to the distant zone of the coordinate system with unit vectors $\mathbf{q}_1$, $\mathbf{q}_2$ and $\mathbf{q}_3$ [1], following formulas connected

with the unit vectors of spherical coordinates $\mathbf{i}_R, \mathbf{i}_\theta, \mathbf{i}_\psi$ by the following formulas

$$\mathbf{q}_1 = \cos\psi\mathbf{i}_\theta - \sin\psi\mathbf{i}_\psi,\ \mathbf{q}_2 = \sin\psi\mathbf{i}_\theta + \cos\psi\mathbf{i}_\psi,\ \mathbf{q}_3 = \mathbf{i}_R,$$

equation (1) is reduced to the form

$$\mathbf{D} = (1 + \cos\theta)\,(N_x\mathbf{q}_1 + N_y\mathbf{q}_2), \qquad (3)$$

where

$$N_x = \iint_S F_x e^{ik\sin\theta\,(x\cos\psi + y\sin\psi)}dS;$$
$$N_y = \iint_S F_y e^{ik\sin\theta\,(x\cos\psi + y\sin\psi)}dS. \qquad (4)$$

Equations (4) been independent. Solving each of them, let us find F_x and F_y - field component in the aperture and the form of aperture. Thus, we reduced the problem of the synthesis of flat antenna with the arbitrary polarization of radiation/emission to the independent even simpler tasks of the synthesis of two flat/plane systems with the plane-polarized fields.

Let us designate $u = \frac{l}{\lambda}\sin\theta\cos\psi,\ v = k\sin\theta\sin\psi,\ \frac{2\pi}{l}x = \mu,\ N(\theta, \psi) = R(u, v),\ 2\pi lF(x, y) = f(\mu, y).$

Then each of equations (4) can be represented in the form

$$R(u, v) = \frac{1}{(2\pi)^2}\int_{-\pi}^{\pi}\int_{y_1(\mu)}^{y_2(\mu)} f(\mu, y)\, e^{i(u\mu + vy)}\, dy\, d\mu, \qquad (5)$$

where $y_1(\mu)$ and $y_2(\mu)$ the curves, which limit aperture.

In this case solution for component f_x we will obtain [1] in the form

$$f_x(\mu, y) = \int_{-\infty}^{\infty} B(\mu, v)\, e^{-iyv}\, dv,$$

where (6)

$$B(\mu, v) = \int_{-\infty}^{\infty} R_x(u, v)\, e^{-iu\mu}\, du.$$

Analogously we find and $f_y(\mu, y)$.

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Antenna aperture coincides with the region, limited by curves $y_2(\mu)$ and $y_1(\mu)$, which are determined from the formulas

$$y_1(\mu) = \lim_{r\to\infty} \frac{\ln|B(\mu, re^{i\frac{\pi}{2}})|}{r}, \qquad y_2(\mu) = \lim_{r\to\infty} \frac{\ln|B(\mu, re^{-i\frac{\pi}{2}})|}{r}. \qquad (7)$$

In work [1] it is shown that so that there would be the exact solution of the task of the synthesis of flat/plane aperture, it is necessary that the components N_x and N_y in the function of the variable/alternating u and v would belong to class W_0 on each of the variable/alternating u and v.

The solution of equations (4) can be also obtained, using, as this is shown in [2], by the theorem of Plancherel and Poy, for the case of two variable/alternating. In this case

$$f_x(\mu, y) = \iint_{-\infty}^{\infty} R_x(u, v)\, e^{-i(\mu u + yv)}\, du\, dv,$$

$$f_y(\mu, y) = \iint_{-\infty}^{\infty} R_y(u, v)\, e^{-i(\mu u + yv)}\, du\, dv.$$

The form of aperture is determined P - by indicator of function R_x and R_y.

Solution (4) can be represented in the form of the series/row

$$f(\mu, y) = \frac{2\pi}{y_2 - y_1} \sum_{-\infty}^{\infty} B\left(u, \frac{2\pi}{y_2 - y_1}, n\right) e^{-in\frac{2\pi}{y_2-y_1}y},$$

where y_1, y_2 and B are determined from formulas (6) and (7).

Somewhat in more detail in the literature are examined the tasks of the synthesis of antenna with the circular aperture.

In work [3] it is proposed flat/plane circular aperture to replace with equivalent linear antenna. For the circular aperture with the axisymmetric radiation pattern, equation (4) they will take the form

$$R(u) = \frac{1}{2\pi} \int_0^1 rf(r) I_0(ur)\, dr. \qquad (8)$$

Since

$$I_0(ur) = \frac{2}{\pi} \int_{-1}^{1} \frac{\cos ury}{\sqrt{1-y^2}}\, dy,$$

that

$$R(u) = \frac{1}{\pi} \int_{-1}^{1} A(y)\, e^{+iuy}\, dy, \qquad (9)$$

where

$$A(y) = \int_y^1 \frac{rf(r)}{\sqrt{r^2-y^2}}\, dr. \qquad (10)$$

Equation (9) is the equation of line-source antenna; the methods of

its solution detailed.

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Equation (10) is the integral equation of Abel. Its solution takes the form

$$f(r)=\frac{2}{\pi}\left[\frac{A(1)}{\sqrt{1-r^2}}-\int_0^1\frac{A(y)\,dy}{\sqrt{y^2-r^2}}\right].$$

Equation has a solution in the only case, if R(u) the integral function of final degree, which satisfies inequality $\int|R(u)|^2\,udu<\infty$, i.e. R(u) must decrease at infinity $\sqrt{u}$ once more rapid than the radiation pattern of linear emitter.

The solution of equation (8) can be still obtained with the help of the transformation of Hankel

$$f(r)=\int_0^\infty uR(u)\,I_0(ur)\,du,$$

and also in the form of the series/row

$$f(r)=\sum_{k=1}^{\infty}\frac{R(\mu_{k,1})}{I_0(\mu_{k,1})}\,I_0(\mu_{k,1},r),$$

where $\mu_{k,1}$ - k root of Bessel function $I_1(x)$.

If the assigned diagram is function of both of the variable/alternating U and ψ:

$$R(u,\psi)=\frac{1}{2\pi}\int_0^1\int_0^{2\pi} rf(r,\varphi)\,e^{iur\cos(\psi,\varphi)}\,d\varphi\,dr,$$

then the function of field distribution according to aperture $f(r,\varphi)$

can be determined according to the formula

$$f(r, \varphi) = \int_0^{\infty}\int_0^{2\pi} uR(u, \psi)\, e^{-iru\cos(\psi-\varphi)}\, d\psi\, du,$$

or is represented in the form of Fourier series

$$f(r, \varphi) = \sum_{-\infty}^{\infty} a_n(r)\, e^{in\varphi},$$

where

$$a_n(r) = \frac{1}{2\pi}\int_0^{\infty} b_n(u)\, I_n(ru)\, u\, du = 2\sum_{k=1}^{\infty} \frac{b_n(\mu_{k,\, n+1})}{I_n^2(\mu_{k,\, n+1})} I_n(\mu_{k,\, n+1} r)$$

$$b_n(u) = \frac{i^{-n}}{2\pi}\int_0^{2\pi} R(u, \psi)\, e^{-in\psi}\, d\psi,$$

$\mu_{k,n+1} - k$ - the root of function $I_{n+1}(x)$.

A special case of the task of the synthesis of flat/plane aperture is the task of the synthesis of the curvilinear emitter, which lies on the plane. If emitter has a form of certain line y=L(x), then equations (4) can be will be registered in the form:

$$N_x = \int_{-l/2}^{l/2} F_x(x)\sqrt{1 + L'^2(x)}\, e^{ik\sin\theta\,(x\cos\psi + L(x)\sin\psi)}\, dx,$$

$$N_y = \int_{-l/2}^{l/2} F_y(x)\sqrt{1 + L'^2(x)}\, e^{ik\sin\theta\,(x\cos\psi + L(x)\sin\psi)}\, dx.$$

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These equations are not independent, since in them enters one and the same unknown function L(x). Consequently, functions N_x and N_y cannot be assigned arbitrarily. Assuming/setting $\frac{l}{\lambda}\sin\theta\cos\psi = u$, $\frac{l}{\lambda}\sin\theta\sin\psi = v$, $\frac{2\pi}{l}x = \mu$, $\frac{2\pi}{l}L(x) = r(\mu)$, $F(a)\sqrt{1+L'^2(a)} = f(x)$, $N(\theta, \psi) = R(n, v)$,

it is possible each of equations (11) to reduce to the form

$$R(u, v) = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(\mu)\, e^{i(vr(\mu)+u\mu)}\, d\mu.$$

Hence

$$f(\mu) = \int_{-\infty}^{\infty} R(u, 0)\, e^{-iu\mu}\, du,$$

$$f(\mu)\, r(\mu) = \int_{-\infty}^{\infty} \frac{\partial}{\partial v} R(u, v)\big|_{v=0}\, e^{-iu\mu}\, du.$$

Solution exists only in such a case, when:

1) $R(u, v) \in W_\pi$ separately on each of the variable/alternating u and v;

2) R(u, v) it satisfies the equation

$$\frac{\partial}{\partial v} R(u, v) = i \int_{-\infty}^{\infty} A(u - x)\, R(x, v)\, dx,$$

where A(x) - certain function of class W_π.

The condition of consistency of both equations (11) is the equality

$$R_y(u, v) = \int_{-\infty}^{\infty} G(u - x)\, R_x(x, v)\, dx,$$

where

$$G(u) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \frac{f_y(\mu)}{f_x(\mu)}\, e^{-iu\mu}\, d\mu.$$

With the solution of the problem of the synthesis of curvilinear emitter it is necessary to have assigned one of the functions $R_x(u, v)$ (or $R_y(u, v)$) and the first member of the expansion in Taylor series of the second function.

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Determination of the spectrum of particles with the help of inversion of data on the diffusion of light.

K. S. Shifrin, A. Ya. Perel'man.

1. Common survey/coverage of problem. Formulation of the complete and limited task.

2. Cases for which complete task has analytical solution:

a) method of small angles

b) inversion of spectral transmittance

c) inversion of indicatrices for "soft" particles.

3. Determination of spectrum of particles from method of small angles (method a).

Method the inversions of integral transparency and indicatrix for the "soft" particles (methods b and c) are based on the conversion of Mellin. They do not contain any arbitrary assumptions about the character of the spectrum of particles and give the response/answer in general form. Is proposed simple network, are given examples of inversion. Are evaluated the field of measurements by wavelengths (to scattering angle) and the accuracy of the measurements of the transparency (indicatrix), necessary for the reliable determination of the spectrum of particles. It is shown that network is stable with respect to the well-aimed measuring errors and calculations and contains the moderate requirements for their accuracy.

Analysis and synthesis of structures with variable surface impedance.

A. F. Chaplin.

Structure with the variable/alternating surface impedance is used into the antenna to technology as the antennas and the decouplers. These structures usually are inscribed in the metallic surface of the specific form. For calculating electromagnetic field, created by this system, it is necessary to consider the radiation/emission of the exciting source, the reflection effect of impedance structure, edge effects on the boundary of impedance structure and metallic surface, and also effect of metallic surface. This it is possible to attain by the method of solution of the electrodynamic task about the excitation of the ideally conducting surface, into which is inscribed the structure with the surface impedance variable/alternating along one of the coordinates. For the purpose of simplification is examined the two-dimensional task: field distribution is considered independent of the coordinate, transverse to the direction of a change in the impedance. Is comprised the integral equation of Fredholm of the second order relative to the functions of the distribution of electrical and magnetic currents on the impedance structure. Cases of TE and TM waves are examined separately. Kernel of integral equation is a product of the function

of the distribution of impedance to Green's function, which satisfies the conditions for radiation/emission at infinity and boundary conditions on the ideally conducting surface. Using known expressions for Green's two-dimensional functions, it is possible to obtain integral equations for the band with the variable/alternating impedance, which lies on the ideally conducting plane or on the face of the ideally conducting wedge; for the disk with the impedance variable/alternating for a radius, inscribed into the ideally conducting plane, for the ring inscribed into the ideally conducting cylinder with the variable/alternating of lengthwise generatrix by impedance, for the segment of the ideally conducting sphere, which has variable/alternating at the meridional direction impedance, etc. It is established/installed, that the simplest and universal method of solving such integral equations is Krylov- Bogolyubov's method. The use/application of this method is demonstrated based on the example of the impedance band, inscribed into the ideally conducting plane.

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Are generalized the results of the analysis of this model on the basis of numerous calculations for different laws of a change in the reactive/jet impedance across the band. The obtained integral equations can be used for the synthesis of impedance structures,

which consists of the determination of the law of a change in the impedance in the specified distribution of electrical or magnetic currents. With the solution of the problem of the synthesis of real systems it is necessary to ensure the pure/clean reactivity of surface impedance. Satisfaction of this condition leads to the redefinition/redetermination of the system of real integral equations relative to the function of the distribution of surface reactance. This redetermined/redefined system is solved by the method of least squares, as a result of which is found the law of a change in the reactance along the surface, which ensures the best root-mean-square approximation to the assigned current distribution with the fixed/recorded outside currents. A more precise approximation of the assigned current distribution is possible during the proper selection of outside currents. Are given the results of calculations according to the method of synthesis proposed for the impedance band, inscribed into the ideally conducting plane.

Wave dissipation by inhomogeneous plasma.

Yu. N. Dnestrovkiy, D. P. Kostomarov.

In the approximation/approach of geometric optic/optics is examined the task of scattering the electromagnetic waves by inhomogeneous plasma. Is placed the inverse problem: to determine the distribution of plasma according to the results of scattering at the different frequencies.

Task is reduced to the integral equation of Volterra of the first order. Are studied the analytical and numerical methods of solving this equation.

Reduction of the form of region by the scattering amplitude.

A. G. Ramm.

1. Is examined task of diffraction of plane wave on surface G of limited region D with Lyapunov boundary in three-dimensional space

$$\Delta u + k_1^2 u = 0 \quad {}^{(1)}\text{в } D,$$
$$\Delta u + k_2^2 u = 0 \quad {}^{(2)}\text{вне } D, \qquad (1)$$

$$u^+ = u^-, \quad \left(\frac{\partial u}{\partial n}\right)^+ = \left(\frac{\partial u}{\partial n}\right)^- \qquad (2)$$

Key: (1). in. (2). outside.

(marks +, they indicate the limiting values of functions respectively from within, also, outside the boundary G);

$$u = e^{ik_2(n,\,x)} + v, \quad n = \frac{\mathbf{k}_1}{k_2}, \qquad (3)$$

where

$$\int\limits_{|x|=R} \left|\frac{\partial v}{\partial n} - ik_2 v\right|^2 dS \underset{R\to\infty}{\longrightarrow} 0. \qquad (4)$$

Let us determine scattering amplitude by the formula

$$f(k_1, k_2, \mathbf{n}, \mathbf{v}) = \lim_{|x|\to\infty \ \arg x = \mathbf{v}} -4\pi |x| e^{-ik_2|x|} v(x, \mathbf{n}, k_1, k_2). \qquad (5)$$

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Using methods of the study of the smoothness of the spectral function of Schroedinger's operator of the spectral parameter and

uniqueness theorems of the theory of the analytic functions of many complex variables, it is possible to demonstrate the theorem:

Theorem 1. If scattering amplitude is known for all unit vectors n, ν, with that fixed/recorded k_1 and k_2, that varies in how it is convenient to the small vicinity k_1, then by these data *form and* position of the scattering region D are uniquely determined.

2. Examining task about scattering on potential of surface, it is possible to demonstrate that scattering amplitude uniquely determines potential and shape of surface.

3. During the study of properties of smoothness of scattering amplitude on energy prove to be useful following theorems:

Theorem 2. A(p) - the locked linear operator (limitedness is not predicted), which functions from B_1 and B_2, where B_1 and B_2 - banach spaces. Let $A^{-1}(p)$ be determined and continuous on B_2 with any fixed/recorded $p \in D$, where D - closed set on the plane complex variable p. The solution of the equation

$$A(p)x = f(p) \qquad (6)$$

will be continuous from the parameter

$$\|x(p+\Delta p) - x(p)\|_{B_1} \xrightarrow[|\Delta p| \to 0]{} 0 \qquad (7)$$

it is even relative to f(p), space B_2 varying on the limited subset,

if

$$\|[A(p+\Delta p)-A(p)]x\|_{B_1} \xrightarrow[|\Delta p|\to 0]{} 0 \qquad (8)$$

it is even relative to x, space B_1, and

$$\|f(p+\Delta p)-f(p)\|_{B_1} \xrightarrow[|\Delta p|\to 0]{} 0. \qquad (9)$$

varying on the limited subset. We assume that the region of the assignment of operator A does not depend on p.

Theorem 3. A(p) - nonlinear operator from B_1 and B_2, who has reverse/inverse, determined on everything B_2 continuous with fixed/recorded p, moreover

$$\|A^{-1}(p)f\|_{B_1} < \Phi(\|f\|_{B_2}), \qquad (10)$$

where $\Phi(u)>0$ continuous function. If are satisfied conditions (8), (9), then occurs conclusion (7) of theorem 2.

The possibility of the optimization of radiation characteristics from waveguide.

M. V. Persikov, A. N. Sivov, I. P. Komik.

1. Is analyzed possibility to improve radiation characteristics from open end/lead of waveguide, selecting proper superposition of waveguide waves. Is examined flat/plane waveguide. Task is placed thus. The open end/lead of the waveguide is approached N of the extending waves, which have cophasal density distribution of current on the opposite plates. It is necessary to find the amplitude ratio of these waves, which ensures the maximum radiant energy in the preset angle 2δ, referred to the energy, applied by N by incident waves.

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This, obviously, corresponds to optimum reflection losses and to the radiation/emission out of the preset angle. It is necessary to also explain the behavior of optimum losses and radiation pattern with a change in number N of the waves considered.

2. In wave zone total field is represented in the form of superposition of radiation fields of waves, which arrive to open end/lead: $u=\sum_{n}^{N} u_n$, where $u_n = z_n \cdot R(r) \cdot \Phi_n(\varphi)$, z_n - amplitude (complex) of n wave. Carrying expression for flow P of energy in the preset angle 2δ to input energy Σ, we obtain

$$\frac{P}{\Sigma} = \frac{A(z,z)}{B(z,z)}, \quad A(z,z) = \sum_{nm}^{N} z_n z_m^* a_{nm}, \quad B(z,z) = \sum_{n}^{N} z_n z_n^* b_n, \qquad (1)$$

$$a_{nm} = \int_{\pi-\delta}^{\pi+\delta} \Phi_n \Phi_m^* d\varphi, \quad b_n = h_n d, \quad h_n = \sqrt{k^2 - \alpha_n^2},$$

d=2a - distance between the plates, $k=2\pi/\lambda$. The requirement of the extremality of relation (1) on phase z_n and possibility with sufficiently large kd to consider it of value Φ_n real leads to the replacement of hermitian quadratic form in numerator (1) of real. Mathematical task is reduced to the determination of characteristic numbers λ_k of the regular beam of quadratic forms, i.e., to finding root λ of equation $|A-\lambda B|=0$. If $\lambda_m = \max \lambda_k$, then λ_m are a maximum of the relation of quadratic forms, and the components of corresponding λ_m main vector $X^m(X_1^m, \ldots, X_N^m)$, $(A-\lambda_m B)X^m = 0$, the essence of wave amplitude, that realize the optimum losses: $1-\lambda_m$ $\left(1-\lambda_m = \min \frac{\bar{P}+\Sigma_R}{\Sigma}, \bar{P}\right.$ - the energy, emitted out of the preset angle, Σ_R - reflection loss).

3. By ETsVM [digital computer] VTs MGU is at the present time produced calculation for case of incidence/drop on discoveries/openings end/lead of waves of type $H_{0,2m-1}$. Calculation is carried out for set of parameters δ, N, ka. The results of

calculations make it possible to do the following conclusions:

a) with insufficiently large $C=ka\cdot\delta$ (approximately/exemplarily to $C\sim 4$) dependence $1-\lambda_m$ on number N of the mixed waves is expressed weakly;

b) with increase of C this dependence sharply grows; so with

$$C=8\ \ \frac{1-\lambda_m(N=1)}{1-\lambda_m(N=3)}=5\cdot 10^{-2};$$

c) however even with large C with increase of N rapidly begins the effect of "saturation" and further increase N is virtually unchanged losses. This independence begins, as a rule, already with N>3. In other words, it is sufficient two-three waves in order to obtain the losses, very close to the radiation losses out of this angle of the antenna, which has in this sense optimum field distribution in the aperture. It is assumed, of course, that this antenna does not have property of "superdirectionality";

d) the analysis of radiation pattern shows that basic part of the losses compose the reflection losses.

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9. Mixed problems in the region with the moving boundary.

Method of solving mixed problems with the changing boundary for the three-dimensional wave equation and its application.

Ye. A. Krasil'shchikova.

Is stated the method of solving mixed problems for the three-dimensional wave equation when the boundary conditions of task, and also the boundary of the region of the assignment of boundary conditions are changed in the time.

By means of the continued method is investigated the spatial problem about the flow around of the wing of the supersonic flow in the presence of moving shock wave.

Is examined the motion of the thin slightly bent/slightly curved wing with the low angle of attack. It is assumed that the motion of wing consists of rectilinear forward motion with the constant supersonic speed and occurs within the infinite volume of the compressible medium. At certain moment of time the wing meets the

weak shock wave whose front is the plane, which moves with the speed of sound.

Considering that the environmental disturbances, excited by the motion of wing and shock wave incoming to it, are small, are made the simplifying assumptions generally accepted in the slender-wing theory, and the task about the flow around of the wing in the presence of shock wave is examined in the linearized setting.

The task of hydrodynamics is reduced to the boundary-value problem for the three-dimensional wave equation when boundary of the region where are assigned boundary conditions, moves over the assigned law.

The solution of problem is obtained in the locked form, and the velocity potential of disturbed flow is found in the form of double quadratures with the region of integration, spread over the wing surface. Velocity potential can be calculated for the wing of arbitrary geometric form with the supersonic edges in the presence of moving shock wave whose front forms arbitrary angle with the plane of the motion of wing.

Wave diffraction on the sphere with varying in time radius.

V. N. Krasil'nikov.

The phenomena, which appear with the wave diffraction on the bodies, form and size/dimension of which vary in the course of time, should be classed as parametric ones. They, generally speaking, are accompanied by the effects of a change in the spectral composition and supply of energy of the field, undergoing diffraction. There is nothing similar in the classical tasks, which examine interaction of waves with the quiescent bodies and the boundaries.

Obvious that the resolution of the tasks of parametric diffraction (condition about this shorter term) is more complicated mathematical problem; therefore at present there are no such solutions, actually, still. In this report is examined, perhaps, the simplest diffraction task of such type. It is placed thus.

The wave field u, which exists in the region out of the sphere with a radius of a(t)arbitrarily varying in the time, is subordinated to the equation:

$$\nabla^2 u - \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} = -f. \qquad (1)$$

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Particular solution (1) with the assigned source function f(r, t)

$$u_0 = \frac{1}{4\pi}\int \frac{f\left(\mathbf{r}, t - \frac{|r|}{c}\right)}{r}\, dV \qquad (2)$$

makes sense of the feeding wave. On the surface of sphere the field satisfies certain linear boundary condition of impedance type:

$$\alpha \frac{\partial U}{\partial r} + \beta U = 0\,|_{r=a(t)}, \qquad (3)$$

where α and β can be the functions of time (in particular, they can be determined by the speed of the motion of the boundary da/dt). At infinity field u is subordinated to usual radiation principle (if necessary in the modified formulation).

Since in the setting indicated the task remains linear, in the principle sufficiently bounding to the case of monochromatic incident field (2).

For the construction of solution should be used an expansion of the unknown field u_1 and the incident field u_0 in the series/rows in spherical functions $Y_n(\theta, \varphi)$:

$$u_0 = e^{-i\omega t}\sum_{n=0}^{\infty} a_n j_n(kr)\, Y_n(\theta, \varphi) \qquad r < r_0 \qquad (2')$$

$$u_1 = \sum_{n=0}^{\infty} u_n(rt)\, Y_n(\theta, \varphi). \qquad (4)$$

Moreover for u_0 (monochromatic field with the known sources,

arranged/located out of the sphere of certain radius r_0) coefficients a_n are known, and $j_n(kr)$ - spherical Bessel function.

However, the coefficients $u_n(rt)$ of series/row (4) depend on time by more complicated form. With the help of the special conversion the task of finding the function $u_n(rt)$ is reduced to the integration of the linear differential equation of the n order with the variable coefficients, which escape/ensues from boundary condition (3).

We will not give here the form of this equation, but let us only point out that in special cases (defining concretely the law of a change in the radius of sphere) it is possible to analytically investigate the properties of the obtained solution.

Task of diffraction in a half-space with the moving boundary.

V. A. Khromov.

Is sought the solution of the nonhomogeneous wave equation

$$\Delta\varphi - \frac{1}{c^2}\frac{\partial^2\varphi}{\partial t^2} = -4\pi g(x, y, z, t) \qquad (1)$$

in region $z > z_s(t)$ for the case when on moving/driving plane $z = z_s(t)$ becomes zero functions φ or its normal derivative. It is assumed that for the moving/driving plane is satisfied the condition

$$\left|\frac{\partial z_s(t)}{\partial t}\right| < C. \qquad (2)$$

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The solution of problem in this case is reduced to finding of the auxiliary function G (Green's function) in region $z > z_s(t)$, of that satisfying the homogeneous wave equation

$$\Delta G - \frac{1}{c^2}\frac{\partial^2 G}{\partial t^2} = 0, \qquad (3)$$

and when $z = z_s(t)$ to the same boundary conditions as function φ ($G_s = 0$ or $\frac{\partial G_s}{\partial n} = 0$). In the particular case is given the expression of the field, which appears with the reflection of monochromatic spherical wave from the plane, which moves with a constant velocity of $\frac{\partial z_s(t)}{\partial t} = v$.

Are considered some other cases of interaction of field with the

moving/driving plane, in particular the radiation/emission of the system, which consists of the oscillating plane and the placed near it stationary source or the flow/discharge.

10. Variation and direct methods.

Proof of the methods of the study of the propagation of electromagnetic vibrations in the irregular waveguides.

A. G. Sveshnikov.

Are developed the direct methods of the solution of the problems about the propagation of oscillations/vibrations in the waveguides with the extended irregular section, with the arbitrary anisotropic filling and the lateral surface of fairly complicated form. These methods must be suitable both for qualitative research of the general/common/total properties of phenomenon and for solving the specific problems with the high accuracy. The latter is achieved by the development of the general/common/total algorithms of solution, which use solution of problem on the electronic computers.

The basis of the method of the construction of approximate solution is reducing of initial electrodynamic task to boundary-value problem for the final system of ordinary differential equations for the method, analogous to the Galerkin method. Integral relationships/ratios, from which is determined approximate solution in this method, are constructed in such a way that approximate

solution would satisfy the same basic energy relationship/ratio, as the exact solution of task. This makes it possible to demonstrate that approximate solution constructed thus with an increase in the order of the system of ordinary differential equations converges in mean to the exact solution of initial task when the latter belongs to space W'_2. In this case it is proved that the coefficient of reflection and the passages of normal waves, obtained from approximate solution, approach their precise values. The proof of the convergence of approximate solution to precise is conducted first for the case when the medium, which fills waveguide, possesses different from zero absorptions. Transition to the case of the absence of absorption is realized with the help of the principle of maximum absorption.

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During the study of the propagation of oscillations/vibrations in the waveguides with the irregular lateral surface to the transition to the system of ordinary differential equations via the selection of the special system of curvilinear coordinates, the determined form of the lateral surface of the waveguide in question, is produced the representation of the initial region of the solution of problem onto regular tube domain of constant cross section. Thereby task is reduced to the equivalent task about the propagation

of oscillations/vibrations in the rectilinear waveguide of constant cross section, but with the anisotropic filling.

The proof of direct methods for the internal tasks of diffraction.

V. V. Nikol'skiy.

The tasks about electromagnetic field in the region of space, by the partially or completely limited conducting surface (in particular, complex form) and containing the inhomogeneous medium (which can be piecewise-discontinuous, and also anisotropic, etc.), according to the character of the wave process in question can be attributed to diffraction. To this group belong, in particular, the tasks about the so-called waveguide transformers - hollow systems, connected with several waveguide channels, characterized by scattering matrix.

In connection with the development of computational technology in recent years substantially increased the value of direct methods in application to such internal tasks. However, in this case in the methods of Galerkin and Ritz it makes sense as the coordinate functions to use systems of its own vector functions, obtained as the solutions of electromagnetic boundary-value problems, i.e., those few from them, for which are known analytical solutions. From certain point of view the development of direct methods looks like the analysis of the effect of irregular factors (complicated boundary,

heterogeneous anisotropic medium, etc.) in electromagnetic field in the computational aspect playing the role of base. This also draws together data of task with the diffraction problem.

It shows that with the approach in question the identical results are obtained as by the method of Galerkin in application to the equations of Maxwell with their boundary conditions (or to the equations of higher order), so also by the method of Ritz during the use of equivalent variation principle. Thus, for instance, if we formulate task relative to Maxwell's operator

$$Mu = \omega\pi u + j,$$

where

$$M = i\begin{pmatrix} 0- & \mathrm{rot} \\ \mathrm{rot} & 0 \end{pmatrix},$$

$$\pi = \begin{pmatrix} \varepsilon & 0 \\ 0 & \mu \end{pmatrix}, \quad u = \begin{pmatrix} \mathbf{E} \\ \mathbf{H} \end{pmatrix} \overset{(1)}{\text{и}} \quad j = i\begin{pmatrix} j \\ 0 \end{pmatrix},$$

Key: (1). and.

that one of the possible functionals, stationary during its solutions, will take the form

$$\Phi = (Mu, \widetilde{u}) - \omega(\pi u, \widetilde{u}) - (u, j) - (j, \widetilde{u}) - i\int_{S_0} [\mathbf{E}, \widetilde{\mathbf{H}}^*]\,\mathbf{n}ds - i\int_{S_\Sigma} [\widetilde{\mathbf{E}}^*, \mathbf{H}]\,\mathbf{n}ds$$

(scalar product it is understood by the volume S_Σ - surface, on which is assigned outside field, S_0 - remaining part of the locked surface). The boundary conditions of task for this functional are

natural, which makes it possible to take the coordinate functions, which do not satisfy them.

Direct methods for the internal tasks of electrodynamics were examined and were applied in the series/row of the previous work of the lecturer and his colleagues.

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Besides some generalizations, the target of this report is the examination of the proofs of these methods. Let us note that recently the beginning to similar efforts/forces was established in A. G. Sveshnikov's doctoral dissertation (MGU, 1963), who established/installed the convergence of the process used by it of bringing the task about the excitation of irregular waveguide to the system of ordinary differential equations. However, in the report is made the attempt to indicate similar results with respect to the methods of Galerkin and Ritz, which were discussed above. Besides the conclusions about the convergence, obtained without the use/application of principle of maximum absorption, apparently, it proves to be possible to raise a question about the initial proof of the Galerkin method, which does not use properties of the zero element/cell of the function space. The latter fact can play structural/design role.

Application of direct methods for calculating the irregular waveguide systems.

D. I. Korniyenko, V. P. Orlov, V. G. Feoktistov.

Large use in shf technology find different plumbing with the use of the ferrites: waveguides with the ferrite, resonators with the ferrite, etc.

From a mathematical point of view the problem is reduced to the resolution of boundary-value problem for the equation of Maxwell, moreover ε and μ - tensors whose separate components - the discontinuous functions of coordinates. With rare exception such tasks (let us name their irregular) do not have analytical solutions; therefore find use the direct methods of solution.

Present report is dedicated to the use/application of direct methods of Galerkin-Ritz's type for the numerical solution of different waveguide tasks: a) the determination of the scattering matrix of X-circulator; b) the calculation of the propagation constants of rectangular waveguide with arbitrarily arranged/located transverse-magnetized ferrite; c) the calculation of the propagation constant of H-shaped waveguide with longitudinal-magnetized ferrite.

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Calculations were produced in electronic computers.

a) X-circulator. In this case is solved the boundary-value problem for the rectangular region, connected with four waveguides and containing the magnetized ferrite cylinder. Device/equipment is completely characterized by its scattering matrix for determination of which Galerkin-Ritz's method is formulated in the inductions. On the diagram of galerkin or Ritz with the coordinate functions, undertaken in the form of the eigenfunctions of the rectangular region indicated, the task is reduced to the system of algebraic equations.

Those obtained during its solution of the representation of fields are projected/designed in the entrances of waveguides, which gives the matrix elements of conductivity, from which then is located the scattering matrix. Calculation was produced according to 52 coordinate functions with the complex permeability. Programming task and calculations are carried out in VTs MGU by I. P. Kotik.

b) Waveguide with the transverse magnetic biasing. For calculating the propagation constants of waveguide with the ferrite the method is used to the equations of Maxwell for the inductions, registered in the waveguide form:

$$\left.\begin{array}{l}\mathrm{rot}_{\perp}\varepsilon^{-1}\mathbf{D} + i\nu\,[\varepsilon^{-1}\mathbf{D}; \mathbf{z}_0] = \omega\mathbf{B}';\\ \mathrm{rot}_{\perp}\mu^{-1}\mathbf{B}' + i\nu\,[\mu^{-1}\mathbf{B}'; \mathbf{z}_0] = \omega\mathbf{D}; \quad \mathbf{B}' = i\mathbf{B}\end{array}\right\}. \qquad (1)$$

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Algebraization (1) leads to the following system:

$$\left\|\begin{array}{c:c} M_{\perp z}\check{X} + \nu' M_{\perp\perp'} & \nu' M_{\perp\perp} \\ \hdashline \frac{1}{\nu'}\check{X}M_{zz}\check{X} + (M_{\perp' z}\check{X} + \check{X}M_{z\perp'}) + \nu' M_{\perp'\perp'} & \check{X}M_{z\perp} + \nu' M_{\perp'\perp}\end{array}\right\| \cdot \begin{pmatrix} b_{\perp'} \\ b_{\perp}\end{pmatrix} = \begin{pmatrix} a_{\perp'} \\ a_{\perp}\end{pmatrix};$$

$$\left\|\begin{array}{cc} \nu' E_{\perp\perp'} & \nu' E_{\perp\perp} \\ \frac{1}{\nu'}\check{X}E_{zz}\check{X} + \nu' E_{\perp'\perp'} & \nu' E_{\perp'\perp}\end{array}\right\| \cdot \begin{pmatrix} a_{\perp'} \\ a_{\perp}\end{pmatrix} = \begin{pmatrix} b_{\perp'} \\ b_{\perp}\end{pmatrix}. \qquad (2)$$

or it is shorter:

$$Mb = a; \quad \partial a = b, \text{ где } a = \begin{pmatrix} a_{\perp'} \\ a_{\perp}\end{pmatrix}, \; b = \begin{pmatrix} b_{\perp'} \\ b_{\perp}\end{pmatrix}. \qquad (3)$$

Here $\nu' = -i\nu/\omega$; X-diagonal matrix/die from the transverse eigenvalues; matrix/die $M_{\alpha\beta}$ and $\partial_{\alpha\beta}\ (\alpha, \beta = z, \perp', \perp)$ have identical structure, for example: $M_{\alpha\beta} = \iint \mathbf{H}_{\alpha n}\mu\mathbf{H}_{\beta n}\,ds$; vector $a_{\perp}$ relates to the electric fields of H-waves, $a_{\perp'}$ - to the electric fields of E-waves, is analogous for $b_{\perp'}$ and $b_{\perp}$.

In the waveguide task the formulation in the inductions is advantageous because of the fact that it makes it possible to exclude longitudinal components for the arbitrary form of tensors ε and μ, i.e., occur the equalities

$$b_z = \frac{1}{\nu'}\check{X}b_{\perp'} \quad \text{и} \quad a_z = \frac{1}{\nu'}\check{X}a_{\perp'}. \qquad (4)$$

Excluding a from the second equation (3), we obtain: ЭMb=b. The conditions of the consistency of system gives equation for the

determination of that infused of propagation $F(v') = \det(EM-I)=0$, which is solved by Newton's method. Matrix/die EM has the 10th order; and μ - complex quantities.

c) N-shaped waveguide. In the comparison with the previous tasks the special feature/peculiarity of data lies in the fact that in the direct method are used as the coordinate ones the functions, which do not satisfy boundary conditions, precisely, the vector eigenfunctions of the rectangular resonator, into which is inscribed the segment of the N-shaped waveguide in question with the ferrite. is solved the problem about the resonator of the N-shaped profile/airfoil, with natural frequencies of which in an obvious manner connected propagation constant of waveguide.

Algebraic formulation can be obtained by both the method of Galerkin and by method of Ritz. In the latter case at the basis of method lies/rests the functional

$$[\omega^2] = \frac{\int_V \operatorname{rot} \mathbf{H}^* \varepsilon^{-1} \operatorname{rot} \mathbf{H}\, dv}{\int_V \mathbf{H}^* \mu \mathbf{H}\, dv}, \qquad (5)$$

steady-state values of which supply/deliver/feed into natural frequencies, and the use/application of the coordinate functions indicated is justified by the fact that the boundary conditions of the boundary-value problem in question are for it natural.

In the implementation of method in the machine were used by 24 coordinate functions.

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Slotted articulation of rectangular waveguides.

I. B. Levinson, P. Sh. Friedberg.

In known works [1, 2] during the solution of the integrodifferential equation for the field on the slot of two covolumes (final or infinite) the slot is assumed to be exponential-narrow ($\ln(\lambda/d)\gg 1$; $\ln(l/d)\gg 1$; l, d - length and the width of slot respectively; λ - wavelength). In the present work the slot is assumed to be the simply narrow ($\lambda/d\gg 1$, $l/d\gg 1$), i.e., in the resolution of kernel of integral equation are held down/retained not only terms of order ln d, but also on the order of 1. Is given variational-iterative method of obtaining the scattering matrix of two volumes, connected through the narrow slot. The obtained result in principle differs from that utilized in [1, 2] the expansions by degrees $\left(\ln\frac{\lambda}{d}\right)^{-1}$ or $\left(\ln\frac{l}{d}\right)^{-1}$.

Vector integral equation [3] for electric field E in the case of narrow slot to be registered in the form

$$\int_{-d/2}^{d/2} du' \int_{0}^{l} dv' X_{uu}(uv, u'v')\, E_u(u'v') = H_v^0(u, v). \qquad (1)$$

where u, v - rectangular coordinates on the slot, u - across the slot from -d/2 to +d/2, v - along the slot from 0 to 1. For narrow slot E_u it is possible to register [1] in the form $E_u(u'v') = U(v')\Phi(u')$, where U(v') - the unknown stress/voltage on the slot, $\Phi(u') = \frac{1}{\pi}[(d/2)^2 - u'^2]^{-1/2}$. Assuming/setting in (1) u=0 (the centerline of slot) and integrating by u', we will obtain one-dimensional integral equation for U(v)

$$\int_0^l dv' N(v, v')\, U(v') = H_v^0(v). \qquad (2)$$

Kernel N(v, v') depends on d and consists of the sum of two averaged admittances, which correspond to joined volumes [3]

$$\eta(v, v') = \int du' \eta_{uu}(0v, u'v')\, \Phi(u'). \qquad (3)$$

Let us isolate dependence $\eta(v, v')$ on d→0, disregarding in this case the terms, which are rotated into zero together with d. With d→0 value $\eta(v, v')$ goes to infinity as ln d; therefore the input admittance of the volume through the narrow slot is represented in the following form:

$$\eta(v, v') = \alpha^{-1} a(v, v') + b(v, v'), \qquad (4)$$

where $\alpha = \left(2 \ln \frac{k_0 d}{4}\right)^{-1}$, and a(v, v') and b(v, v') do not depend on d, $k_0 = 2\pi/\lambda$. Since the singular part of Green's function does not depend on the geometry of system, then independent of the geometry of slot in the volume we have

$$a(v, v') = -\frac{1}{2\pi i}\frac{\eta_0}{k_0}\left[k_0^2 + \frac{\partial^2}{\partial v^2}\right]\delta(v - v'), \qquad (5)$$

η_0 - a conductivity of free space. Kernel N(v, v') takes the form

$$N(v, v') = \alpha^{-1}A(v, v') + B(v, v'), \qquad (6)$$

where

$$A(v, v') = 2a(v, v'), \quad B(v, v') = b_1(v, v') + b_2(v, v'). \qquad (7)$$

Terms $b_1(v, v')$ and $b_2(v, v')$ depend on frequency and geometry of the joined volumes. The presence in (6) of logarithmic factor makes it possible to examine the approximation/approach of exponential-narrow slots $|\alpha|<<1$.

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In this case it is possible to disregard B(v, v') and integral equation (2) is reduced to the differential equation of Ya. N. Fel'd:

$$\left(\frac{\partial^2}{\partial v^2} + k_0^2\right) U(v) = \pi\alpha(-i\omega\mu_0) H_v^0(v) \qquad (8)$$

with the boundary conditions U(0)=U(1)=0, ω - angular frequency, μ_0 - magnetic permeability of free space.

Let us produce the calculation of the nonlogarithmic term b(v, v') based on the example to the transverse slot in the wide wall of infinite multimodal waveguide. Using a tensor function of Green of infinite waveguide and differentiating it formally, let us find a component η_{uu} transverse-transverse relative to slot of the tensor of admittance η. Assuming/setting further z=0, y'=0 and neutralizing on z' with the help of $\Phi(z')$, we will obtain

$$\eta(x, x', y) = \eta_0 \int \frac{dk}{2\pi i} \sum_{m,\, n=0}^{\infty} \frac{\varepsilon_m \varepsilon_n}{abk_0} \frac{k^2 + \beta_n^2}{\alpha_m^2 + \beta_n^2 + k^2 - k_0^2} \times$$

$$\times J_0\left(\frac{1}{2} kd\right) \sin \alpha_m x \sin \alpha_m x' \cos \beta_n y, \qquad (9)$$

where x, y, z - rectangular coordinates, x - along a, y - along b, z - along the axis of waveguide, a and b - sizes/dimensions of cross section,

$$\alpha_m = \frac{m\pi}{a}, \quad \beta_n = \frac{n\pi}{b}, \quad \varkappa_{mnk}^2 = \alpha_m^2 + \beta_n^2 + k^2,$$

$$\varepsilon_m = \begin{cases} 1, & m = 0 \\ 2, & m = 1, 2\ldots \end{cases}$$

integration for k is conducted with the circuit/bypass of poles on the negative real semi-axis on top, and on the positive real semi-axis - from below. Integral-series obtained as a result of this formal procedure diverges. For determining the method of its addition let us consider more attentively the sense of passages to the limit. Twofold formal differentiation of Green's function, strictly speaking, is illegal. For the justification for this procedure it is possible to conduct under the sign of integral-sum the factor of convergence $e^{-\Delta\varkappa_{mnk}} \sim e^{-\Delta|k|} e^{-\Delta\alpha_m} e^{-\Delta\beta_n}$. This is equivalent to the examination of Green's pre-limit function, which is the solution of equation with the pre-limit δ-function, "smeared" in the region with the linear dimensions of order Δ. Green's this function can be used, if all distances in question much more than Δ. Such distance in our task is distance y from the slot to the point, where is computed the field,

which is to lace with $y \to +0$. Therefore a passage to the limit must be realized, by directing first $\Delta \to 0$ and only then $y \to +0$. It is not difficult to see that all this as a result is a physical justification for the addition of series/rows in Abel-Poisson's sense. After fulfilling in (9) by the method indicated addition and representing result in the form (4), let us find the unknown expression b(v, v') of infinite multimodal waveguide with the transverse slot on the wide machine tool. It is analogously obtained by b(v, v'), also, for the multimodal semi-infinite waveguide with the slot in the end/face.

The articulation of two volumes is completely characterized by scattering matrix. Using (3), it is possible to write variation principle for the matrix elements of scattering S. In this case naturally will enter two test stresses/voltages. As these test stresses/voltages can be undertaken the solutions of integral equation (2) with B=0, i.e., actually the solution of equation (8) for exponential-narrow slot.

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The given theory is valid both for the resonance ones and for the nonresonant slots. For resonant slot (8) gives sinusoidal field distribution with the infinite amplitude; however, in the variation

principle of amplitude they are reduced, and field distribution in this case (8) gives relatively correctly.

Analogously can be generalized the theory of exponential-thin radiators/resonators/elements in the case of thin radiators/resonators/elements.

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11. Slight disturbances are long waves.

Averaged boundary conditions on the surface of wirings with the rectangular nuclei and the diffraction of electromagnetic waves on such grids.

M. I. Kontorovich, M. I. Astrakhan, M. N. Spirina.

This report is the survey/coverage of the works, dedicated to the study of the electrodynamic properties of wire gauze in the form of the systems of the intersected leads/ducts (in essence they are intended of grid with the square and rectangular nuclei).

Are examined the shielding, reflecting and retarding properties of different types of grids. Solution is carried out by the method of the averaged boundary conditions. Its essence consists in the fact that instead of the real field E, which passed through the grid or reflected from it, is sought the "smoothed" field $\hat{E}$. This field is subordinated to the Maxwell equations, and mesh surface is substituted by the plane, on which are satisfied some boundary conditions, which depend on the structure of grid; these boundary conditions are called the averaged boundary conditions. The equivalency of true and "smoothed" of fields is established/installed

by their comparison with the averaged field which is obtained from by the true course of its averaging on the region, determined by the structure of grid.

The averaged boundary conditions make it possible by usual methods to solve the problems of diffracting the electromagnetic waves on the mesh surfaces.

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Were found the averaged boundary conditions on the surface of grid with the rectangular nuclei (sizes/dimensions of nucleus axb):

$$\begin{aligned}\hat{E}_x &= -E_{x\,\text{ст}} + \frac{2i\omega b}{c^2}\ln\frac{b}{2\pi r_0}\left\{\left[1+\frac{\mu f(s)}{4\ln\frac{b}{2\pi r_0}}\right]j_x + \right.\\ &\left.+\frac{1}{k^2\left(1+\frac{a}{b}+\chi_x\right)}\frac{\partial}{\partial x}\left(\frac{a}{b}\operatorname{div}\mathbf{j}+\chi_x\frac{\partial j_x}{\partial x}\right)\right\};\\ \hat{E}_y &= -E_{y\,\text{ст}} + \frac{2i\omega a}{c^2}\ln\frac{a}{2\pi r_0}\left\{\left[1+\frac{\mu f(s)}{4\ln\frac{a}{2\pi r_0}}\right]j_y + \right.\\ &\left.+\frac{1}{k^2\left(1+\frac{b}{a}+\chi_y\right)}\frac{\partial}{\partial y}\left(\frac{b}{a}\operatorname{div}\mathbf{j}+\chi_y\frac{\partial j_y}{\partial y}\right)\right\};\end{aligned} \qquad (1)$$

$$f(s)=\begin{cases}0 & {}^{(1)}\text{при } \sigma=0\\ \left.\begin{array}{ll}1-\frac{i}{s^2} & {}^{(2)}\text{для малых } s\\ \frac{1-i}{s} & {}^{(3)}\text{для больших } s\end{array}\right\} & {}^{(1)}\text{при } \sigma\neq 0\end{cases}\quad s=\frac{r_0\sqrt{\pi\omega\sigma\mu}}{\sqrt{2}\,c};\quad k=\frac{\omega}{c}.$$

Key: (1). with. (2). for small ones. (3). for large ones.

Formulas (1) are obtained on the assumption that in the plane of

grid $E_{ст}$ little is changed for the elongation/extent of one nucleus and the parameters of grid are subordinated to the conditions

$$r_0,\ l \ll a,\ b;\ a,\ b \ll \lambda, \tag{2}$$

where r_0 - radius of leads/ducts, l - distance between the leads/ducts intersected in the node/unit.

Function f(s) considers the final conductivity of the material of leads/ducts and the phenomenon of skin-effect in them.

Coefficients χ_x and χ_y depend on the character of the contact between the leads/ducts in the nodes/units of nuclei. It is shown that

$$\chi_x = 2i\omega C_0^* za, \quad \chi_y = 2i\omega C_0^* zb,$$

$$\frac{1}{C_0^*} = 4\ln\left[\frac{\sqrt{ab}}{\pi r_0}\left(1 + 4{,}9\frac{l^2}{a^2} + 4{,}4\frac{l^4}{a^4}\right)^{1/4}\left(1 + 4{,}9\frac{l^2}{b^2} + 4{,}4\frac{l^4}{b^4}\right)^{1/4}\right], \tag{3}$$

if between the leads/ducts of nuclei are connected impedances z, or

$$\chi_x = \frac{2C_0^* a}{C - C_0^* \frac{a+b}{2}}, \qquad \chi_y = \frac{2C_0^* b}{C - C_0^* \frac{a+b}{2}},$$

where C - the available capacity, which falls to one nucleus, if between the intersected leads/ducts in the nodes/units of nuclei resistance is not included.

From formulas (1)as special cases, it is easy to obtain the averaged boundary conditions on the surface of grid with the square nuclei and for the system of parallel leads/ducts.

Similarly obtained the averaged boundary conditions on the surface of two closely spaced grids with the square nuclei.

The averaged boundary conditions are made it possible comparatively simply to examine the different kind of the task of electrodynamics for the systems with the grids.

Is examined an incidence in plane electromagnetic wave of arbitrary polarization at arbitrary angle on the grid (for three types of the grids: with the square nuclei, with the rectangular nuclei, parallel leads/ducts). Are found expressions for the reflection coefficients and are investigated the dependences of the reflecting and shielding properties of grids in the dependence on the parameters of the incident wave and parameters of grid.

The same problem is solved for two parallel grids with the square nuclei.

Furthermore, is examined the propagation of the electromagnetic waves between two infinite networks, along one grid, in the waveguide

of the delay/retarding/deceleration of electromagnetic waves on the parameters of the systems being investigated and transmission mode.

The comparison of calculated results with the experimental ones, conducted in the majority of tasks, showed a good coincidence.

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Problem of diffraction for the lattice from contours of arbitrary form.

I. Ye. Tarapov.

Is examined the decision of the exterior problems of Neumann and Dirikhle for the equation of Helmholtz in the case of lattice with the smooth ducts/contours of arbitrary form.

Solution is found out in the form of the potential of simple layer (Neumann's task) and potential of the double layer (Dirichlet problem), folded with the plane wave, which falls at certain angle to the lattice. The intensities of layers are the solution of the integral equations in which is contained the integration only for one of congruent contours. In the kernels of these equations easily is selected their "hydrodynamic part" (not depending on the length of the incident wave).

Is examined approximation/approach in the case of very long waves. Are obtained formulas for the solution in the first approximation, and are given estimations of error.

In comparison with other methods the proposed method possesses the advantage that, in the first place, it makes it possible to consider error and, in the second place, allows/assumes obtaining further approximations/approaches with the arbitrary duct/contour of lattice.

Diffraction on the group of bodies.

I. V. Smirnova, L. A. Cherches.

Let us consider diffraction on group n of the diverse bodies, for each of which individually the solution of task u_k is known. Let us take as the supporting/reference solution without taking into account interaction between the bodies

$$u_0 = \sum_n u_k. \tag{1}$$

The decision of the boundary-value problem of Neumann can be represented as follows:

$$u = u_0 - \frac{1}{4\pi} \int_S \frac{\partial u_0}{\partial n} \bigg|_S G dS, \tag{2}$$

where $S = \sum_n S_k -$ bounding surface.

In the class of the group of the bodies, for which is correct the asymptotic representation of Green's function,

$$G = G_n [1 + o(1)], \tag{3}$$

where $G_n -$ Green's function without taking into account interaction of bodies, o(1)→0 when $P = \frac{1}{kd_{min}} \to 0$, d_{min} — the minimum distance between two elements/cells of group, we have

$$u = u_0 - \frac{1}{4\pi}\int_S \frac{\partial u_0}{\partial n}\Big|_S G_n dS + o\left(\int_S \frac{\partial u_0}{\partial n}\Big|_S G_n dS\right). \quad (4)$$

This class includes, for example, the group of belts or disks (is not compulsory identical form), located in one plane.

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In certain cases, with the nonfulfillment of condition (3), Green's function is expedient to represent as follows:

$$G = G_g + G_p = G_g + G_{pn}\,[1 + o(1)], \quad (3a)$$

where G_g — is determined by geometric optic/optics, G_p — the component of scattering, G_{pn} — the component of scattering without taking into account interaction of bodies.

As an example let us give result in distant zone ($kr\to\infty$) for two diverse half-planes (wide slot in the flat/plane shield) during incidence in the plane wave of the single amplitude

$$u = u_1 + u_2 - \frac{1}{2^{3/2}\pi}\frac{e^{ik(r+d)}}{(kr)^{1/2}(kd)^{1/2}}\left\{\frac{\operatorname{tg}\frac{\vartheta}{2}}{\cos\frac{\vartheta}{2}}\frac{\sin\frac{\theta}{2}e^{i\frac{kd}{2}\cos\theta}}{1+\cos\theta} + \right.$$
$$\left. + \frac{\operatorname{ctg}\frac{\vartheta}{2}}{\sin\frac{\vartheta}{2}}\frac{\cos\frac{\theta}{2}e^{-i\frac{kd}{2}\cos\theta}}{1-\cos\theta}\right\} + o\left([kd]^{-3/2}\right), \quad (5)$$

where ϑ - angle between the direction of propagation of the incident wave and the plane of shield, θ - the angle between the radius-vector

into observation point and the plane of shield, d - the width of slot.

Expression (5) coincides with the result, obtained in the literature by the method of singular equations.

Diffraction on the weakly deformed bodies.

L. A. Cherches.

Let us consider diffraction on the body with bounding surface, deformed in comparison with the reference body, for which the solution of task u_0 is known. The amount of strain we will characterize by the low parameter p.

For the certainty let us dismantle/select the boundary-value problem of Neumann $\left(\frac{\partial u}{\partial n}\Big|_S = 0\right)$ the equation of Helmholtz (acoustically rigid body).

Solution can be represented in the following form:

$$u(M) = u_0(M) - \frac{1}{4\pi}\int_S \frac{\partial u_0}{\partial n}\Big|_S G(M|S)\,dS. \qquad (1)$$

For Green's function the correctly asymptotic representation

$$G = G_0[1 + o(1)], \qquad (2)$$

where G_0 - Green's function for the reference body, $o(1)\rightarrow 0$ with $p\rightarrow 0$.

As a result we have

$$u = u_0 - \frac{1}{4\pi} \int_S \frac{\partial u_0}{\partial n} G_0 dS + o\left(\int_S \frac{\partial u_0}{\partial n} G_0 dS \right). \qquad (3)$$

From expression (1) it is possible to obtain relationships/ratios of type (3) and more efficient. For this Green's function it is expedient to represent in the form

$$G = G_g + G_{po} [1 + o(1)], \qquad (2a)$$

where G_g — is determined by geometric optic/optics, G_{po} — the component of scattering by reference body.

Let us note also that in the asymptotic representation of Green's function instead of the reference body to sometimes more conveniently use other surfaces S_k, touching with individual sections S.

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For illustration let us give several characteristic examples with the reference body - infinitely thin half-plane.

1. Half-plane of final thickness t (short-wave asymptotic behavior). With normal incidence in the plane wave of single amplitude for distant zone (kr→∞) we will obtain

$$u = u_0 + \frac{e^{-i\pi/4}}{(kr)^{1/2}} \frac{e^{ikr}}{2\pi^{1/2}} \cos\frac{\theta}{2} \cdot kt \ln\frac{\pi}{kt} + o\left(kt \ln\frac{\pi}{kt}\right), \qquad (4)$$

where the angle θ is counted off from the half-plane.

2. Bent "half-plane" (short-wave asymptotic behavior). With the bending radius $R \to \infty$ we obtain the estimation

$$u = u_0 + O\left([kR]^{-1/3}\right). \tag{5}$$

3. Wedge with small aperture angle α.

With $\alpha \to 0$ we obtain the estimation

$$u = u_0 + O(\alpha). \tag{6}$$

Algorithmic method of solving the boundary-value problems for low-frequency electromagnetic field.

D. B. Gurvich, Ye. A. Svyadoshch.

The analytical difficulties, connected with the integration of the equations of Maxwell in their common format, i.e., without any simplifying assumptions, are at present in principle surmounted only in six systems of curvilinear orthogonal coordinates, which substantially limits the circle of the tasks for which can be obtained the exact solutions. At the same time in the vast region of questions of applied electrodynamics, fields, connected with the use, which are slowly changed in the time, the necessities of engineering practice satisfy also solutions in the quasi-stationary and quasi-static approximations/approaches. Thus, for low-frequency electromagnetic field, excited by sources both of the electrical and magnetic type, the system of equations of Maxwell can be approximately substituted by the equations of the following form:

$$\mathrm{rot}\,\mathbf{F} = \mathbf{Q}, \quad \mathrm{div}\,\mathbf{F} = 0, \qquad (1)_1$$

$$\mathrm{rot}\,\mathbf{Q} = 0, \quad \mathrm{div}\,\mathbf{Q} = 0, \qquad (1)_2.$$

where by $\vec{F}$ and $\vec{Q}$ indicate the vectors of magnetic and electrical intensity/strength, and also current density.

The determination of vector $\vec{Q}$ does not present fundamental difficulties, since formulas ensuing from (1),

$$Q = -\operatorname{grad}\Phi, \quad \Delta\Phi = 0$$

is reduced to the boundary-value problem for the equation of Laplace, the methodology of solution of whom detailed. In the so-called dividing systems of coordinates ξ_1, ξ_2, ξ_3 there are solutions of the form

$$\Phi = X_1(\xi_1)X_2(\xi_2)X_3(\xi_3), \qquad (3)$$

each of the functions $x_i (i = 1, 2, 3)$ satisfying the equation

$$\frac{d}{d\xi_i} f_i \frac{dX_i}{d\xi_i} + (\lambda p_i + \tau q_i) X_i = 0, \qquad (4)$$

where λ and τ - separation constant, and f_i, p_i, q_i — function ξ_i, characteristic for this coordinate system. The general solution of the equation of Laplace is obtained by the imposition of functions (3) with the different values of the parameters λ and τ.

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In contrast to $\vec{Q}$ the determination of the field of vector F runs into the difficulty of fundamental character. Usually by the use/application of a vector potential the solution of system $(1)_1$ is reduced to the equation, which contains the Laplacian of vector,

which, however, does not make it possible to obtain separate equations for each component. Because of this solution of problem by the method of vector potential it proves to be feasible to the end/lead only in that special case when the considerations of symmetry make it possible to previously conclude that the vector potential has only one different from zero component.

For the purpose of overcoming this difficulty the authors propose the following algorithm by which the construction of the field of vector F is implemented automatically only on the basis of the solution of the equation of Laplace for potential Φ without any further calculations, namely: components F_i along axes (ξ_i) are expressed by the following formulas analogously with components Q_i through functions X_i, of them derivatives X_i' of ξ_i and coefficients $C(\lambda_n, \tau_m)$ of the expansion of potential in functions (3):

$$F_1 = \pm \frac{q_1}{h_1} f_2 f_3 \sum_{(n)} \sum_{(m)} \frac{C(\lambda_n, \tau_m)}{\lambda_n} X_1(\lambda_n, \tau_m, \xi_1) X_2'(\lambda_n, \tau_m, \xi_2) X_3'(\lambda_n, \tau_m, \xi_3); \quad (5)_1$$

$$F_2 = \pm \frac{q_2}{h_2} f_1 f_3 \sum_{(n)} \sum_{(m)} \frac{C(\lambda_n, \tau_m)}{\lambda_n} X_1'(\lambda_n, \tau_m, \xi_1) X_2(\lambda_n, \tau_m, \xi_2) X_3'(\lambda_n, \tau_m, \xi_3); \quad (5)_2$$

$$F_3 = \pm \frac{q_3}{h_3} f_1 f_2 \sum_{(n)} \sum_{(m)} \frac{C(\lambda_n, \tau_m)}{\lambda_n} X_1'(\lambda_n, \tau_m, \xi_1) X_2'(\lambda_n, \tau_m, \xi_2) X_3(\lambda_n, \tau_m, \xi_3). \quad (5)_3$$

Here q_i, f_i — the functions, entering in (4), h_i — the metric coefficients of the selected coordinate system. The necessary and sufficient conditions for the applicability of formulas (5) are the following:

$$f_1 \frac{h_1}{h_2 h_3} (p_2 q_1 - p_1 q_2) = \pm 1 \quad (6)$$

and the two additional equalities, obtained from that written by the cyclic permutation of indices, and also

$$f_i q_i = \text{const}, \quad i = 1, 2, 3. \qquad (7)$$

Direct checking makes it possible to ascertain that the written conditions are satisfied in all eleven systems of three-dimensional/space orthogonal coordinates ξ_i, in which, as is known, is divided the equation of Laplace.

Thus, system (1), which gives the satisfactory approximate description of the wide circle of electromagnetic phenomena, proves to be solvable with the help of the proposed algorithm in a considerably larger quantity of coordinate systems, than the general/common/total equations of Maxwell.

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12. Functionally invariant solutions. Analytic functions.

Some self-similar problems of the dynamic theory of elasticity.

B. V. Kostrov.

Are examined the following tasks:

- the task about the depression with the constant velocity of rigid wedge into the elastic half-space;

- task about the depression with the constant velocity of rigid cone into the elastic half-space;

- two-dimensional problem about the crack propagation of tangential interruption/discontinuity;

- task about the propagation of the circular crack of normal interruption/discontinuity;

- task about the propagation of the circular crack of tangential

interruption/discontinuity;

All problems are solved by the single method: with the help of the method of the functionally invariant decisions of V. I. Smirnov and S. L. Sobolyev are reduced to finding of some analytic complex variable functions, which possess the assigned special features/peculiarities. In this case substantially is used the fact that the solutions of problems must be the uniform functions of coordinates and time. During the solution of spatial problems it is used the superposition of flat/plane solutions by the method of Smirnov - Sobolyev.

Are obtained explicit solutions in the quadratures. It is shown that the solutions do not exist, if the velocity of propagation of the edge of crack or the speed of the displacement/movement of the edge of die/stamp over the surface of half-space exceeds the speed of the ground waves of Rayleigh. Are obtained the equations, which are determining the velocity of propagation of cracks in the dependence on the load and the properties of medium.

Tasks of diffracting the shock waves, which are reduced to the boundary-value problems for the analytic complex variable functions.

S. I. Ter-Minasyants.

Is examined the class of the self-similar problems of the weak diffraction of the shock waves whose solutions are obtained by single method - via their reducing to boundary-value problems for the analytic complex variable functions. This of the task:

1). Task about the diffraction of plane wave on the wedge, which moves with supersonic speed.

2). Task about an instantaneous small change in supersonic speed of the motion of wedge.

3). Task about the diffraction of the plane slantwise incident shock wave on the dull wedge.

4). The task about the insertion/immersion of dull wedge into compressible liquid with the arbitrary speed whose vector can compose final angle with the flat/plane floating surface.

5). Task about the flow around delta wing with the small sweep angle at the final angle of attack and with the final slip angle.

6). Some other tasks.

In all named tasks the region of diffraction is localized by solid wall, circular arcs of Mach with the center on this wall and by slightly bent impact front conditions at which in view of linearity are carried to the secant, which has by points of intersection with Mach's circle/circumference the end-points of the region of diffraction.

The system of equations of the flat/plane unsteady self-similar flow of gas is reduced to the equation of Laplace for the compressive disturbance p, and presenting impact front secant passes in the circular arc, orthogonal to the unit circle of Mach.

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Boundary conditions on this arc take the form

$$\frac{\partial p}{\partial n} \Big/ \frac{\partial p}{\partial s} = (A \operatorname{tg}\theta - B \operatorname{ctg}\theta)/\sqrt{1 - m^2 \sec^2\theta}, \quad \frac{\partial v}{\partial y} = \frac{B}{y} \cdot \frac{\partial p}{\partial y}.$$

Here s and n - coordinate along the tangent and the normal to the form of impact front; y - coordinate of lengthwise aforementioned secant, calculated off the perpendicular, omitted to it from the center of circle of Mach; θ - the vectorial angle, calculated off the same perpendicular, and m, A and B - constants, determined by the initial nondiffracted shock wave and the angle between its front and wall. These relationships/ratios for the isentropic exponent $\varkappa = 1.4$ were obtained by Lighthill; we repeated his conclusion/output in the case of any $\varkappa$. Boundary conditions on wall and circles/circumferences of Mach either are represented respectively in the form dp/dn=0 and dp/ds=0 or in the right side of any one of them figures δ - function, caused either by the fracture of wall (when it within the diffraction region), or by the meeting of Mach front (from the source, which is located out of the diffraction region) with Mach's circle/circumference. The position of the points, at which $\delta(s) \neq 0$, easily is determined during the solution of each specific problem.

The conformal conversion

$$z = \sigma + i\tau = \ln\,[(\lambda - \lambda_2)/(\lambda - \lambda_1)] - \lambda_3,\ \lambda = Re^{i\theta}$$

(λ_1, λ_2, λ_3 - known complex constants) maps diffraction region to the rectangle $-l<\sigma<l$, $0<\tau<\pi$, and the latter by function Sn(z/C, k), where modulus/module k and constant C are determined by the conformity of points, to the upper half-plane of plane $\omega = \varphi + i\psi$. The right vertical side of the rectangle indicated, which corresponds to

impact front, passes in the segment $1<\varphi<1/k$ of real axis, boundary condition in which is obtained from the condition on the side of the rectangle indicated:

$$\frac{\partial p}{\partial s} \Big/ \frac{\partial p}{\partial n} = \frac{C_1 (m_0 - M \cos \tau) \sin \tau}{A_1 (m_0 - M \cos \tau)^2 - B_1 (M - m_0 \cos \tau)^2} = b(\tau).$$

(A_1, B_1, C_1, m_0, M - known constants) by substitution instead of τ the elliptical integral of the first kind. conditions on the remaining elements/cells of boundary do not vary their form.

Since in plane ω $\partial p/\partial n = \partial p/\partial \psi$ and $\partial p/\partial s = \partial p/\partial \varphi$, then the system of our boundary conditions can be considered as the boundary condition of the task of Riemann - Gilbert/Hilbert with the discontinuity coefficients for function $\Gamma = \frac{\partial p}{\partial \varphi} + i\frac{\partial p}{\partial \psi}$:

$$a(\varphi)\frac{\partial p}{\partial \varphi} + b(\varphi)\frac{\partial p}{\partial \psi} = c(\varphi),$$

where $a=0$ with $-1/k<\varphi<-1$ and $a=1$ on the remaining part of the real axis; $b=0$ everywhere, besides cutting off $-1/k<\varphi<-1$, where $b=1$, and cutting off $1<\varphi<1/k$, where $b[\tau(\varphi)]$ it is defined, as noted above.

$c(\varphi) = C_2\delta(\varphi-\varphi_0)$, C_2=const to us is known. The obtained task with the discontinuity coefficients is reduced to the task with the continuous coefficients by the introduction to new unknown function $\Gamma_1(\omega)$:

$$\Gamma(\omega) = \Gamma_1(\omega)\cdot\Omega(\omega), \text{ где } \Omega(\omega) = [(\varphi+1)(k\varphi+1)]^{-1/2}.$$

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For function $\Gamma_1(\omega)$ we will obtain boundary-value problem with the continuous coefficients: $a_1=1$, $b_1=b\ [\tau,\ (\varphi)]$ with $1<\varphi<1/k$ and b=0 out of this segment. It is easy to perceive, that the index of function a_1+ib_1 is equal to zero. The solution of initial problem we obtain in the form

$$\Gamma(\omega) = i\beta\Omega(\omega)\exp F(\omega)\,[1 + L(\varphi_0 - \varphi)],$$
$$F = \frac{1}{\pi}\int_1^{1/k} \operatorname{arc\,tg} b\,[t(\zeta)]\,\frac{d\zeta}{\zeta - \omega}$$

(L - it is known); constant β is determined from the second boundary condition.

Hence by integration we obtain solution for the pressure

$$p = \operatorname{Im}\left[i\int_{\omega_0}^{\omega}\Gamma(\omega)\,d\omega\right] = \operatorname{Im}\left\{-\int_{\omega_0}^{\omega}|\Omega(\omega)|\exp\,[i\arg\Omega(\omega) + F(\omega)]\times\right.$$
$$\left.\times\left[1 + \frac{L}{\varphi_0 - \omega}\right]d\omega\right\} + p_0;$$

p_0 - here known pressure at any point ω_0. Calculation formula is obtained by the distribution of the alleged and real parts of this expression. According to the Sokhotskiy formulas it is easy to obtain the distribution of pressure at the wall and the impact front, and also the form of this front.

Wave field, which arose fr⁄m the interruption of continuity on the interface of two elastic media (two-dimensional problem).

L. M. Flitman.

The elastic medium, comprised of the half-spaces with the different elastic properties and the densities to t=0, was found in the uniform stressed state and in the rest.

With t=0 on the interface is cracked of tangential interruption/discontinuity on surface of which remain continuous the normal components of displacement and stress/voltage.

Let us introduce rectangular coordinate system (x, y), directing x axis along the interface. Task is reduced to the construction of the solution of the dynamic equations of the theory of the elasticity

$$(\lambda_i + 2\mu_i)\,\mathrm{grad\,div}\,u_i - \mu_i\,\mathrm{rot\,rot}\,u_i = \rho_i\,\frac{\partial^2 u_i}{\partial t^2},\quad i = 1, 2,$$

i=1 for y>0 and i=2 for y<0,

λ and μ - coefficients of Lame, u - displacement vector, projection on x axis and in which are u and v. Initial conditions are zero. With

y=0 condition they take the form

$$v_1 = v_2,\ \sigma_{1yy} = \sigma_{2yy},\ \tau_{1xy} = \tau_{2xy}$$
$$(-\infty < x < \infty).$$

Furthermore, on crack $\tau_{1xy} = \tau_{2xy} = -\tau_0$ (τ_0— shearing stress, available of up to the interruption/discontinuity), out of crack $u_1=u_2$.

Cracks are examined two kinds: either semi-infinite y=0, x>0, that is formed instantly or growing with a constant velocity C from point x=0, y=0.

In the tasks of stress/voltage and speed of displacement in question there are similar functions of the zero kind of coordinates and time. This makes it possible to use the method of the functionally invariant decisions of Smirnov - Sobolyev.

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All unknown functions are expressed as one analytic function, which is the solution of the problem of Gilbert/Hilbert. Is obtained exact solution. Are investigated the frontal zones of waves.

As a special case is examined wave field from the crack of tangential interruption/discontinuity, which was being formed along the rigid boundary.

13. Regions with the uneven boundaries. Sommerfield's integral.

On the smoothness of the solutions of elliptic equations in the regions with the angular and conical points.

V. A. Kondrat'yev.

We will examine the solution of the equation

$$\sum_{i,j=1}^{n} a_{ij}(x)\, u_{x_i x_j} + \sum_{i=1}^{3} b_i u_{x_i} + cu = f \qquad (1)$$

in G region which becomes zero on boundary Γ and it belongs to space $W^1{}_2(G)$. We will consider that equation (1) - elliptical equation with the infinitely differentiated coefficients, and we will study a question about the smoothness of solution $u(x_1, x_2, x_3)$ in closed domain G depending on smoothness f and on the structure of domain G.

Let us assume first the boundary of domain G is infinitely differentiated everywhere, besides a finite number of conical points.

Point (0, 0, 0) is called conical, if in certain vicinity its equation of bounding surface takes the form

$$x_3^2 = \sum_{i,j=1,2} a_{ij} x_i x_j + o\left(\sqrt{x_1^2 + x_2^2}\right),$$

where $\Sigma a_{ij}x_ix_j$ — positive-definite square form.

It is proven, that if $f \in W_2^k$, the solution in vicinity (0, 0, 0) is represented in the form

$$u = \sum_{\frac{2-n}{2}<\lambda_n<\frac{2k+4-n}{2}} (\ln^k r)\, r^{\lambda_n} \Phi_{nk}(\varphi, \theta) + w(x_1 x_2 x_3), \tag{2}$$

where $w(x_1x_2x_3) \in W_2^{k+2}$, function $\Phi_{nk}(\varphi, \theta)$ — the infinitely differentiated functions of polar coordinates, λ_n — certain set of real numbers which is uniquely determined by values $a_{ij}(0), \alpha_{ij}$. Studying dependence λ_n on $a_{ij}(0), \alpha_{ij}$, it is possible to show that

$$\min \lambda_n > K \inf \frac{\Sigma \alpha_{ij} \zeta_i \zeta_j}{\Sigma \zeta_i^2}, \tag{3}$$

where K - constant, which depends on $a_{ij}(0,0,0)$. Expansion (2) and inequality (3) show that with the infinitely differentiated right side the solution is not infinitely differentiated, but smoothness increases with the decrease of the solution/opening of cone.

It is possible to show that if $f \in W'_2$, then so that the solution u would belong W^3_2, it is necessary and sufficient so that the solution/opening of cone would not exceed 2π.

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Let boundary Γ consist of two surfaces Γ_1 and Γ_2, which intersect along line 1. Intersection angle of their $\omega(s)$, where s - parameter on curved l. It shows that if $f \in L_2$, then so that would occur insertion $u \in W_2^2$, is necessary and is sufficient executing of inequality $\omega(s) \leq \pi$. At the same time it is possible to show that if $\omega(s) < \pi$, then solution has first-order derivatives, continuous up to the boundary. If $\omega(s) > \pi$, then this is erroneous.

Is studied also the smoothness of the solution, which satisfies conditions: $u|_{\Gamma_1} = 0$, $\frac{du}{dn} - g(x_1, x_2, x_3)u|_{\Gamma_2} = 0$. It is shown that if $f \in L_2$, $\omega(s) < \pi$, the such solution belongs to class $C^{1/2}$. Without changing inequality $\omega(s) < \pi$, larger smoothness than $u \in C^{1/2}$, cannot be attained not with what smoothness f.

Diffraction on the polygons and the polyhedrons.

V. A. Borovikov, A. F. Philipp.

Formulation of the stationary and unsteady problems of diffraction.

Connection/communication between the asymptotic with $k \to \infty$ function of Green of stationary task and the interruptions/discontinuities at the wave fronts of Green's function unsteady task. Ray expansions.

Principle of locality for the wave equation and its role in the determination of Green's function unsteady task. Green's function of unsteady task in the angular range. Concept of secondary cylindrical wave.

Green's function for the polygon. Propagation of wave fronts. Concept of optical path. Cylindrical waves, which arose as a result of repeated diffraction at the apexes/vertexes of polygon.

Formulas for the ray resolution of secondary cylindrical wave. Recursion formulae for the ray resolution of cylindrical wave, which

corresponds to this optical path.

Straight/direct formulas for the ray resolution of such waves and their integral representation.

Region of penumbra. Quasi-ray expansions. Region of penumbra in the distant zone. Fresnel's integrals and their unsteady analogs.

Three-dimensional tasks. Diffraction of plane wave at the polyhedral angle and reducing of this task to the equation of Laplace.

Representation of Green's function for the polyhedral angle by the explicit formula through the solution of the corresponding problem of diffracting the plane wave.

Green's function for the dihedral angle.

Green's function for the infinite prism. Concept of spindle-shaped wave. Recursion formulae for the ray resolution of such waves.

Green's function for the polyhedron.

Electromagnetic task of diffracting the plane wave on the cone.

Unresolved tasks and prospect. Green's tensor for the electromagnetic task of diffraction on the polyhedral/multifaceted and on the dihedral angles. Asymptotic behavior of Green's function unsteady task with $t \to \infty$. Incidence in the plane wave on the open end/lead of the waveguide.

Limits of the applicability of the theory presented.

Diffraction from the half-plane of waves, formed on the surface of liquid and on the interface in the laminar liquid by the periodically functioning source.

S. S. Voyt.

Half-space z<0 is filled with the liquid, which consists of the layers of the liquid of density ρ_1, that floats on the surface of the infinitely deep homogeneous liquid of density ρ. In the liquid is immersed the flat/plane semi-infinite vertical wall whose vertical boundary coincides with axis Oz, and horizontal - with axis Ox.

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Is studied wave diffraction, which are formed on the floating surface and the interface as a result of acting the periodic source with the frequency σ, that is located in the thickness of liquid.

The task in question is the generalization of the work of the author, carried out by it earlier for the case of homogeneous liquid (applied math. and mech., Vol. 25, Is. 2, 1961).

In the present work the solution is constructed also by Sommerfield's method and is produced the asymptotic analysis of the obtained formulas for the wave amplitudes on the floating surface and the interface.

Are investigated different cases, when source is located in the upper layer and in the thickness of infinitely deep liquid.

It shows that the part of the increase, obliged to diffraction, consists of two systems of waves. The waves of the first system have larger amplitude on the floating surface, while the waves of the second system have larger amplitude on the interface, the ratio of the amplitudes of the second systems of waves growing with the decrease of the difference in the densities of upper and lower liquid.

Are indicated the boundaries, in which are applicable different asymptotic expressions.

Diffraction of ground wave with oblique incidence on the fracture of impedance plane.

M. S. Bobrovnikov, V. N. Kislitsyn, V. G. Myshkin, R. P. Starovoytova.

Is examined the task about the diffraction of the scalar surface ground wave, which encounters on one of the faces to the edge/fin of impedance wedge at angle θ*. The three-dimensional function, which describes field, is sought in the form

$$U(r, \varphi, z) = R(r, \varphi)\, e^{i\Gamma\beta_+ \sin\theta z}.$$

The edge/fin of wedge coincides with z axis of cylindrical coordinate system r, φ, z; β_+ - delay/retarding/deceleration $\left(\frac{c}{v_\Phi}\right)$ on the face, on which attacks the wave; β_- - delay/retarding/deceleration on the second face.

Function R(r, φ) in wedge region $r > 0, -\Phi < \varphi < \Phi$ satisfies wave equation $\Delta R + \Gamma^2 R = 0$, on the faces - to boundary conditions $\frac{1}{2}\frac{\partial R}{\partial \varphi} \mp i\Gamma \sin\theta_{\pm} R = 0, (\varphi = \pm\Phi)$ and it can be represented with the help of Sommerfield's integral as with normal incidence (θ=0) in the wave on the edge/fin of the wedge:

$$R(r, \varphi) = \frac{1}{2\pi i}\int_{\gamma} e^{-i\Gamma r\cos\alpha}\, S(\alpha + \varphi, \theta, \Phi)\, d\alpha, \qquad (1)$$

where $\Gamma = k\sqrt{1-\beta_{\pm}^{2}\sin^{2}\theta}$; $\sin\vartheta_{\pm} = -\frac{ikQ_{\pm}}{\Gamma}$ - some equivalent impedances of faces, $Q_{\pm}$ - positive real numbers, characterizing the purely reactive impedance respectively of the first and second faces with normal incidence in the ground wave on the edge/fin of wedge and connected with the delays/retardings/decelerations with relationship/ratio $Q_{\pm} = \sqrt{\beta_{\pm}^{2} - 1}$.

Function $S(\alpha - \varphi, \vartheta_{+}, \Phi)$ is found from the functional equations, which are obtained as a result of substitution (1) into the boundary conditions.

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Here values Γ and ϑ in contrast to the normal incidence are the functions of angle of incidence θ. This fact leads to some special features/peculiarities in the behavior of wave field.

With a change of the angle of incidence in the interval $0 \leqslant \theta \leqslant \theta_1 = \arcsin 1/\beta_{+}$ the diffraction of ground wave with oblique incidence qualitatively barely differs from diffraction with normal incidence ($\theta = 0$). But when $\Gamma^2 < 0$ $(\theta > \theta_1)$ the domain of existence of integral (1) will be changed. As this follows from the analysis of

solution, sky wave in the direction, perpendicular to the edge/fin of wedge, it proves to be that not running and is the field, localized near the edge/fin of wedge. In this field of ground wave on the second face with $Q_+ \leqslant Q_-$ it will be that extending. But in the case of $Q_+ > Q_-$ and $\Gamma^2 < 0$ in the interval of angles of incidence $\pi/2 > \theta > \theta_2 = \arcsin \beta_-/\beta_+$ ground wave on the second face will not transfer energy. This field also is localized near the edge/fin of wedge. Thus, in the latter case the incident ground wave will be reflected completely, analogous with the total reflection of plane waves upon transfer of optically the more solid medium to optically less dense.

For the ground waves with oblique incidence on the edge/fin of wedge with the jump of impedance it is possible to speak about the analog of the Snell laws for these waves. The angle of refraction not depending on the flare angle of wedge.

Further during inclined incidence in the ground wave, as with the normal incidence, there is such series of flare angles of wedge $2\Phi = \frac{\pi}{2n}$ (n=1, 2, 3 ...), in which the ground wave will be completely reflected at any angles of incidence θ.

Are calculated the graphs of the coefficients of reflection, passage depending on angle of incidence for the series of the flare angles of wedge $2\Phi = \frac{5\pi}{4}$, π, 0.4 π.

Excitation of electromagnetic horn by flat wide waveguide.

V. V. Malin, Ye. I. Nefedov.

1. Electromagnetic horn is one of basic elements of broadband trunk line of communications (so-called quasi-optical line). Therefore the electrodynamic calculation of horn is appropriate. Below we will consider the method of the construction of Green's function of the system, which consists of the horn, excited by flat/plane waveguide. The case when horn is degenerated into the infinite flange of flat/plane waveguide, is a special case of the present of tasks.

2. Let us assume flat/plane (d/dz=0) wide (ka>>1, k=2π/λ - wave number, a - width of waveguide) waveguide ($x \in [-\infty, 0]$) at points O and O' have fractures to angle +-ϑ, i.e., it passes into horn. Of the surfaces of the angular ranges, which form system waveguide - horn, let us designate through S_1 and S_2. To the left, from the side of waveguide, on the horn falls one of its own waves of flat/plane waveguide $u_1 = \sin\alpha_1 y \exp\{ih_1 x\}$. Temporary/time dependence is selected in the form exp { -iωt }.

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The complete field

$$u_t = u_i + \bar{u} \tag{1}$$

satisfies the equation of Helmholtz

$$(\Delta + k^2) u_t(\mathbf{r}) = 0, \quad (\mathbf{r} = (r, \varphi)), \tag{2}$$

to Dirichlet boundary condition (for the certainty)

$$u_t(\mathbf{r})|S = 0, \; S = S_1 + S_2 \tag{3}$$

and to the principle of Foch-Malyuzhints extinguishability.

$$u_t(\mathbf{r}_0, k) < \infty. \tag{4}$$

For the solution of problem we will use Green's formula

$$\int_V (u\Delta\Gamma - \Gamma\Delta u)\, dV = \int_S (\Gamma u_n' - u\Gamma_n')\, dS, \tag{5}$$

where n - internal to the V standard/normal. As function $\Gamma(\mathbf{r}, \mathbf{r}_0)$ let us take Green's function, which satisfies the nonhomogeneous equation of Helmholtz

$$L\Gamma(\mathbf{r}, \mathbf{r}_0) \equiv (\Delta + k^2)\Gamma(\mathbf{r}, \mathbf{r}_0) = \delta(\mathbf{r} - \mathbf{r}_0) \tag{6}$$

and boundary condition $\Gamma(\mathbf{r}, \mathbf{r}_0) = 0, \; \mathbf{r} \in S$.

Substitution (5) and (4) leads us to the following relationship/ratio:

$$u(\mathbf{r}_0) = -\int_S u(\mathbf{r})\Gamma_n'(\mathbf{r}, \mathbf{r}_0)\, dS, \tag{7}$$

which gives complete solution, if is known the function of Green $\Gamma(\mathbf{r}, \mathbf{r}_0)$ of task (6) in question.

3. For construction of function $\Gamma(\mathbf{r}, \mathbf{r}_0)$ we will use method of consecutive diffractions which had extensive application in theory of wide waveguides, and also by fan-junctions of Green $\Gamma_{1,2}(\mathbf{r}, \mathbf{r}_0)$ of wedge regions $S_{1,2}$.

Let us assume $\Gamma_{11}(\mathbf{r}, \mathbf{r}_0)$ — the solution of the following task

$$L\Gamma_{11}(\mathbf{r}, \mathbf{r}_0) = \delta(\mathbf{r} - \mathbf{r}_0), \; \Gamma_{11}|_{S_1} = 0 \qquad (8)$$

and represent Green's unknown function in the form $\Gamma = \Gamma_{11} + \varphi$. Is obvious that $L\varphi = \delta(r - r_0)$ satisfies the boundary conditions: (a) $\varphi|_{S_1} = 0$, (b) $\varphi|_{S_2} = -\Gamma_{11}|_{S_2}$. We will seek the solution for φ, which satisfies only condition (b), and let us designate this solution through $\Gamma_{12} = \int \Gamma_2' \Gamma_{11} dS$. Since Γ_{12} condition (a) does not satisfy, then $\Gamma_{11} + \Gamma_{12}$ is not a solution of task Γ. Therefore further again we write/record, then $\Gamma = \Gamma_{11} + \Gamma_{12} + \varphi$ and (a') $\varphi|_{S_1} = -\Gamma_{12}|_{S_1}$, (6') $\varphi|_{S_2} = 0$. And we again seek the solution for φ, which satisfies only condition (a'). We will obtain $\Gamma_{21} = \int \Gamma_1' \Gamma_{12} dS$ and so forth.

The general solution for $\Gamma(\mathbf{r}, \mathbf{r}_0)$, thus, will be

$$\Gamma(\mathbf{r}, \mathbf{r}_0) = \sum_{n=1}^{\infty} (\Gamma_{n1} + \Gamma_{n2}), \qquad (9)$$

where

$$\Gamma_{n1} = -\int\limits_{S_1} \Gamma_1' \, \Gamma_{n-1,2} \, dS, \quad n \geqslant 2,$$
$$\Gamma_{n2} = -\int\limits_{S_2} \Gamma_2' \, \Gamma_{n,1} \, dS, \quad n \geqslant 1. \tag{10}$$

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Using Green's function (9), (10) and taking into account (7), it is easy to obtain the solution of the problem of diffraction for incident field u_i. The solution of the problem of diffraction in this case takes the form

$$u_t = u_i + \sum_{n=1}^{\infty} (u_{n1} + u_{n2}), \tag{11}$$

where

$$u_{n1}(\mathbf{r}_0) = \int\limits_{-\infty}^{\infty} u_{n-1,2}(\mathbf{r}) \, \Gamma_1'(\mathbf{r}, \mathbf{r}_0) \, dr, \quad r \in S_1,$$
$$u_{n2}(\mathbf{r}_0) = \int\limits_{-\infty}^{\infty} u_{n-1,1}(\mathbf{r}) \, \Gamma_2'(\mathbf{r}, \mathbf{r}_0) \, dr, \quad r \in S_2, \tag{12}$$

moreover here $u_{01} = u_{02} = u_i$, a $u_{11} = \int\limits_{0}^{\infty} u_i(\mathbf{r}) \, \Gamma_1'(\mathbf{r}, \mathbf{r}_0) \, dr, \quad r \in S_1$.

The posed problem in general form is solved by formulas (9)-(12), that present field at the arbitrary point r_0.

In the report are examined different representations of solution, and also are considered data of some numerical calculations.

One method of solving the functional equations of Malyuzhints for some special cases of diffracting the plane acoustic wave in the touching liquid and elastic wedges.

V. Yu. Zavadsky.

Malyuzhints [1], [2], [3] proposed and developed the method of the solution of the problems of diffraction in the angular ranges, applied to the wedge of arbitrary solution/opening with the ideal and impedance faces, to the sector media, which present the system of wedges with one common edge/fin and general/common/total faces, to the elastic wedges. This method is based on the representation of field in the angular range with the integral of Sommerfield and reduces the task of diffraction to the functional equations for the integrands. The solutions of functional equations were obtained in many interesting cases. However, for the series of problems, in particular for the sector media, comprised of the wedges with the different speeds of sound, equations were not solved. The variable coefficients, which figure in the functional equations in this case, impede the use of known methods and integral transforms of the type of Fourier, Laplace for the determination of the solution of equations. In the reported work examined one generalization of the

integral Laplace (Fourier) is proposed the method of solving the equations of Malyuzhints for the tasks of diffraction in the touching angular ranges when several wedges have one common edge/fin and general/common/total internal faces, on which the pressure and normal speed are continuous. On the external faces are examined uniform boundary conditions of the type of the Dirichlets, Neumann.

In the work is used the representation

$$\sigma(\alpha) = \int_{-\infty}^{+\infty} t(\alpha, \beta)\, e^{-\alpha\beta}\, d\beta$$

$$(\sigma(\alpha) \div t(\alpha, \beta)),$$

which is the generalization of Laplace's bilateral integral [4] (Fourier) and represents the function of one argument $\sigma(\alpha)$ through the function of two arguments $t(\alpha, \beta)$. If $t(\alpha, \beta)$ does not depend on α, then this representation passes into Laplace's integral.

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Convenience in this representation lies in the fact that in some tasks function $t(\alpha, \beta)$ it is possible, without coming into conflict with the conditions of task, to consider it periodic on the argument α, while function $\sigma(\alpha)$ is not periodic. For example, if $\sigma(\alpha)$ satisfies the functional equation

$$\sigma(\alpha) = q(\alpha)\, \sigma(\alpha + \Phi) + f(\alpha),$$

where $q(\alpha)=q(\alpha+\pi)$ - periodic function with the period π, $f(\alpha)$ is represented in the form $f(\alpha)=\int_{-\infty}^{\infty}F(\beta)e^{-\alpha\beta}d\beta$. A sufficient condition for $S(\alpha)$ on the left side of representation $S(\alpha)\doteqdot T(\alpha, \beta)$ was converted into zero, is inversion into zero for any α, β function $T(\alpha, \beta)$. Then, after using representation $\sigma(\alpha)\doteqdot t(\alpha, \beta)$, we obtain for $t(\alpha, \beta)$ the equation

$$t(\alpha, \beta)=q(\alpha)e^{-\Phi\beta}t(\alpha+\Phi, \beta)+F(\beta).$$

The solution of this equation can be registered in the form

$$t(\alpha, \beta)=F(\beta)L_{\Phi}[q(\alpha)e^{-\Phi\beta}],$$

where $L_{\Phi}[q(\alpha)e^{-\Phi\beta}]=1+\sum_{k=0}^{\infty}\prod_{\nu=0}^{k}q(\alpha+\nu\Phi)e^{-\Phi\beta}$. On the basis of the fact that $q(\alpha)=q(\alpha+\pi)$, function $L_{\Phi}[q(\alpha)e^{-\Phi\beta}]$ is periodic with the period π. In special cases (assuming that the series/row descends, for example, when $|q(\alpha)|<1$, $-\infty<\alpha<+\infty$, $\Phi\beta>0$), we have

$$L_{\pi}[q(\alpha)e^{-\Phi\beta}]=\frac{1}{1-q(\alpha)e^{-\Phi\beta}},$$

$$L_{\pi\frac{m}{l}}[q(\alpha)e^{-\Phi\beta}]=\frac{1+\sum\limits_{\mu=0}^{l-2}\prod\limits_{\nu=0}^{\mu}q\left(\alpha+\frac{\pi m}{l}\nu\right)e^{-\Phi\beta}}{1-\prod\limits_{\nu=0}^{l-1}q\left(\alpha+\frac{\pi m}{l}\nu\right)e^{-\Phi\beta}}.\quad (m,l=1,2\ldots).$$

Function $t(\alpha, \beta)$ has also a period π in the argument α, i.e., the same as coefficient of $q(\alpha)$ in the initial functional equation. This fact makes it possible to use a representation $\sigma(\alpha)\doteqdot t(\alpha, \beta)$ during the solution of functional equations with the periodic coefficients of sufficiently general view and to find the solutions of some problems of diffraction in the wedge regions.

One of the tasks of such type is the following. In the cylindrical coordinate system (r, ϑ, z) we examine the case of wave motion when sound pressure harmonically ($e^{-i\omega t}$) depends on time and does not depend on coordinate z. Are examined two homogeneous isotropic media by density ρ, ρ_1(ρ=mρ_1) and by speed of sound c, c_1(c=nc_1), the filling respectively two wedges with the aperture angles Φ, $\Phi_1 : 0 \leqslant \vartheta \leqslant \Phi; -\Phi_1 \leqslant \vartheta \leqslant 0$.

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The acoustic pressures p, p_1 in the wedges, which satisfy wave equations $(\Delta+k^2)p=0$, $(\Delta+k^2n^2)p_1=0$, are represented by Sommerfield's integrals:

$$p = \frac{1}{2\pi i}\int_\gamma e^{-ikr\cos(\vartheta-\alpha)} s(\alpha)\, d\alpha \qquad (0 < \vartheta < \Phi,\ 0 \leqslant r \leqslant \infty);$$

$$p_1 = \frac{1}{2\pi i}\int_\gamma e^{-ikrn\cos(\vartheta-\alpha)} s_1(\alpha)\, d\alpha \qquad (-\Phi_1 \leqslant \vartheta \leqslant 0,\ 0 \leqslant r \leqslant \infty).$$

where γ - Sommerfeld duct/contour of the integration on the plane α=Re α+i Im α. As the incident wave is chosen the plane wave of single amplitude $e^{-ikr\cos(\vartheta-\vartheta_0)}$, falling from infinity (r=∞) to edge/fin r=0 of wedge (ρ, c) at angle ϑ_0 $(0 \leqslant \vartheta_0 < \Phi)$ to the general/common/total face of wedges ϑ=0. On the general/common/total face of wedges with ϑ=0 the pressures and normal speeds are considered continuous. External faces when $\vartheta = \Phi$, $\vartheta = -\Phi_1$ are assumed to be free ones, and on them pressure must become zero. From the properties of Sommerfield's

integral [1] it follows that for satisfaction of the enumerated conditions function $s(\alpha)$ must have a special feature/peculiarity of form $1/\alpha-\vartheta_0$ in band $0 \leqslant \operatorname{Re}\alpha \leqslant \Phi$, function $s_1(\alpha)$ must be regular in band $-\Phi < \operatorname{Re}\alpha < 0$ and $s(\alpha)$, $s_1(\alpha)$ must satisfy the system of the functional equations

$$s(\alpha+\Phi) = s(-\alpha+\Phi);\ s_1(\alpha-\Phi_1) = s_1(-\alpha-\Phi_1);$$
$$s(\alpha) - s(-\alpha) = \tau(\alpha)\{s_1[\varphi(\alpha)] - s_1[-\varphi(\alpha)]\};$$
$$s(\alpha) + s(-\alpha) = n\{s_1[\varphi(\alpha)] + s_1[-\varphi(\alpha)]\},$$

where $\varphi(\alpha) = \arccos\frac{\cos\alpha}{n}$ – the branch of the multiple-valued function $\arccos(\cos\alpha)/(n)$ (branch point $\alpha=\pm\arccos n+k\pi$ ($k=0, \pm1$ ...) are connected in pairs by sections/cuts), that satisfies condition $\varphi(\alpha)=\alpha$ with $n=1$, $\tau(\alpha) = \frac{d\varphi(\alpha)}{d\alpha} = \frac{\sin\alpha}{\sqrt{n^2-\cos^2\alpha}}$. Functions $\varphi(\alpha)$, $\tau(\alpha)$ possess properties [1]: $\varphi(-\alpha)=-\varphi(\alpha)$; $\tau(-\alpha)=\tau(\alpha)$; $\varphi(\alpha+\pi)=\varphi(\alpha)+\pi$; $\tau(\alpha+\pi)=\tau(\alpha)$. Functional equations for $s(\alpha)$, $s_1(\alpha)$ are a special case of functional equations for sector media [1].

We use for $s(\alpha)$, $s_1(\alpha)$ representations $s(\alpha)\doteq t(\alpha,\beta)$; $s_1[\varphi(\alpha)] \doteq t_1(\alpha,\beta)$ and we will consider that $\Phi_1 = \frac{\pi}{2}l$ $(l=1, 2, 3)$. Continuing representation $s(\alpha)\doteq t(\alpha,\beta)$ for the pole $\alpha=\vartheta_0$, we will obtain by analogy with the results of work [1]:

$$s(\alpha+\Phi) = \int_{-\infty}^{+\infty} [t(\alpha+\Phi,\beta) + e^{\vartheta_0\beta}]\, e^{-\alpha\beta}\, d\beta.$$

Since $\tau(\alpha)=\tau(\alpha+\pi)$, then functions $t(\alpha,\beta)$, $t_1(\alpha,\beta)$ have on α the period π. Taking into account everything said, we will obtain for $t(\alpha,\beta)$, $t_1(\alpha,\beta)$ the functional equations:

$$[t(\alpha+\Phi,\beta)+e^{\beta u_0}]\,e^{-\Phi\beta}=[t(-\alpha+\Phi;-\beta)+e^{-\beta u_0}]\,e^{\Phi\beta};$$

$$t_1(\alpha,\beta)=t_1(-\alpha,-\beta)\,e^{-2\Phi_1\beta};$$

$$t(\alpha,\beta)-t(-\alpha,-\beta)=\tau(\alpha)\,[t_1(\alpha,\beta)-t_1(-\alpha,-\beta)];$$

$$t(\alpha,\beta)+t(-\alpha.-\beta)=m\,[t_1(\alpha,\beta)+t_1(-\alpha,-\beta)],$$

whence

$$t(\alpha,\beta)=[t(\alpha+2\Phi,\beta)\,e^{-2\Phi\beta}+2e^{-\Phi\beta}\,\mathrm{sh}\,\beta\,(\vartheta_0-\Phi)]\,\frac{m-\tau(\alpha)\,\mathrm{th}\,\Phi_1\beta}{m+\tau(\alpha)\,\mathrm{th}\,\Phi_1\beta}.$$

The solution of latter/last equation can be registered in the form

$$t(\alpha,\beta)=2e^{\Phi\beta}\mathrm{sh}\beta\,(\vartheta_0-\Phi)\left\{L_{2\Phi}\left[e^{-2\Phi\beta}\frac{m-\tau(\alpha)\,\mathrm{th}\,\Phi_1\beta}{m+\tau(\alpha)\,\mathrm{th}\,\Phi_1\beta}-1\right]\right\}.$$

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If, for example, $\Phi=\Phi_1=\frac{\pi}{2}$, then

$$t(\alpha,\beta)=\frac{2\,\mathrm{sh}\,\beta\,(\vartheta_0-\pi/2)\left[m\,\mathrm{ch}\,\frac{\pi}{2}\beta-\tau(\alpha)\,\mathrm{sh}\,\frac{\pi}{2}\beta\right]}{[m+\tau(\alpha)]\,\mathrm{sh}\,\pi\beta},$$

whence

$$s(\alpha)=\frac{\cos\vartheta_0\pi}{\sin\alpha-\sin\vartheta_0}+\frac{m-\tau(\alpha)}{m+\tau(\alpha)}\cdot\frac{\cos\vartheta_0\pi}{\sin\alpha+\sin\vartheta_0}.$$

The poles of function $s(\alpha)$, located in band $-\pi<\alpha<\pi$ with $\alpha=\pm\vartheta$, $\alpha=\pm(\pi-\vartheta_0)$, make it possible to compute the integral of Sommerfield and to obtain field in wedge $(0, \pi/2)$ in the form of the sum of four plane waves.

If we assume that on face $\vartheta=\frac{\pi}{2}\left(\Phi=\frac{\pi}{2}\right)$ the pressure is equal to zero, and on face $\vartheta=-\frac{\pi}{2}\left(\Phi_1=\frac{\pi}{2}\right)$ normal speed becomes zero, then,

functioning similarly, we will have

$$s(\alpha)=\frac{1}{\sin t_2}\left[\gamma\frac{\sin\left[\frac{\vartheta_0-\alpha}{\pi}(\pi-t_2)\right]}{\sin(\vartheta_0-\alpha)}+\gamma\frac{\sin\left[\frac{\vartheta_0+\alpha-\pi}{\pi}(\pi-t_2)\right]}{\sin(\vartheta_0+\alpha)}+\right.$$
$$\left.+\frac{\sin\frac{\vartheta_0+\alpha}{\pi}(\pi-t_2)}{\sin(\vartheta_0+\alpha)}+\frac{\sin\frac{\vartheta_0-\alpha-\pi}{\pi}(\pi-t_2)}{\sin(\vartheta_0-\alpha)}\right],$$

where $\gamma=\frac{m-\tau(\alpha)}{m+\tau(\alpha)}$; $\cos t_2=\gamma\ (0\leqslant t_2\leqslant\pi)$. Terms (by way of recording) have pole with $\alpha=-\pi+\vartheta_0$; $\alpha=-\vartheta_0$; $\alpha=\pi-\vartheta_0$; $\alpha=\vartheta_0$.

The method proposed makes it possible to solve the systems of the Malyuzhints functional equations, which correspond to diffraction in the wedge of arbitrary aperture angle, to both faces of which adjoin the wedges with the aperture angles, multiple $\pi/2$, in the liquid wedge (or two wedges) of arbitrary aperture angle, which lies on the elastic half-space, and also when four rectangular wedges are in contact by faces and have one common edge/fin.

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14. Use of particular solutions of special form.

Green tensor functions of Maxwell equations for tube domains.

B. A. Panchenko.

The solution of the equations of Maxwell

$$\nabla \times \mathbf{H} = j\omega\varepsilon\mathbf{E} + \mathbf{j}^{э}, \quad -\nabla \times \mathbf{E} = j\omega\mu\mathbf{H} + \mathbf{j}^{м} \tag{1}$$

in certain region it is possible to represent with the help of the tensor functions of Green

$$\begin{aligned} \mathbf{E}(\mathbf{r}) &= \int_{V'} [\Gamma_{11}(\mathbf{r}, \mathbf{r}')\,\mathbf{j}^{э}(\mathbf{r}') + \Gamma_{12}(\mathbf{r}, \mathbf{r}')\,\mathbf{j}^{м}(\mathbf{r}')]\, dv', \\ \mathbf{H}(\mathbf{r}) &= \int_{V'} [\Gamma_{21}(\mathbf{r}, \mathbf{r}')\,\mathbf{j}^{э}(\mathbf{r}') + \Gamma_{22}(\mathbf{r}, \mathbf{r}')\,\mathbf{j}^{м}(\mathbf{r}')]\, dv', \end{aligned} \tag{2}$$

V' - the volume of the distribution of outside currents $j^{э}$ and $j^{м}$.

Depending on the calibration of field it is possible after substitution (2) in (1) to obtain two possible representations for tensors $\Gamma(\mathbf{r}, \mathbf{r}')$:

$$\Gamma_{11} = \frac{1}{j\omega\varepsilon}(k^2 \mathbf{e} + \nabla\nabla)\, G^{э}, \quad \Gamma_{12} = -\nabla \times G^{м}$$

$$k = \omega\sqrt{\varepsilon\mu}, \quad \mathbf{e} — \text{ед. тензор}$$

$$\Gamma_{21} = \nabla \times G^{э}, \quad \Gamma_{22} = \frac{1}{j\omega\mu}(k^2 \mathbf{e} + \nabla\nabla)\, G^{м}. \tag{3}$$

Key: (1). un. tensor.

or

$$\Gamma_{11} = -j\omega\mu G_l^{э}, \ \Gamma_{12} = -\nabla \times G_l^{м},$$
$$\Gamma_{21} = \nabla \times G_l^{э}, \ \Gamma_{22} = -j\omega\varepsilon G_l^{м}.$$

Functions $G^{э,м}$ and $G_l^{э,м}$ satisfy in this case the affinor equations of the following form

$$\nabla^2 G(\mathbf{r}, \mathbf{r}') + k^2 G(\mathbf{r}, \mathbf{r}') = -\delta(\mathbf{r} - \mathbf{r}')\, E. \quad (4)$$

Further is examined the solution of equations (4) in the systems of orthogonal coordinates for which one of the axes is Cartesian. Functions $G^{э}$ and $G^{м}$ are decomposed/expanded in the series/row according to the vector eigenfunctions of the homogeneous vector equation of Helmholtz - $\mathbf{L}_{mn}, \mathbf{M}_{mn}$ and $\mathbf{N}_{mn}$. Functions $G_l^{э}$ and $G_l^{м}$ are decomposed/expanded along systems $\mathbf{M}_{mn}$ and $\mathbf{N}_{mn}$ and are called transverse. For example, expansion $G^{э}$ takes the form

$$G^{э}(\mathbf{r}, \mathbf{r}') = \sum_{m,n} \left\{ \mathbf{a}_3\mathbf{a}_3 \frac{1}{\Lambda_{mn}^n} \chi_{mn}(\mathbf{r}) \chi_{mn}(\mathbf{r}') g_{mn} + \frac{1}{(k_{mn}^2)^2 \Lambda_{mn}^2} [\mathbf{a}_3 \times \nabla\psi_{mn}(\mathbf{r})] \times [\mathbf{a}_3 \times \nabla\psi_{mn}(\mathbf{r}')] f_{mn} + \frac{1}{(k_{mn}^n)^2 \Lambda_{mn}^n} [\nabla\chi_{mn}(\mathbf{r})] [\nabla\chi_{mn}(\mathbf{r}')] f_{mn} \right\}, \quad (5)$$

where $\mathbf{a}_3$ - unit vector along the Cartesian axis (axis Oz); $\nabla = \mathbf{a}_1 \frac{1}{h_1}\frac{\partial}{\partial\zeta_1} + \mathbf{a}_2 \frac{1}{h_2}\frac{\partial}{\partial\zeta_2}$; ζ_1, ζ_2 - curvilinear coordinates in the cross section of the region in question with Lame's coefficients h_1 and h_2;

functions χ_{mn} and ψ_{mn} are the mutually orthogonal their own scalar functions of the two-dimensional equations

$$\nabla^2\chi_{mn}(\zeta_1\zeta_2)+(k^h_{mn})^2\chi_{mn}(\zeta_1\zeta_2)=0,\quad \nabla^2\psi_{mn}(\zeta_1\zeta_2)+(k^2_{mn})^2\psi_{mn}(\zeta_1\zeta_2)=0.$$

On the conducting boundaries of the region must be implemented the boundary conditions

$$\chi_{mn}=\frac{\partial}{\partial n}\psi_{mn}=0,$$

Λ^h_{mn} and Λ^2_{mn} - the norm of eigenfunctions; functions f_{mn} and g_{mn} satisfy the ordinary differential equations

$$(\partial^2/\partial z^2)f_{mn}-\gamma^2_{mn}f_{mn}=-\delta(z-z')\quad \gamma_{mn}=\sqrt{k^2_{mn}-k^2}.$$

Being given different boundary conditions for f_{mn} and g_{mn}, it is possible to obtain the infinite, semi-infinite and limited regions along coordinate z.

For the external boundary-value problems when the domain of definition of solution is unconfined, the spectrum of eigenvalues k^h_{mn} and k^2_{mn} continuous. Green's corresponding function is obtained by the replacement of series/row in (5) to the integral according to the wave numbers.

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After are found the components of tensors $G^э$ and $G^м$ or $G^э_t$ and $G^м_t$, tensors Γ are determined by expressions (3). In this case in general form it is possible to convert (without passing to the

concrete/specific/actual coordinate system). As a result are obtained the scalar components of each of the tensors Γ. For example,

$$\Gamma_{21}(\mathbf{r}, \mathbf{r}') = \sum_{m,n} \Big\{ \mathbf{a}_1\mathbf{a}_1 \Big[-\frac{1}{\Lambda^h_{mn}} \cdot \frac{1}{(k^h_{mn})^2} \cdot \frac{1}{h_2} \frac{\partial}{\partial \zeta_2} \chi_{mn}(\mathbf{r}) \frac{1}{h'_1} \frac{\partial}{\partial \zeta'_1} \chi_{mn}(\mathbf{r}') f_{mn} +$$

$$+ \frac{1}{\Lambda^2_{mn}(k^2_{mn})^2} \frac{1}{h_1} \frac{\partial}{\partial \zeta_1} \psi_{mn}(\mathbf{r}) \frac{1}{h'_2} \frac{\partial}{\partial \zeta'_2} \psi_{mn}(\mathbf{r}') f_{mn} \Big] +$$

$$+ \mathbf{a}_1\mathbf{a}_2 [\ldots] + \mathbf{a}_1\mathbf{a}_3 [\ldots] + \ldots\ldots\ldots\ldots + \mathbf{a}_3\mathbf{a}_2 [\ldots] + \mathbf{a}_3\mathbf{a}_3 \cdot 0 \Big\}$$

Are determined functions χ_{mn} and ψ_{mn}, of their norm and eigenvalues for the numerous regions in the cylindrical coordinate systems. In many instances instead of the labor-consuming integration of the squares of eigenfunctions for determining the norms was used the method of Titchmarsh, which gives the expansion of arbitrary function in the series/row or integral of orthonormal sets of the functions of differential equations of Sturm-Liouville's type.

The method of Green's tensor functions gives in the single form solution for a whole series of the internal and external electrodynamic tasks, which have wide practical use/application. The symbolism of Green's tensors is very convenient during composition and solution of vector integral equations. At the variation formulation of boundary problems such important parameters of electrodynamic devices/equipment as equivalent resistance and the conductivity of heterogeneities in the waveguides, resonance frequencies of resonators, impedance characteristics of antennas, efficient diameter of scattering with diffraction, etc., are expressed in the form of the bilinear functionals, which include Green's tensors.

The particular integral of the equation of thermal conductivity and equations of the nonlinear vibrations of anisotropic bodies with variable physical characteristics.

L. M. Galonen.

The equation of the thermal conductivity of anisotropic bodies with the variable/alternating thermal characteristics is given, as is known, to the form

$$\frac{\partial u}{\partial t}=\frac{\partial}{\partial x}\left(a\frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial y}\left(b\frac{\partial u}{\partial y}\right)+\frac{\partial}{\partial z}\left(c\frac{\partial u}{\partial z}\right)+f. \qquad (1)$$

During the warm-up of humid materials, heating of metals, etc. the coefficients of thermal diffusivity a, b and c vary with a change in the temperature and equation (1) becomes nonlinear.

If the power of the internal thermal source f is also function from the temperature, then for finding the integral of equation (1) let us replace with its equivalent system of equations with four unknown functions:

$$\left.\begin{aligned}
&T=\frac{\partial}{\partial x}(ap)+\frac{\partial}{\partial y}(bq)+\frac{\partial}{\partial z}(cl)+f(u),\\
&\frac{\partial u}{\partial x}=p;\ \frac{\partial u}{\partial y}=q;\ \frac{\partial u}{\partial z}=l;\ \frac{\partial u}{\partial t}=T,\\
&\frac{\partial p}{\partial y}=\frac{\partial q}{\partial x};\ \frac{\partial p}{\partial z}=\frac{\partial l}{\partial x};\ \frac{\partial p}{\partial t}=\frac{\partial T}{\partial x};\ \frac{\partial q}{\partial z}=\frac{\partial l}{\partial y},\\
&\frac{\partial q}{\partial t}=\frac{\partial T}{\partial y};\ \frac{\partial T}{\partial z}=\frac{\partial l}{\partial t}.
\end{aligned}\right| \qquad (2)$$

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Taking those integrals of system (2), for which functions p, q, 1 T depend only on u, and joining to it equation in total differentials:

$$du = p\,dx + q\,dy + l\,dz + T\,dt, \qquad (3)$$

and integrating, we will obtain equation for determining of p, and then the solution of equation (1).

In particular, for the unsteady process of thermal conductivity without the internal thermal source, i.e., with f=0, solution will be:

$$\int \frac{a(u) + \alpha^2 b(u) + \beta^2 c(u)}{t + \varepsilon}\, du = \delta(\alpha x + \beta y + \gamma z + \delta t) + \rho. \qquad (4)$$

The integral of the equation of stationary task with the source takes the form

$$\int \frac{a(u) + \alpha^2 b(u) + \beta^2 c(u)}{\sqrt{\varepsilon - \int (a + \alpha^2 b + \beta^2 c) f(u)\, du}}\, du = x + \alpha y + \beta z + \delta. \qquad (5)$$

Constants $\alpha, \beta, \gamma, \delta, \varepsilon, \rho$ are determined from the boundary conditions. For example, for the one-dimensional stationary task with the boundary first-order conditions

$$u(0) = u_0, \quad u(R) = u_1$$

solution will be obtained by exception/elimination C from the equations:

$$\left.\begin{aligned}\int_{u_0}^{u}\frac{a(u)\,du}{\sqrt{C-2\int a(u)\,f(u)\,du}}&=x,\\ \int_{u_0}^{u_1}\frac{a(u)\,du}{\sqrt{C-2\int a(u)\,f(u)\,du}}&=R.\end{aligned}\right\}$$

Analogously is solved both problem with the boundary conditions of the second and third kind and also task with the nonlinear boundary conditions. Equation (1) can have periodic solution, if $\delta=-m^2$. Method is applicable to the integration of the equation of the nonlinear vibrations of the anisotropic media of the form

$$\frac{\partial^2 u}{\partial t^2}=\frac{\partial}{\partial x}\left[a(u)\frac{\partial u}{\partial x}\right]+\frac{\partial}{\partial y}\left[b(u)\frac{\partial u}{\partial y}\right]+\frac{\partial}{\partial z}\left[c(u)\frac{\partial u}{\partial z}\right]. \qquad (6)$$

In this case the solution of unsteady problem with p, q, l, t, which depend on u, will be

$$\int[\gamma^2-a(u)-\alpha^2 b(u)-\beta^2 c(u)]\,du=\delta(x+\alpha y+\beta z+\gamma t)+e.$$

The integral of equation (6) for the stationary task of forced oscillations under the action of force f(u) takes the form

$$\int\frac{a(u)+\alpha^2 b(u)+\beta^2 c(u)}{\sqrt{C-2\int(a+\alpha^2 b+\beta^2 c)\,f(u)\,du}}\,du=x+\alpha y+\beta z+\delta.$$

The periodicity of process can be provided with the appropriate selection of arbitrary constants.

Problem with the boundary conditions is solved analogously with the equation of thermal conductivity.

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Reasonings easily are generalized to the case of multidimensional region. The solution of the problem of thermal conductivity, for example, will be in this case

$$\int \frac{a_1(u) + \sum_{2}^{n} \alpha_i^2 a_i(u)}{t + \varepsilon} = \delta\left(\sum_{i=1}^{n} \alpha_i x_i + \delta t\right) + \varepsilon.$$

The independent propagation of longitudinal and transverse waves in some elastic inhomogeneous media.

V. Yu. Zavadsky.

In the Cartesian system of coordinates xyz is examined the elastic lamellar-heterogeneous medium in which the density ρ and the parameters of Lame λ, μ are changed only depending on coordinate z as continuous, differentiated at least twice, function $\rho(z)$, $\lambda(z)$, $\mu(z)$. Displacement vector U(x, y, z, t) is considered that located in plane xz, not depending on y and depending on x, t according to factor exp $[-i\omega t+i\xi x]$. Accordingly equalities $U=\nabla\varphi+\nabla\times(\psi e_y)$ are introduced scalar φ and vector ψe_y the potentials, connected in accordance with displacement $U_e=\nabla\varphi$ in longitudinal waves $(\nabla\times U_e=0)$ and in the transverse ones: $U_t=\nabla\times(\Psi e_y)(\nabla U_t=0)$. Moreover $U=U_e+U_t$. Longitudinal and transverse waves correspond to the strains, connected only with a change in the volume of the element/cell of medium or only with a change in its form. It is known that in the dynamic theory of elastic media the method of the representation of the field of displacement in the form of the sum of two independent fields, which relate to different types of the strain of medium, has high value. In particular, in the homogeneous elastic medium waves of both types are

propagated independently, interacting with each other only on the boundaries, free (rigidly attached) or separating/liberating homogeneous media with different properties. In some layered inhomogeneous elastic media longitudinal and transverse waves are also propagated independently.

In the reported work it is shown that in the elastic medium with the power law of a change in the parameters ρ, μ: $\rho(z)=\rho(z_0)(z/z_0)^{-2}$, $\mu(z)=\mu(z_0)(z/z_0)^{-1}$ and the arbitrary function $\lambda(z)$ (continuous and differentiated twice) can extend such transverse waves, which are not converted into the longitudinal ones. In this case the potential φ is identically equal to zero, and potential ψ takes the form

$$\psi = z\left[C_1 W_{\frac{\nu}{2},\frac{3}{2}}(2\xi z) + C_2 W_{-\frac{\nu}{2},\frac{3}{2}}(-2\xi z)\right]$$

where $\nu = \omega^2\rho(z_0)\, z_0 \xi^{-1}[\mu(z_0)]^{-1}$, $W_{k,s}(x)$ - Whittaker function; C_1, C_2 - arbitrary constants.

The opposite case when $\psi \equiv 0, \varphi \not\equiv 0$, is also feasible for some layered inhomogeneous elastic media with the continuous and twice differentiated values ρ, λ, μ. Connections/communications, superimposed on ρ, λ, μ, are reduced to the equations

$$\frac{\mu'}{\rho} = \alpha = \text{const}; \quad (\lambda + 2\mu)\mu'' - 2\mu'^2 = 0 \quad \left(' = \frac{d}{dz}\right).$$

Under these conditions the potential φ is defined as the general solution of the differential the second order equation:

$$\varphi'' + \frac{\rho'}{\rho}\varphi' + \left(\frac{\omega^2}{2\alpha}\frac{\rho'}{\rho} - \xi^2\right)\varphi = 0.$$

As examples are examined some special cases of the elastic layered inhomogeneous media in which one type waves can extend without the conversion into the waves of another type.

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15. Computational methods.

Development of one computational method in a theory of diffraction.

G. D. Malyuzhinets, A. V. Popov, Yu. N. Cherkashin.

The proposed by one of the authors [1] method of calculating the diffraction fields with the short waves is based on the numerical solution of the proposed by them [2] equation

$$ik\left[2\frac{\partial u}{\partial\xi}+\frac{\partial\ln(nh_\eta h_\zeta)}{\partial\xi}u\right]+\frac{1}{nh_\eta h_\zeta}\left[\frac{\partial}{\partial\eta}\left(\frac{h_\zeta}{nh_\eta}\frac{\partial u}{\partial\eta}\right)+\frac{\partial}{\partial\zeta}\left(\frac{h}{nh_\zeta}\frac{\partial u}{\partial\zeta}\right)\right]=0, \quad (1)$$

of that describing in ray coordinates ξ, η, ζ the transverse diffusion of wave amplitude $\left(h_\xi=\frac{1}{n}, n\right.$ - refractive index, $k=\omega/c_0$; c_0 - velocity of propagation at the point where is arranged/located source). It is obtained by the rejection of the corresponding terms in the precise equation

$$\left(\frac{\partial^2 u}{\partial\xi^2}+2ik\frac{\partial u}{\partial\xi}\right)+\frac{\partial\ln nh_\eta h_\zeta}{\partial\xi}\left(\frac{\partial u}{\partial\xi}+iku\right)+\frac{1}{nh_\eta h_\zeta}\left[\frac{\partial}{\partial\eta}\left(\frac{h_\zeta}{nh_\eta}\frac{\partial u}{\partial\eta}\right)+\frac{\partial}{\partial\zeta}\left(\frac{h_\eta}{nh_\zeta}\frac{\partial u}{\partial\zeta}\right)\right]=0$$

for amplitude u, entering the expression of satisfying equation $p=ue^{ik\xi}$, Helmholtz's wave field

$$\Delta p+k^2n^2p=0 \quad (p\sim e^{-i\omega t}).$$

Function p represents only one of the components/terms/addends of the unknown solution of diffraction problem, allowing/assuming

introduction of the ray coordinates, to which value $e^{ik\xi}$ is the main oscillating factor for p with $k \to \infty$.

Instead of equation (1) it is sometimes more convenient (see below) to use equation [2, 3]

$$2ik\frac{\partial v}{\partial \xi}+\frac{1}{\sqrt{nh_\eta h_\zeta}}\left[\frac{\partial}{\partial \eta}\left(\frac{h_\zeta}{nh_\eta}\frac{\partial}{\partial \eta}\frac{v}{\sqrt{nh_\eta h_\zeta}}\right)+\frac{\partial}{\partial \zeta}\left(\frac{h_\eta}{nh_\zeta}\frac{\partial}{\partial \zeta}\frac{v}{\sqrt{nh_\eta h_\zeta}}\right)\right]=0 \qquad (2)$$

for the "ray" amplitude (function of weakening), obtained from (1) by the substitution

$$u=\frac{v}{\sqrt{nh_\eta h_\zeta}}.$$

In the case of two-dimensional problem when $n=n(\xi,\eta)$, $h_\zeta=1$, $\frac{\partial u}{\partial \zeta}=0$, the corresponding terms in (1) and (2) vanish.

For the amplitude of field in the shadow zones after the caustics occur the analogous differential equations, registered in the complex ray coordinates.

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Into the sphere of the applicability of method enter the tasks of diffraction only in such regions and for such functions of initial data in source, for which complete field in the final subregion, interesting us, which contains source, can be with the sufficient accuracy represented by the final sum of the partial fields (wave: falling, reflected primarily, for a second time and so forth from the

boundaries of the region or from the caustic curves, that penetrate abroad of geometric shadow or for the caustic curve), which have their subregions of the existence where these fields allow/assume the introduction of the single-valued system of ray coordinates. In each of such subregions for the partial field after the isolation/liberation from it of the main oscillating factor can be registered the equation of transverse diffusion. The set of these equations taking into account the conditions of coupling of partial fields of amplitudes on the boundaries, which divide the subregions indicated, makes it possible to examine transverse diffusion at the united fronts ξ-const, which intersect entire set of the subregions, into which these fronts are continued. In the case of point source, arranged/located at point $\xi=0$ and the boundaries of the region, which are cambered inwards, such united fronts remain locked (although they can branch on the caustics), beginning from $\xi=0$ up to this value $\xi=\xi_1$, where certain ray/beam $\eta=\mathrm{const}, \xi=\mathrm{const}$ concerns boundary of the region, forming the initial point-geometric shadow. Beginning from such points with the continuations of similar rays/beams they are some geodetic covering the shaded part of the boundary of the region. The here united fronts ξ-const are finished, but the initially assigned uniform boundary condition on the shaded part of the boundary it proves to be sufficient for the single-valued determination of the field of amplitudes. In this case the assignment of the value of the amplitude of field on $\xi=\xi_0$ near the source plays

the role of initial condition. The replacement of the equations of transverse diffusion by difference equations, with the use of the corresponding conditions for coupling on the boundaries between the subregions, makes it possible to produce with certain space on ξ the consecutive calculation of fields at the united fronts with the dispersion method.

Thus is computed in the approximation/approach of transverse diffusion the field of amplitudes at the fronts, which is zero approximate and is designated through u_0. With the necessity to obtain following correction u_1 from that found u_0 is computed the function

$$-\left(\frac{\partial^2 u}{\partial \xi^2} - \frac{1}{nh_\eta h_\zeta}\frac{\partial (nh_\eta h_\zeta)}{\partial \xi}\frac{\partial u_0}{\partial \xi}\right),$$

being the right side of the equation for u_1, while left side retains the same form, that in (1). With the help of this equation calculation u_1 is produced just as for u_0, with the difference that the initial condition for u_1 with $\xi=\xi_0$ is taken by zero. In perfect analogy is produced the calculation also of the following corrections. Sum $u = \sum_{i=0}^{N} u_i$ presents solution for the amplitude at the front in the necessary approximation/approach. For the determination of the resulting field (but not amplitude) at certain assigned point of space x, y, z it is necessary to determine all corresponding to it and lying/horizontal on different fronts points, to multiply the values of amplitude in them for appropriate factors $e^{ik\xi}$ and finally

to sum all products. This sum gives the resulting diffraction field.

For obtaining the solution of the unsteady problem of diffraction for the sufficiently smoothly modulated oscillations/vibrations of source according to the law $f(t)e^{i\omega t}$ is required a very small change in the method. During the use, for example, only of zero approximation u_0 it relates only to the latter/last stage and is reduced to the additional multiplication of the separate members of the sum, which presents the resulting steady-state solution, to the appropriate factors $f(t-\xi/C_0)$.

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Absence at present sufficiently complete proof with the estimations of error in the method described above causes the necessity for the experimental check based on the examples, which allow/assume convenient comparison with a strict solution. In the report are given the results of this comparison for the case of two-dimensional problem of diffraction on the wedge under the zero boundary condition on the faces. Since the fields of the amplitudes of the incident and geometrically reflected wave in this case are accurately known (1 and -1), according to the calculation on computer(s) undervent only the wave, scattered by the edge/fin of wedge. The ray amplitude v for this wave is approximately

subordinated to equation (2), which in this case obtains the form

$$2ik\,\frac{\partial v}{\partial r}+\frac{1}{r^2}\,\frac{\partial^2 v}{\partial \varphi^2}=0. \qquad (3)$$

The right side of the task are the assigned magnitudes of jumps under coupling conditions on the rays/beams, which correspond to the boundaries of shadow for the incident and geometrically reflected wave. The united fronts are the circular arcs r=const, which are finished on the faces of wedge, along which slip extreme rays/beams φ=const.

For replacing equation (3) by the finite-difference was selected the six-point implicit diagram whose stability in application to this equation specially was investigated. The asymptotic values of field calculated with the help of computer(s) in fraunhofer region [3] proved to be in a good agreement with a strict solution of Sommerfield.

Calculation by the same method of diffraction fields the inhomogeneous medium needs the preliminary determination of ray coordinates and coefficients of Lame, expressed in these coordinates, entering equation (1) or (2), which by itself represents complex problem. Two-dimensional problem of this type for the arbitrary refractive index, which depends only from one Cartesian coordinate, also is examined in the report. For the purpose of convenience in the use/application to the calculations of the transverse diffusion of

amplitude the task is placed as reverse/inverse with respect to the usual task of geometric optic/optics. namely, instead of the calculation of fronts ξ=const and rays/beams η=const in coordinates x, y are computed values x, y, which correspond to the values of ray coordinates ξ, η.

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Method of steady state for the multidimensional integrals.

M. V. Fedoryuk.

1. Contribution from boundary.

The integrals of the form

$$F(k) = \int_D e^{ikf(x)} \varphi(x)\, dx, \qquad (1)$$

where D - region in E^n, $x = (x_1, \ldots, x_n)$, $\partial D = \Gamma$, frequently are encountered in the theory of diffraction.

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The contribution from the boundary to F(k) it is called

$$F_1(k) = \int_{D_\Gamma} e^{ikf(x)} \varphi(x) \varphi_1(x)\, dx, \qquad (2)$$

where $D_\Gamma \subset D$ - ridge lengthwise Γ, $\varphi_1(x) \equiv 1$ when $x \in \Gamma$, $\varphi_1(x) \equiv 0$ on remaining boundary D_Γ together with all derivatives, $\varphi_1(x) \in C^\infty$. Let us assume f, φ, Γ is infinitely differentiated, $\nabla f \neq 0$ on Γ, and Γ does not have singular points. Then with $k \to +\infty$

$$F_1(k) = -\frac{i}{k} \int_\Gamma e^{ikf(x)} \varphi(x)\, \omega + O(k^{-2}); \qquad (3)$$

$$\omega = (-1)^j dx_1 \ldots d\hat{x}_j \ldots dx_n \cdot \left(\frac{\partial f}{\partial x_j}\right)^{-1}. \qquad (4)$$

It is actually sufficient final smoothness f, φ, Γ. Contribution

from the boundary was computed in [1] - [3] under more particular assumptions (3), (4) somewhat simpler than formulas of [1].

2. Contribution from close saddle points.

Let f, φ depend on the real low parameter α, $A(x, \alpha= \left\| \frac{\partial^2 f(x, \alpha)}{\partial x_i \partial x_j} \right\|$, and when $\alpha=0, \nabla f(x^0, 0)=0$ rank r of matrix/die A(x^0, 0) it is less than n. Case of r=n-1 cm. [4]. If r=n-2, then the task about asymptotic behavior F(k) with $k \to +\infty$, $|\alpha|<\delta$ is reduced to the research of the double integral of form (1), where D - plane (x_1, x_2), x^0=(0, 0), f=f_0, $f_0(x, \alpha)=x^3_1+ax^2_1x_2+bx_1x^2_2+x^3_2-\alpha(\alpha_1x_1+\alpha_2x_2)$, if we reject/throw members on the order of 4 and it is above in expansion f_0 in the Taylor series. Let a, b be such, that f_0 does not contain multiple factors. The integral

$$F_0 = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{ikf_0} dx_1 dx_2$$

formally diverges, but it it is possible to make by that converging, after shifting range of integration into complex domain. Let us consider case of k>>1, α<<1, $\alpha k^{2/3}$>>1 and will designate x^j, $1 \leqslant j \leqslant 4$ saddle points f_0. The contribution from x^j to asymptotic behavior F_0 is equal to

$$2\pi k^{-1} D_j^{-\frac{1}{2}} \exp\left(-\frac{2}{3} i\alpha k (\alpha_1 x_1^j + \alpha_2 x_2^j)\right), \qquad (5)$$
$$D_j = \det \| A(x^j, \alpha) \|.$$

Asymptotic behavior F_0 is equal to the sum of the contributions from points x^j, for which exponent in (5) is not growing. Finally with n=2

the contribution from close saddle points to F(k) is equal to

$$(x^j(0) = x^0)\, \varphi(x^0, 0) \exp(ikf(x_0, 0)) \times F_0(k, \alpha).$$

Analogously it is possible to investigate case of r<n-2.

It is possible to still simplify F_0. By the linear replacement of the variable/alternating (for example, see [5]) it is possible to reduce the cubic part of f_0 to form $y^3_1+y^3_2$ (replacement can be complex), and F_0 will be reduced simply to the product of Airy's functions. With n-r<2 this it is not possible to do.

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Matrix methods in tasks of the electromagnetic excitation of the bodies of piecewise-coordinate form.

D. M. Sazonov.

Is proposed the universal algorithm of the numerical solution of the tasks of diffraction and arbitrary excitation of electromagnetic waves on the compounds whose form consists of the correct sections of coordinate surfaces of any orthogonal coordinate system, which allows/assumes separation of variables in the equation of Helmholtz. As basis is assumed the solution of the simpler problem about the determination of the wave scattering matrix of arbitrary order for the joint of two uniform lines of transmission with the different value of cross section. The problem about the joint is solved with the help of the new method, called "mirror-iterative". The essence of this method consists of the repeated use of results of two auxiliary tasks. In the first of them (task I) is assigned the requirement of the inversion into zero of complete tangential electric field on the entire interface of two waveguides, after which is realized the join of complete tangential magnetic fields in the overall section of interface. As a result the amplitude of any incoming to the joint

(incident) wave in second region B_n^I by the relationship/ratio of the form A_ν^I

$$B_n^I = \alpha_n^I \sum_{\nu=\nu_1}^{\nu_{\text{макс}}} \beta_\nu^I A_\nu^I L_{\nu n}, \tag{1}$$

where α_n^I, β_ν^I - some coefficients, depending on the location of joint lengthwise, ν and n - eigenvalues of transmission modes, $L_{\nu n}$ - the scalar product of the vector functions of field distributions in the cross section of waveguides for the harmonics with numbers ν and n. The amplitudes of waves b_ν^I and a_n^I reflected from the joint are expressed as the known coefficients of reflection p_ν^I and p_n^I for the ideal metallic piston in the form

$$b_\nu^I = p_\nu^I A_\nu^I, \quad a_n^I = p_n^I B_n^I. \tag{2}$$

In another auxiliary task II is assigned the requirement of the inversion into zero of complete tangential magnetic field on the entire interface of two waveguides, after which is realized the join of complete tangential electric fields in the general/common/total section of interface with the use of condition $E_{tg} = 0$ on the continuous abutting surface. As a result of join the amplitude of any incoming for the joint (incident) wave in first region A_ν^{II} proves to be connected with all amplitudes of the incident waves in the second region:

$$A_\nu^{II} = \alpha_\nu^{II} \sum_{n=n_1}^{n_{\text{макс}}} \beta_n^{II} B_n^{II} L_{\nu n}. \tag{3}$$

Simultaneously with the help of the coefficients of reflection p_ν^{II} and p_n^{II} from the ideal magnetic piston can be found the amplitudes of waves reflected from the joint in task 2:

$$b_\nu^{II} = p_\nu^{II} A_\nu^{II}, \quad a_n^{II} = p_n^{II} B_n^{II}. \tag{4}$$

Relationships/ratios (1)-(4) are used for the solution of the complete problem of exciting the same joint by primary sources with the known matrix/die of amplitudes $\|A_\nu\|$ as follows.

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First with the help of formulas (1)-(2) are solved auxiliary problem 1, for sources $\|A_\nu^I\| = q\|A_\nu\|$ where q - complex weight coefficient, moreover in the solution appear primary sources $\|B_n^I\|$ according to (1). For their compensation is constructed the auxiliary task II, in which sources $\|B_n^{II}\| = -\|B_n^I\|$ lead to the appearance in the first region of primary sources with matrix/die $\|A_\nu^{II}\|$ according to (3). Then is realized the superposition of auxiliary tasks in the diagram:

$$\begin{array}{l}
+\begin{array}{l}
\|b_\nu^I\| \leftarrow (2) \leftarrow \|A_\nu^I\| \rightarrow (1) \rightarrow \|B_n^I\| \longrightarrow (2) \rightarrow \|a_n^I\| \\
\|b_\nu^{II}\| \leftarrow (4) \leftarrow \|A_\nu^{II}\| \leftarrow (3) \leftarrow \|B_n^{II}\| \longrightarrow (4) \rightarrow \|a_n^{II}\|
\end{array} \\
\hline
\|b_\nu^I + b_\nu^{II}\|;\ \|A_\nu^I + A_\nu^{II}\| = \|A_\nu\| + \underbrace{\|\Delta A_\nu(q)\|}_{\text{(с обратным знаком)}};\ 0;\ \|a_n^I + a_n^{II}\|;
\end{array} \quad (5)$$

(1)

Key: (1). (with the opposite sign).

As a result proves to be known as much as desired precise (if we consider a sufficient number of terms in the series/rows) solution of the problem about the excitation of joint by sources $\|A_\nu^I + A_\nu^{II}\|$,

differing from the given ones to certain addition $\|\Delta A_v(q)\|$, depending on the selection of the complex coefficient q. Forming the norm of this addition and requiring its minimum, we find the necessary value q. After this, if the norm of addition $\|\Delta A_v(q)\|$ is insufficiently small in comparison with the norm of initial sources $\|A_v\|$, should be repeated the process of solution by diagram (5), using instead of the initial sources addition $\|\Delta A_v\|$ with the opposite sign. Thus, the process of calculations acquires cyclic character, and its convergence is guaranteed by the continuous decrease of the norm of uncompensated for remainder/residue $\|\Delta A_v\|$, which is strictly shown in the work. After the calculation of the scattering matrices of two adjacent joints easily can be realized their association with the help of the matrix methods of the theory of the shf circuits. Then is considered the third joint, the fourth and so on. As a result becomes possible the calculation of the arbitrary electromagnetic excitation of comparatively complicated piecewise-coordinate bodies, the calculations occurring consecutively/serially on one and the same comparatively simple standard subroutines. Together with the joints within the framework of matrix method easily is considered heterogeneous magnetic-dielectric filling of any waveguide, assigned in the gaps/intervals between the joints in the form of any number of following after each other uniform layers with piecewise-varying parameters. The use/application of the proposed method is checked in a number of examples: the joints of rectangular waveguides, the

excitation of final ones according to a radius of metallic wedge and band, the calculation of biconical radiator/resonator/element, the arbitrary excitation of disk with opening/aperture, etc. Method is convenient for the machine calculation with the maximum sizes of bodies $\sim(5-7)\lambda$, and also it can be successfully used for obtaining the asymptotic formulas in the long-wave approximation/approach. By small modification method can be spread to the case of the transmission lines with the continuous spectrum of eigenvalues.

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Transient wave processes of the deformation of rods, plates and shells, caused by pulsating load.

U. K. Nigul.

The practice of the design of special constructions/designs put forth the task of the analysis of the transient wave processes of deformation, caused by sharp pulsating load. Report is devoted to the use/application of methods of integral transforms and to their combination with the net point method.

1. Rods under action of longitudinal impulse/momentum/pulse. It is experimentally proved that the one-dimensional theory of state of plane stress, which leads to the integration of the equation of string, gives unsatisfactory results, since actually occurs the dispersion. More acceptable results are obtained by Davis on the basis of Rayleigh-Love's approximate theory, which considers transverse inertia, and Miklovich on the basis of Mindlin-Herman's theory, which considers also the effect of transverse shift/shear (two dispersive mode). Numerical data are acquired by integration for the coasts of sections/cuts and asymptotic methods of the inversion

of contour integrals. Skalak and other authors investigated semi-infinite rod on the basis of three-dimensional theory with the help of the method of twofold integral transforms. After the first inversion appeared formal solutions in the form of the infinite sum of the contour integrals, of which the first were calculated by the steepest descent method for the high values of time t. In this way theoretically proved the acceptability of the refined theories with large t, but information about the behavior of construction/design with small t barely was increased.

2. Plates/slabs under action of transverse impulse. Closed solutions can be obtained on the basis of the Kirchhoff theory, but they correctly reflect reality only with large t far from the fronts. Is more wide the range of the applicability of Timoshenko's theory, used for the first time by Ya. S. Uflyand and further by Dengler, Bola, Miklovich, etc. The calculation of contour integrals by integration for the coasts of shear/sections proved to be very labor-consuming, but for the limiting cases Flyuge and Zayak indicated the efficient approximation methods. On the basis of three-dimensional theory are constructed the stressed states: a) in the proximity of epicenter - by method of the falling/incident potentials (I. N. Vekua) and with the method of Kanyar (G. I. Petrashen', K. I. Ogurtsov, Davids, Broberg, Miklovich, etc.); b) when $t\to\infty$ - by next inversion of contour integrals with the

enlistment of the steepest descent method; c) near the fronts - by ray series/rows. It is established/installed, that the use/application of Timoshenko's theory is substantiated with sufficiently large t in the region, which they had time to pass the ground waves of Rayleigh.

3. Shells. Semi-infinite circular cylindrical shell is investigated by Berkovich during axisymmetric zero moment deformation and N. A. Alumyae's Lifetime on the basis of theory of Timoshenko's type with the axisymmetric and during the cyclic centrally symmetrical deformation. The start of spherical shell under the action of concentrated impulse/momentum/pulse is investigated by physician on the basis of the theory of slightly curved shells. The methods of integral transforms proved to be the efficient instrument of research of the complicated equations of the dynamics of the shells, which contain the low parameter and, as a rule, which have variable coefficients. They in many instances made it possible to explain the possibilities of the separation of the stressed state into the zero moment and into the edge effects.

4. Conclusions/outputs from experiment of use/application of methods of integral transforms. Within the framework of the approximate theories are investigated the processes with one space coordinate, and according to the three-dimensional theory - with two

coordinates. The displaced coefficients of equation are the substantially complicating factor. It is comparatively easy to carry out qualitative analysis and to obtain data for the limiting cases (with $t \to \infty$, with $t \to 0$, for the points near the fronts and the place of the application of load), but at the "average/mean" values of t and basic coordinate numerical results are obtained with difficulty only for the semi-infinite objects on the basis of the approximate theories.

5. On use/application of methods of integral transforms in combination with net point method. Net point method works well in that region where the use/application of methods of integral transforms is difficult.

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In the institute of cybernetics of AS of E.S.S.R are worked out the programs of solution by the net point method of systems of two or three hyperbolic second order equations for the analysis of transient wave processes. Were preliminarily isolated the particular solutions, which transfer the interruptions/discontinuities of the unknown functions and their derivatives, which are encountered in the equations. One of the latter/last programs, analyzing in the course of computation the curvature of the computed functions, divides/marks

off region into the adequate/approaching s-bands different space. Is solved series of problems for the plate/slab and the shells with the finite dimensions. With the isolation/liberation of particular solutions and during the determination of the conditions for reflection from the supports, and also for the analysis of wave process with $t \to 0$ and $t \to \infty$ were used the methods of integral transforms.

Numerical integration of the oscillating functions by the Filon-Nikolayeva method.

L. I. Bogin, A. G. Zhuravleva

In the report is examined the approximate calculation, with the help of the Filon-Nikolayeva method, the integrals of the form

$$\int_a^b f(x)\, e^{i\psi(\alpha x)}\, dx, \qquad (1)$$

$$\int_a^b f(x)\, e^{\psi(\alpha x)}\, dx. \qquad (2)$$

Integrals of such type frequently are encountered in the theory of wave propagation. At the small values of the parameter α the integrals can be computed, for example, according to the method of Simpson or Gauss. However, when the value of the parameter is great, calculation with the help of the methods indicated becomes difficult, since it is necessary to preliminarily divide the gap/interval of integration into a large number of parts. The process of calculation even more is complicated, when appears the need of calculating the integrals for the series/row of the values of the parameter. Considerably less labor-consuming is the numerical integration of expressions (1) and (2) for the method of Filon [1] and Nikolayeva [2], which consists of the following: by any method, for example, by the method of replacing the variable/alternating, integrals (1) and (2) are reduced to the form

$$J(\omega) = \int_0^1 y(t)\, e^{i\omega t}\, dt, \qquad (3)$$

or

$$J(q)=\int_0^1 y(t)\,e^{-qt}\,dt. \qquad (4)$$

The idea of method lies in the fact that with the help of the interpolation polynomials is approximated not all integrand, but only comparatively slowly varying factor y(t).

Phylo and Nikolayev considered only the integrals of form (3). In the report their method is applied to the case when integrals (1) and (2) more conveniently to reduce to the form

$$\mathcal{J}(\omega)=\int_0^1 y(t)\,e^{i\omega t^2}\,dt \qquad (5)$$

or to the form

$$\mathcal{J}(q)=\int_0^1 y(t)\,e^{-qt^2}. \qquad (6)$$

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The gap/interval of integration [0, +1] is divided/marked off on 2m the equal intervals with a length of h=1/2m.

On each pair of intervals ($t_n \div t_{n+2}$, where n - even) function y(t) is approximated by the polynomial of the second power

$$y(t)=\frac{\alpha_n}{2h^2}t^2+\frac{\beta_n}{2h}t+\frac{\gamma_n}{2}. \qquad (7)$$

In the interpolation points ($t_0 \div t_{2m}$) are computed ordinates $y_n=y(t_n)$, with the help of which from equation (7) are determined

coefficients α_n, β_n, γ_n. The latter prove to be equal to

$$\alpha_n = y_n - 2y_{n+1} + y_{n+2},$$
$$\beta_n = \Delta_n + \Delta_{n+1} - 2(n-1)\alpha_n, \qquad (8)$$
$$\gamma_n = 2y_{n+1} - (\Delta_n + \Delta_{n+1})(n+1) + (n+1)^2\alpha_n,$$

where

$$\Delta_n = y_{n+1} - y_n; \quad \Delta_{n+1} = y_{n+2} - y_{n+1}.$$

Formulas (8) determine parabola in section $(t_n \div t_{n+2})$. The calculation of integrals (5), (6) is reduced to the determination of the sum of three integrals of $t^2e^{-q^2t^2}$, $te^{-q^2t^2}$ and $e^{-q^2t^2}$ (or $t^2e^{-i\omega t^2}$, $te^{i\omega t^2}$, $e^{i\omega t^2}$). i.e.

$$I(q) = \frac{1}{2h^2}\sum_{n=0}^{2m-2}\alpha_n\int_{t_n}^{t_{n+2}} t^2e^{-q^2t^2}\,dt + \frac{1}{2h}\sum_{n=0}^{2m-2}\beta_n\int_{t_n}^{t_{n+2}} te^{-q^2t^2}\,dt + \\ + \frac{1}{2}\sum_{n=0}^{2m-2}\gamma_n\int_{t_n}^{t_{n+2}} e^{-q^2t^2}\,dt, \qquad (9)$$

where n - only even. This expression is reduced to the form, convenient for the calculations.

Analogous expression can be written, also, for $I(\omega)$.

The approximate calculation of oscillating integrals (1) or (2) according to the Filon-Nikolayeva method and comparison with the calculations of these integrals according to the method of Simpson or Gauss makes it possible to do the following conclusions:

1. At the larger value of the parameter α the volume of calculations according to the Filon-Nikolayeva method is considerably

less than according to the method of Simpson or Gauss.

2. Since with the help of interpolation polynomials is approximated factor y(t), not depending on parameter, then at different values of parameter α is not required changes in space h.

3. Significant part of intermediate results does not depend on form of the function y(t). This makes it possible to shorten work on the calculation of the integrals, which are characterized by only the form of the function y(t).

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II. SPECIFIC PROBLEMS OF THE THEORY OF DIFFRACTION.

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Diffraction in open resonators with confocal mirrors.

L. A. Vaynshchteyn.

The reported work consists of three parts. The first part is dedicated to natural oscillations in the open resonator with the confocal cylindrical mirrors (radius of curvature of mirrors is equal to distance 21 between them). The study of oscillations in this system is reduced to the solution of the integral equation

$$f(t) = \sqrt{\frac{c}{2\pi}}\, e^{i\left(\varkappa - \frac{\pi}{4}\right)} \int_{-1}^{1} e^{-ictt'} f(t')\, dt' \qquad (1)$$

for eigenfunction f(t), which is determining current distribution on the mirror. Through c is designated the basic parameter of the task

$$c = \frac{ka^2}{2l} \qquad (2)$$

(2a - the width of mirrors). Complex parameter χ (more precise $e^{i\chi}$) there is eigenvalue of integral equation (1), which is determining complex natural vibration frequency according to the formula

$$2kl = \pi q + \varkappa, \qquad (3)$$

where q - large integer. Parameter χ can be represented in the form

$$\varkappa = \left(m + \frac{1}{2}\right)\frac{\pi}{2} - i\,\frac{\Lambda}{2}$$
$$(m = 0, 1, 2, \ldots), \qquad (4)$$

where Λ is determined damping the m oscillation due to the radiation

losses (diffraction fading). Because of these losses the vibrational energy is reduced in e of times for time $\frac{\tau}{\Lambda}$, where τ - time for which the light/world passes segment 21.

As is known, integral equation (1) is equivalent to the differential equation

$$(1-t^2)\frac{d^2f}{dt^2}-2t\frac{df}{dt}+c^2(\theta-t^2)f=0 \qquad (5)$$

together with the condition of finiteness f with t=+-1.

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The basic content of the first part of the work consists in the asymptotic integration of equation (5) by the method of standard equation. As the standard equation is undertaken the equation

$$\frac{d^2g}{du^2}+\frac{1}{4}\left(1+\frac{4\nu}{u}+\frac{1}{u^2}\right)g=0, \qquad (6)$$

solution of which is expressed as the confluent hypergeometric function. Independent the variable/alternating t and u, and also functions f and g are connected by usual relationships/ratios, then value Λ, being of greatest practical interest, it is obtained in the form

$$\Lambda=\ln(1+e^{-2\pi\nu}). \qquad (7)$$

Depending on the value of parameter c this oscillation with index m can belong to one of three types: 1) oscillation with caustic curve

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(Λ is small), 2) transient type oscillation, 3) oscillation without the caustic curve. Are given graphs for value Λ in the dependence on c with m=0, 1, ..., 9 and $c \leq 15$.

The results, obtained in the first part, make it possible to calculate diffraction losses in the resonators with right-angled spherical mirrors. In the second part are investigated the resonators with confocal spherical mirrors of circular form. Integral equation (1) for them is substituted by the following:

$$f_m(t) = ce^{i\left[\chi-(m+1)\frac{\pi}{2}\right]}\int_0^1 J_m(ctt')\, f_m(t')\, t'dt', \qquad (8)$$

the index m=0, 1, 2, ... determining the azimuth dependence of natural oscillation ($\cos m\varphi$ or $\sin m\varphi$). Equation (8) is equivalent to the differential equation

$$(1-t^2)\frac{d^2f}{dt^2} - 2t\frac{df}{dt} + \left[c^2(\theta - t^2) - \frac{m^2-\frac{1}{4}}{t^2}\right]f = 0 \qquad (9)$$

for the function

$$f(t) = \sqrt{t}\, f_m(t). \qquad (10)$$

Differential equation (9) is again solved by the method of standard equations and thus are computed the diffraction losses of natural oscillations in this system.

The third part of the work is dedicated to the calculation of the norm of natural oscillations examined earlier for which the

general theory of the excitation of the open resonators gives the expressions, barely suitable for the practical calculations.

Control of the radiation of plane-parallel layer with the help of metallic lattices.

O. A. Tret'yakov, S. S. Tret'yakova, V. P. Shestopalov.

As the simplest model of lasers and generators can serve the plane-parallel layer of the optically active substance, which possesses the negative absorption coefficient. For the precomputations this model is very convenient; the value of diffraction losses, caused by the finite dimensions of the ends/faces of the layer of real instrument, it is possible then to compute by different known methods.

Plane-parallel layer with the arbitrary refractive index $N = N_0 + i\chi$ is investigated in the series/row of works.

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It is discovered, in particular, that with the given ones to the wavelength of radiation/emission, the absorption coefficient and to the thickness of the layer of the condition of self-excitation and the energy radiation characteristics are determined only by

refractive index N_e. Here we will consider control capability of these characteristics with the help of the lattices, replaced to the ends/faces of layer, we investigate the dependence of these values from the method of depositing the lattices.

We will consider that to the plane-parallel layer of thickness a with the refractive index N strip/tape metallic of lattice can be replaced by the following three methods: a) layer with the lattice on one of the walls; b) layer with the identical strip/tape lattices, arranged/located on both walls strictly one under another; c) layer with the lattice on one of the walls and with the ideal shield on another. Strips/films and shield are assumed to be those ideally conducting, the thickness of belts is not considered; l - period of lattices, d - width of slots - they are arbitrary.

General/common/total for these three cases is the fact that the field emergent caused by the induced radiation of the substance of layer will be periodical with period l inside, also, out of the layer, therefore, it are conveniently sought in the form of Fourier series. The value of Fourier coefficients field in each case is determined by finding the solutions of the equations of Maxwell, which are subordinated to precise boundary conditions on one of the periods of structure. The obtained boundary-value electrodynamic problems are reduced then to the boundary-value problem of

Riemann-Hilbert for certain analytic function. The solution of problem in each case is obtained in the form of the infinite uniform system of linear algebraic equations, which allows/assumes obtaining solution by the method of reduction. The condition for existence of the nontrivial solution of system is the condition of the self-excitation of plane-parallel layer. Energy radiation characteristics are found as a result of the enlistment of the nonlinear theory of the optical properties of plane-parallel layers.

In the particular case when $l/\lambda \lesssim {}^1/_2\sqrt{N_0}$ and $\lambda \ll a$, the conditions of self-excitation take the sufficiently simple form:

$$\left.\begin{array}{ll}
\text{a)} & R_a e^{-Ka} = 1;\ 2\pi N_0 a/\lambda + \gamma/2 = \pi s, \\
\text{b)} & R_б e^{-Ka} = 1;\ 2\pi N_0 a/\lambda + \gamma/2 = \pi s, \\
\text{c)} & R_в e^{-2Ka} = 1;\ 4\pi N_0 a/\lambda + \gamma/2 = \pi s,
\end{array}\right\} \qquad (1)$$

where

$$R_a = \sqrt{r}\sqrt{\frac{r+t^2}{1+t^2}};\quad R_б, R_в = \sqrt{\frac{r+t^2}{1+t^2}};\quad t \simeq \frac{2\lambda}{l(N_0+1)\ln\frac{1+u}{2}},$$

$$u = \cos\frac{\pi d}{l},$$

$$r' \simeq \frac{N_0-1}{N_0+1};\quad \gamma = \operatorname{arctg} t/\sqrt{r} + \operatorname{arctg} t + \tilde{\gamma};\quad s = 0, 1, 2, \ldots$$

$\tilde{\gamma}$ - small correction.

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Are obtained also expressions for the amplitude of the radiant

flux $\vec{P}$ and density of the electromagnetic energy U within layer (-a<z<0).

$$\text{а)}\quad \left.\begin{aligned} P &= -\frac{c}{\alpha N_0}\,\frac{K_0 a - \ln R_а}{R_а - 1}\sqrt{r}\,\frac{2R_а}{R_а + r}\,\mathrm{sh}\left(\frac{z}{a}\ln R_а + \ln\frac{R_а}{r}\right),\\ U &= \frac{1}{\alpha}\,\frac{K_0 a - \ln R_а}{R_а - 1}\sqrt{r}\,\frac{2R_а}{R_а + r}\,\mathrm{ch}\left(\frac{z}{a}\ln R_а + \ln\frac{R_а}{r}\right); \end{aligned}\right\} \quad (2)$$

$$\text{б)}\quad \left.\begin{aligned} P &= \frac{c}{N_0 \alpha}\,\frac{K_0 a - \ln R_б}{R_б - 1}\,R_б\,\mathrm{sh}\left[\left(\frac{z}{a} + \frac{1}{2}\right)\ln R_б\right],\\ U &= \frac{1}{\alpha}\,\frac{K_0 a - \ln R_б}{R_б - 1}\,R_б\,\mathrm{ch}\left[\left(\frac{z}{a} + \frac{1}{2}\right)\ln R_б\right]; \end{aligned}\right\} \quad (3)$$

$$\text{в)}\quad \left.\begin{aligned} P &= -\frac{c}{N_0 \alpha}\,\frac{K_0 2a - \ln R_в}{R_в - 1}\,R_в\,\mathrm{sh}\left[\left(\frac{z}{2a} + \frac{1}{2}\right)\ln R_в\right],\\ U &= \frac{1}{\alpha}\,\frac{K_0 2a - \ln R_в}{R_в - 1}\,R_в\,\mathrm{ch}\left[\left(\frac{z}{2a} + \frac{1}{2}\right)\ln R_в\right]. \end{aligned}\right\} \quad (4)$$

Here α - the parameter of the nonlinearity of substance, c - the speed of light. The Umov-Poynting vector P is directed always perpendicular to the walls of layer.

As it follows from (2)-(4), in the absence of lattices (u=-1) the radiation/emission always begins from the middle of layer. If lattices are plotted to the walls of layer symmetrically (case b), then the center of radiation/emission also is located on the middle of layer, in the remaining cases the center of radiation/emission is shifted/sheared.

The amplitude of radiant flux P, energy density u are determined respectively by values $R_а$, $R_б$, $R_в$, which in turn, depend on the

sizes/dimensions of lattice d, l. Consequently, energy radiation characteristics it is possible to manage with the help of the lattices, replaced to the walls of layer. The carried out calculations show that this control can be very efficient.

Open resonators with mirrors, which possess variable reflection coefficient.

N. G. Vakhitov.

In the present work with the help of the asymptotic method, used works [1-2], examine the open resonators with the mirrors, which are characterized by variable reflection coefficient.

Are obtained the equations for determining the natural frequencies and the formulas, which establish/install field distribution between the mirrors in the case of flat/plane and concave mirrors.

Is at first examined the two-dimensional task about the natural oscillations between two limitless parallel planes $-\infty<x<\infty$, $-\infty<y<\infty$, $|z|<l$, where is sought the solution of the equation of Helmholtz

$$\Delta\Phi + k^2\Phi = 0, \qquad (1)$$

satisfying on planes z=+-1 impedance boundary condition,

$$\frac{\partial\Phi}{\partial z} + ikg\Phi = 0 \quad \overset{(1)}{\text{при}}\ z = \pm l, \qquad (2)$$

Key: (1). with.

moreover

$$g = \frac{1-e^{-\frac{2x^2}{a^2}}}{1+e^{-\frac{2x^2}{a^2}}}, \qquad (3)$$

where a - certain positive real constant.

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Let us represent solution $\Phi(x, z)$ in the form

$$\Phi(x, z) = W(x, z)\, e^{ikz} - (-1)^q W(x, -z)\, e^{-ikz}, \qquad (4)$$

where each component/term/addend must satisfy equation (1), and we will seek the asymptotic solution of stated problem under the assumption

$$\frac{1}{k}\left|\frac{\partial^2 W}{\partial z^2}\right| \ll \left|\frac{\partial W}{\partial z}\right|, \quad \frac{1}{k}\left|\frac{\partial W}{\partial z}\right| \ll |W|. \qquad (5)$$

After substituting (4) in (1) and (2) and using conditions (5) for determining of W(x, z), we come to the parabolic equation

$$\frac{\partial^2 W}{\partial x^2} + 2ik\frac{\partial W}{\partial z} = 0 \qquad (6)$$

and the boundary condition

$$W(x_1 - l) = e^{-\frac{2x^2}{a^2}}\, W(x, l)\, e^{i(2kl-\pi q)}. \qquad (7)$$

With the help of Green's function

$$\Gamma(x - x', z - z') = \sqrt{\frac{k}{2\pi(z - z')}}\, \exp i\left[\frac{k(x - x')^2}{2(z - z')} - \frac{\pi}{4}\right] \qquad (8)$$

task (6), (7) is reduced to the integral equation

$$W(x, l) = e^{i(2kl-\pi q)} \int_{-\infty}^{\infty} \Gamma(x - x', z - z')\, e^{-\frac{2x'^2}{a^2}} W(x', l)\, dx', \qquad (9)$$

making it possible to determine field distribution on the surfaces of mirrors.

Integral equation (9) succeeds in solving strictly. As a result for determining the natural frequencies of the resonator examined we obtain the equation

$$\cos\frac{\chi}{m+\frac{1}{2}}=1+\frac{i}{M^2}, \qquad (10)$$

where are introduced designations $\chi=2kl-\pi q,\ M=\sqrt{\frac{ka^2}{4l}}$, and field distribution on the mirrors is determined by the formula

$$W_m(x,\ l)=c_m H_m\left[2\sqrt{M(1-i)}\frac{x}{a}\right]e^{-(M-1)\frac{x^2}{a^2}}e^{iM\frac{x^2}{a^2}}, \qquad (11)$$

where H_m is Hermite's polynomials.

Results easily are generalized to the three-dimensional case. Natural frequencies are determined by the equation

$$\cos\frac{\chi}{m+n+1}=1+\frac{i}{M^2}. \qquad (12)$$

Field distribution on the mirrors is given by the formula

$$W_{mn}(x,\ y,\ l)=W_m(x,\ l)\,W_n(y,\ l). \qquad (13)$$

Obtaining results relate to the resonators with the limitless mirrors. In the case of the limited mirrors it is possible to conduct the appropriate evaluations and to show that formulas (12) and (13) will be valid approximately for those natural oscillations, for which value $W(a_0)$ is respectively low, where $2a_0$ - width of mirror.

In the work analogously investigated the natural oscillations of the open resonators with the concave (spherical) mirrors.

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Diffractions of plane electromagnetic wave in cylindrical conductor in plane layer of dielectric.

V. A. Kaplun, A. A. Pistol'kors.

In limitless plane layer of dielectric without the losses is included the infinite ideally conducting thin lead/duct whose axis lies/rests at the mean section of layer. Is examined the diffraction of plane electromagnetic wave, which falls at arbitrary angle to the layer, but so that the vector of the electric field of the incident wave and lead/duct lie/rest at the plane of incidence.

With the solution of problem first is located the propagation constant of electromagnetic waves along the lead/duct, that corresponds to the absence of external excitation, and then with the help of the obtained results are examined the conditions of exciting the lead/duct by the incident wave. During the determination of the diffracted fields the cylindrical field of lead/duct is decomposed/expanded into the plane waves which in turn, are divided into the components, the perpendicularly and in parallel polarized to the interfaces. Each of the components then is examined separately.

As a result of using the boundary conditions on the surface of lead/duct for the sum of the longitudinal components of electric field is obtained the equation for the determination of propagation constant. The entering the equation integral of the complex function is computed numerically.

Are given the graph/diagrams of the dependence of the calculated propagation constant on the parameters of layer and lead/duct (thickness of the layer, the diameter of lead/duct, etc.).

Further is examined the excitation of conductor by an outside source - incident to the layer plane electromagnetic wave. Is conducted the analysis of the obtained expression for the current in the lead/duct. It shows that with the coincidence of propagation constant, created by outside wave, with its own propagation constant of system "layers - the lead/duct" the current of lead/duct with the ideal material approaches infinity. This phenomenon in practice can be observed with the dielectric constant of the material of layer less than 1 (for example, in the layer of plasma).

Radiowave propagation in the waveguide channel Earth - ionosphere.

T. I. Volodicheva, E. M. Gyunninen, I. N. Zabavina, S. T. Rybachek.

Let r, θ and φ - spherical coordinates with the beginning in the center of terrestial globe. In cavity a<r<s between the Earth (r<a) and ionosphere (r=c) is sought the field of the electric dipole, arranged/located at point r=b, θ=0 and directed along r, with the moment/torque $p = p_0 e^{-i\omega t}$ [1].

FOOTNOTE [1]. a<b<c. ENDFOOTNOTE.

For electrical $\vec{E}$ and magnetic $\vec{H}$ vectors of the field are considered carried out the boundary conditions

$$E_\theta = -Z_0\delta H_\varphi, \quad E_\varphi = Z_0\delta H_\theta \quad \text{при } r = a \tag{1}$$

и

$$E_\theta = Z_0(\delta_{11} H_\theta + \delta_{12} H_\varphi), \quad E_\varphi = -Z_0(\delta_{21} H_\theta + \delta_{22} H_\varphi) \quad \text{при } r = c \tag{2}$$

Key: (1). with. (2). and.

twisting the ionosphere it is assumed the anisotropic as a result of the magnetic field Earth; the given surface impedances δ and δ_{ik} (i,

k=1, 2) are assumed to be known ones, $Z_0 = \sqrt{\frac{\mu}{\varepsilon}}$ — characteristic impedance for air.

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δ_{ik} are found from the solution of the separat problem about an incidence in the plane wave on the flat/plane ionospheric layer, heterogeneous in height, and during the arbitrary orientation of the magnetic field of the Earth and refractivity gradient of the ionosphere relative to each other and incident direction in the wave. Solution is constructed as expansion in terms of the separate rays/beams, moreover to each ray/beam are compared their impedances δ_{ik}, found at the same angle of incidence in the plane wave on the boundary of layer, that also in this ray/beam. In the terms r-yx the component P and $\widehat{P}$ of Hertz's electrical and magnetic vectors solution is represented in the form

$$\left\|\begin{matrix}\Pi \\ \tilde{\Pi}\end{matrix}\right\| = \sum_{m=0}^{\infty} \left\|\begin{matrix}\Pi^{(m)} \\ \tilde{\Pi}^{(m)}\end{matrix}\right\|, \tag{3}$$

(1) где

$$\Pi^{(0)} = \frac{p_0}{4\pi\varepsilon} \cdot \frac{i}{kb^2} \sum_{k=0}^{\infty} \left(n + \frac{1}{2}\right) \xi_n^{(1)}(kb) \cdot Z_n^{\parallel}(kr) \cdot P_n(\cos\theta), \tag{4}$$

$$\tilde{\Pi}^{(0)} = 0$$

Key: (1). where.

- solution for the spherical Earth without the ionosphere, and

$$\left\| \begin{matrix} \Pi^{(1)} \\ \tilde{\Pi}^{(1)} \end{matrix} \right\| = \frac{p_0}{4\pi\varepsilon} \cdot \frac{i}{kb^2} \sum_{n=0}^{\infty} \left(n + \frac{1}{2}\right) Z_n^{\cdot} (kb)\, P_n (\cos\theta) \cdot \hat{g}_n^{(1)} (kr), \qquad (5)$$

$$\left. \begin{matrix} \Pi^{(m)} \\ \tilde{\Pi}^{(m)} \end{matrix} \right\| = \frac{p_0}{4\pi\varepsilon} \cdot \frac{i}{kb^2} \sum_{n=0}^{\infty} \left(n + \frac{1}{2}\right) \cdot Z_n^{\cdot} (kb) \cdot P_n (\cos\theta) \cdot \hat{g}_n (kr) \cdot [\hat{\alpha}_n]^{m-2} \cdot \hat{\beta}_n, \quad m > 2, \qquad (6)$$

where matrices/dies $\hat{\alpha}_n, \hat{\beta}_n$ and $\hat{g}_n$ are assigned by the formulas

$$\hat{\alpha}_n = \left\| \begin{matrix} {}_\parallel r_\parallel \cdot r_\parallel, & {}_\perp r_\parallel \cdot r_\parallel \\ {}_\parallel r_\perp \cdot r_\perp, & {}_\perp r_\perp \cdot r_\perp \end{matrix} \right\|, \qquad (7)$$

$$\hat{\beta}_n = \left\| \begin{matrix} {}_\parallel r_\parallel \cdot r_\parallel \\ {}_\parallel r_\perp \cdot r_\perp \end{matrix} \right\|, \qquad (8)$$

$$\hat{g}_n = \left\| \begin{matrix} {}_\parallel r_\parallel \cdot Z_n^\parallel (kr), & {}_\perp r_\parallel Z_n^\parallel (kr) \\ {}_\parallel r_\perp \cdot Z_n^\perp (kr), & {}_\perp r_\perp Z_n^\perp (kr) \end{matrix} \right\|, \qquad (9)$$

and $\tilde{g}_n^{(1)}$ differs from $\hat{g}_n$ in terms of the fact that the elements/cells of the second column are different zero.

Through $r_\parallel$ and $r_\perp$ are designated "spherical" reflection coefficients from the Earth for the vertical and horizontal polarizations, through ${}_\parallel r_\parallel$, ${}_\parallel r_\perp$, ${}_\perp r_\parallel$ and ${}_\perp r_\perp$ the analogous reflection coefficients from the ionosphere, the first mark designating the polarization of the incident wave and the second - polarization of that reflected. For the brevity is marked

$$\begin{aligned} Z_n^\parallel (x) &= \zeta_n^{(2)} (x) + r_\parallel \zeta_n^{(1)} (x), \\ Z_n^\perp (x) &= \zeta_n^{(2)} (x) + r_\perp \zeta_n^{(1)} (x) \end{aligned} \qquad (10)$$

and $\zeta_n^{(1)}$ and $\zeta_n^{(2)}$ – "spherical" Hankel functions, $P_n(\cos\theta)$ – Legendre's polynomials.

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Series/rows (5) and (6), registered in the form of contour integrals, are represented (in the case of straight/direct rays/beams) in the approximation/approach of reflecting formulas and (in the case of diffraction rays/beams) they are computed by numerical contour integration, close one and duct/contour of the fastest descent. In the latter case the magnetic field of the Earth is not considered, since its effect proves to be small.

In the second part of the work the solution (without taking into account the magnetic field of the Earth) is represented in the form expansion in terms of the waveguide modes. Eigenvalues were located by the numerical solution of the transcendental equation

$$1 - A_{\nu-\frac{1}{2}} B_{\nu-\frac{1}{2}} = 0 \qquad (11)$$

relatively ν, where

$$A_n = \frac{\tilde{\zeta}_n^{(1)\prime}(kc) + i\delta_{12}\tilde{\zeta}_n^{(1)}(kc)}{\tilde{\zeta}_n^{(2)\prime}(kc) + i\delta_{12}\tilde{\zeta}_n^{(2)}(kc)}, \qquad (12)$$

$$B_n = \frac{\tilde{\zeta}_n^{(2)\prime}(ka) + i\delta\tilde{\zeta}_n^{(2)}(ka)}{\tilde{\zeta}_n^{(1)}(ka) + i\delta\tilde{\zeta}_n^{(1)}(ka)}, \qquad (13)$$

moreover function $\tilde{\zeta}_n^{(1)}$ and $\tilde{\zeta}_n^{(2)}$ (and also their derivatives on n, necessary for the determination fields) they were computed with the help of the integral representations of Sommerfield for the Hankel functions. Integration was conducted according to the ducts/contours of the fastest descent.

Results of asymptotic calculations in the problem about the propagation of electromagnetic waves of low frequency in the waveguide Earth - ionosphere.

Ye. G. Guseva, D. S. Fligel'.

1. Examination of formulas for vertical component of electric field and equation of poles for spherical and flat/plane waveguide (vertical dipole, uniform Earth and ionosphere).

2. Comparison of results of calculating poles (wave numbers) for flat/plane waveguide Earth - ionosphere (from works of E. L. Alpert, P. Ye. Krasnushkin, J. R. Waita) in the range of frequencies from 1 kHz to 30 kHz.

3. Comparison of results of calculations of poles for flat/plane and spherical waveguides (from 50 Hz to 30 kHz) and comparison of amplitude spectra calculated according to these data.

4. Comparison of results of theoretical calculations with experimental data:

a) amplitude dependences of field on distance (at fixed/recorded frequencies),

b) phase speed.

Accelerating system with drift tubes for superhigh frequencies.

V. B. Krasovitskiy, V. I. Kurilko.

The use of large electric intensities, obtained at present with the help of the quantum generators, for accelerating the charged/loaded particles requires the developments of circuits for the waves of the corresponding frequency band [1]. As one of such systems can serve the cylindrical diffraction grating proposed by Ya. B. Faynberg, which is the analog of the accelerating system with the drift tubes, utilized in the linear accelerators.

In the work is examined the excitation of this lattice, formed by the infinitely thin conducting rings of radius a and width 1/2(1 - period of structure), placed into the dielectric with $\varepsilon \sim 1$.

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Excitation is realized with the help of the axially symmetrical convergent wave. Is examined the case, when wavelength $\lambda \underset{\sim}{\ } l < a$.

Solutions for the stray fields in the system can be sought in

the form of Fourier series with the period, equal to the period of lattice. The amplitudes of Fourier's components are found from the boundary conditions for E_z and H_φ with the help of the method, proposed in [2].

It is shown that for the frequencies, determined by the condition

$$\omega_p = \frac{c}{a\sqrt{\varepsilon - 1}}\left[n\pi - \frac{\pi}{4} + \operatorname{arctg}\left(6\sqrt{\varepsilon - 1}\right)\right]; \; n \sim \frac{a}{l},$$

the amplitude of the fundamental harmonic whose phase speed $v_\phi = \frac{\omega}{k_{z_1}} = \frac{lc}{\lambda}$, is close to the speed of light when $\frac{l-\lambda}{\lambda} \ll 1$, 3-4 times it exceeds the amplitudes of adjacent harmonics. For these frequencies the amplitude of the fundamental harmonic on the axis of system, in reference to the amplitude of the incident wave on the surface of lattice, proves to be equal to

$$\frac{E_z^{(1)}(r=0)}{E_z^{(n)}(r=a)} = \pi i \sqrt{\frac{\varepsilon}{l}} (\varepsilon - 1)^{3/4}.$$

Thus in the presence of dielectric simultaneously with the delay/retarding/deceleration of wave can occur the focusing of field near the axis in the region with the sizes/dimensions of the order of wavelength. The radial force, which functions on relativistic particle $(\beta\sqrt{\varepsilon} > 1)$, which moves near the axis of system, it proves to be focusing:

$$f_r = e(E_r - H_\varphi) = \frac{ie}{\varepsilon} E_z^{(1)}(0) J_1\left(\lambda_p \frac{r}{a}\right);$$
$$J_0(\lambda_p) = 0$$

There are examined also the dispersive properties of this system, moreover it is shown that for the frequencies, close to the resonance $\left(\omega_p = \frac{c\lambda_p}{a\sqrt{\varepsilon}}\right)$, the fading, caused by radiation/emission through the slots, is small $\left(\sim \frac{nl}{L}, n -\right.$ the number of harmonic, L - length of lattice).

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The question of propagation of electromagnetic waves in waveguides with a large number of longitudinal slots.

V. V. Meriakri, M. V. Persikov, A. N. Sivov.

1. Is examined regular system, formed by circular smooth metal tube and lattice, placed into it, convoluted into round cylinder in such a way, that its conductors would be parallel to axis of external duct (z axis). In this case external and internal cylinders coaxial, and their radii are equal with respect to b and a. Lattice is periodical in azimuth bearing (φ) with period p which is assumed small in comparison with the wavelength λ. To the transverse sizes/dimensions of conductors with respect to the period no limitations are superimposed.

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2. For analysis of conditions for propagation of electromagnetic waves is used set of boundary conditions, obtained in [1]. In this system the most separate possible existence of symmetrical magnetic and electrical waves. In this case boundary conditions in the cylindrical coordinate system (r, φ, z) take the form: for the

electrical waves

$$E_{z2} - E_{z1} = il_2 \frac{\alpha^2}{k} (H_{\varphi 2} + H_{\varphi 1}),$$
$$E_{z2} + E_{z1} = il_3 \frac{\alpha^2}{k} (H_{\varphi 2} - H_{\varphi 1}); \quad (1)$$

for the magnetic waves:

$$H_{z2} - H_{z1} = -D \frac{\alpha^2}{k^2} (E_{\varphi 2} + E_{\varphi 1}),$$
$$E_{\varphi 2} - E_{\varphi 1} = ikl (H_{z2} + H_{z1}); \quad (2)$$

here $k = \frac{2\pi}{\lambda}$, $\alpha = \sqrt{k^2 - h^2}$, h — phase constant,

$D = ikl_1 \cdot M$, $M = \frac{1}{1 - (\alpha 3)^2 \cdot \Delta_2}$, l_i $(i = 1, 2, 3)$ and Δ_2 - lattice parameters, which depend on the form and the relative sizes/dimensions of conductors. Indices "1" and "2" relate to the regions 0<r<a, a<r<b respectively. Boundary conditions (1) reduce to the dispersion equation for the symmetrical electrical waves

$$J_0(x) - \frac{Q_1 (l_2 + l_3) + \alpha^2 l_2 l_3 \cdot 2}{\alpha (l_2 + l_3) + 2Q_2} \cdot J_1(x) = 0, \quad (3)$$

but conditions (2) give equation for the magnetic waves

$$J_1(x) - \frac{Q_2 (1 + \alpha^2 l l_1 M) + 2\alpha l}{(1 - \alpha^2 l l_1 M) + 2Q_2 \alpha l_1 M} \cdot J_0(x) = 0, \quad (4)$$

x=αa, Q_i — some combinations of the Bessel functions and Neumann. Equations (3), (4) ensure correct transition, in particular, during the unlimited approach of the conductors of lattice.

According to equation (4) by ETsVM is produced the calculation of eigenvalues of waves of the type H_{0n} for large set of parameters of system.

3. Is obtained expression for amplitudes A_n of waves of type

H_{0n}, appearing at joint of system in question and smooth waveguide of radius a; during incidence on it in wave H_{0i} from smooth waveguide:

$$A_n = \frac{N_i \alpha_i \int\limits_0^a J_1(\alpha_i r) I_1(\alpha_n r)\, r dr}{\alpha_n \left\{ \int\limits_0^a I_1^2(\alpha_n r)\, r dr + \int\limits_0^a [V_n I_1(\alpha_n r) - W_n N_1(\alpha_n r)]\, r dr \right\}}, \quad (5)$$

$\alpha_i = \frac{\mu_i}{a}$, $J_1(\mu_i) = 0$; $\alpha_n = \frac{x_{0n}}{a}$; V_n and W_n — some combinations of Bessel functions and Neumann.

4. Experimental research of system with joint gave good coincidence with results of calculation according to formulas (4) and (5).

5. Analysis of waveguides conducted with large number of longitudinal slots made it possible to create series/row of efficiently functioning waveguide elements/cells (variable/alternating attenuators, filters and couplers) for circuits, which work on wave H_{01}.

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Waveguide bend.

B. I. Volkov, S. Ya. Sekerzh-Zen'ko.

Two rectilinear semi-infinite waveguides of the round cross section of an identical radius are connected by the bent section. The form of the cross section of transition differs little from the circle. Surface of waveguide ideally conducting. Is placed the task of determining the amplitudes of the reflected and scattered waves with the incidence/drop in the bent section of the waveguide of the assigned normal wave. Boundary-value problem for the equations of Maxwell is reduced to the boundary-value problem for the system of ordinary differential equations relative to the unknown amplitudes.

Is conducted qualitative research of the obtained system, and also are examined the methods of the numerical solution, which use the high speed computers. Define the boundaries themselves of the applicability of asymptotic formulas.

Propagation of electromagnetic vibrations in waveguides with irregular lateral surface.

A. S. Ilinskiy.

Is examined the task about the propagation of stationary electromagnetic field in the heterogeneous waveguides. As the typical is selected the task about the agreement of circular and rectangular waveguides with the help of the transition section.

Is constructed approximate solution of task in the form of series/rows along certain complete system of the vector functions, which satisfy boundary conditions on the lateral surface of waveguide. The determination of the coefficients of expansion is reduced to the solution of boundary-value problem for the system of ordinary differential equations. The solution of the latter is conducted numerically on EVTsM. is investigated the dependence of the gear ratios/transmission factors and reflection from the form of transition and frequency. Is carried out comparison with the asymptotic results.

Approximation method of calculating field distribution in a sectoral horn with impedance walls.

D. V. Shannikov.

The task about the excitation of wedge impedance regions is examined in a whole series of the work of many authors. Usually the authors are limited to the case when wedge angle is equal to $\frac{2m-1}{2n}\pi$, where to m and n - integers. This is connected with the very great mathematical difficulties, which appear with the solution of the problem about the wedge. Furthermore almost in all works it is discussed the impedance, connected with the poor conductivity of the walls of wedge ($z_{\text{пов}}$ it is small). However, in a whole series of practical tasks it proves to be interesting to consider the case when $z_{\text{пов}}$ is great. This occurs, for example, when the vector of electric field is perpendicular to the metallic wall, covered with dielectric layer, or when it is parallel to wall, and dielectric layer has a thickness, close to the critical

$$\left(t = \frac{\lambda}{4\sqrt{\varepsilon - 1}}\right).$$

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In this case it proves to be possible to represent the solution of problem in the form of series/row according to the degrees of the low parameter α:

$$E = \sum_{n=0}^{\infty} \alpha^n E_n(r, \vartheta), \qquad (1)$$

where E - transverse component of electric field, parallel to the surface of wedge, and α - coefficient, entering the boundary ones the condition

$$\frac{\partial E}{\partial \vartheta} \pm \alpha k r E = 0; \quad \vartheta = \pm \beta, \qquad (2)$$

where $\vartheta = \pm \beta$ - coordinates of the faces of wedge.

We will consider that E satisfies homogeneous wave equation everywhere, with exception of the apex/vertex of the wedge where is arranged/located the filamentary source whose intensity does not depend on value α. This gives grounds to register solution for $E_0(r, \vartheta)$ in the form

$$E_0(r, \vartheta) = H_0^{(2)}(kr), \qquad (3)$$

where source strength is placed equal to one, and dependence on the time is undertaken in the form $e^{i\omega t}$.

The substitution of expansion (1) in the equation and into boundary conditions (2) gives

$$\Delta E_n + k^2 E_n = 0 \qquad (4)$$

$$\frac{\partial E_{n+1}}{\partial \vartheta} \pm kr\, E_n = 0; \quad \vartheta = \pm \beta, \qquad (5)$$

i.e. the solution of problem is reduced to the solution of the

equation of Helmholtz under the heterogeneous boundary conditions. Its solution is conducted with the help of the transformation of kontorovica- Lebedev:

$$\overline{E}_n(\vartheta, \nu) = \int_0^\infty E_n(r, \vartheta) \frac{H_\nu^{(2)}(kr)}{r} dr. \quad (6)$$

The transformation of boundary conditions and equation leads to the following relationships/ratios:

$$\frac{\partial^2 \overline{E}_n}{\partial \vartheta^2} + \nu^2 \overline{E}_n = 0, \quad (7)$$

$$\frac{\partial \overline{E}_n}{\partial \vartheta} \pm \int_0^\infty E_{n-1}(r, \vartheta) H_\nu^{(2)}(kr) d(kr) = 0, \quad (8)$$

solution of which taking into account the symmetry of task relatively $\vartheta=0$ gives

$$\overline{E}_n(\vartheta, \nu) = A_n(\nu) \cos \nu\vartheta. \quad (9)$$

For n=1 it is possible to obtain

$$E_1(r, \vartheta) = \frac{i}{2} \int_{-i\infty}^{i\infty} \frac{e^{-i\frac{\pi}{2}\nu} \sin \nu\pi H_\nu^{(2)}(kr) \cos \nu\vartheta \, d\nu}{(\cos \pi\nu - 1) \sin \nu 3}. \quad (10)$$

Latter/last integral for large kr can be calculated approximately by the method of steady state, for which Hankel's function is substituted by its integral representation. It should be noted that after the appropriate transformations it is evident that near the way of integration are arranged/located the poles of integrand. Thus, in the final formula is used function $I = \int_{-\infty}^{\infty} e^{-p^2} \frac{dp}{\alpha - p}$, calculated in the work of V. A. Foch.

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Final result for $E_1(r, \vartheta)$ takes the following form:

$$E_1 = i\left\{\sqrt{\frac{2}{\pi kr}} A(\vartheta) e^{-i\left(kr-\frac{\pi}{4}\right)} - \frac{e^{-ikr}}{\pi}\left(i\pi + 2\pi\int_0^a e^{i\xi^2} d\xi\right) e^{-a^2}\right\}, \quad (11)$$

where $a = \sqrt{\frac{kr}{2}}(\beta - \vartheta) e^{-i\frac{\pi}{4}}$; $A(\vartheta)$ — auxiliary function, calculated in the referred work.

Besides expression (11) for E_1 it is possible to obtain expansion in Bessel functions, for which it is necessary to use the inversion formula, obtained by M. I. Kontorovich and N. N. Lebedev, into which enters not the function of Hankel, but Bessel function. Using this form of solution, it is possible to show that E_1 with tendency r toward zero also it vanishes, i.e., it does not have special feature/peculiarity in zero.

Determination E_n, where n>1, presents great computational difficulties. Therefore the method, analogous to calculation E_1, produced estimation E_2.

It turned out that when $\alpha < \sqrt{\frac{\pi}{2kr}}$ the third term of expansion (1) is less than the second. Thus, the obtained formulas it is possible to use only at the sufficiently low values α, moreover at the high values of kr it is necessary that α would be sufficiently little. In

the work are carried out the calculations for the different flare angles of wedge and values $\alpha<0.1$. Results are compared with the experiment.

Resonance diffraction of electromagnetic waves on a sphere (cylinder) being deformed.

A. A. Andronov.

The electromagnetic resonance properties of the limited objects are defined, as is known, by their electrodynamic and geometric characteristics. However, the ponderomotive forces, which appear with the diffraction of electromagnetic waves change the parameters of similar bodies, moreover ponderomotive forces are especially great in the presence of the resonance and, therefore, they attempt to deduce object from the resonance.

In the report is examined quasi-static dipole resonance in the sphere (cylinder)being deformed. Is given the simple concentrated model, which illustrates all basic features of the task in question. Are investigated the solid sphere (cylinder) and the plasma sphere (cylinder), placed into the elastic shell, during a small strain. Is found the dependence of the scattering cross section of electromagnetic waves on the amplitude of the incident field. Is examined Raman scattering of electromagnetic waves on the oscillations of sphere (cylinder).

Efficient diameter of the backscattering of meteor trails, commensurate with the size of Fresnel zone.

Yu. M. Zhidko, V. N. Kopaleyshchvili.

To the study of reflections from the meteors and meteor traces are devoted many works. In the larger part of these works the scattering of the meteor trails is examined in the approximation/approach of Fraunhofer diffraction. However, this examination is not always correct: in a number of cases the sizes/dimensions of meteor trails prove to be such large (several ten kilometers)that at the length of trace are placed several Fresnel zones. These cases are examined in the report.

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Problem is solved on the assumption that the meteor trail is dielectric cylinder with variable/alternating (along the axis, also, on a radius) dielectric constant ε, slightly different from the dielectric constant of environment ε_0. This gives the possibility to use Born approximation. It is assumed also,that the transverse size/dimension of cylinder is much lower than the Fresnel zone, and

dependence on the coordinates takes form $\varepsilon(z, \nu) = f_1(z)\cdot f_2(\nu)$, where ν and z - coordinates along a radius and an axis respectively. In this case the field at observation point, caused by the reflection of the spherical wave incident to the meteor trail, takes the form

$$E = A\int_{-l/2}^{+l/2} f_1(z)\,\zeta(\vartheta_z)\,\frac{e^{2ik\rho}}{\rho^2}\,dz, \qquad (1)$$

where A - constant, which depends on the electron concentration in the trace, $\varphi(\vartheta)$ - the diagram of scattering the element/cell of trace by a length of d/z (disk with a thickness of dz), ϑ_z — the angle between the axis of trace and the direction from one observation point to the next on the axis of trace with coordinate z: ρ - distance from observation point to the elementary section of trace with coordinate z; 1 - length of trace; $k = \frac{2\pi}{\lambda}$, λ - wavelength. In the approximation/approach of Fraunhofer ρ=R+zsinϑ, $\vartheta_z = \vartheta$, where R and ϑ the value of values ρ and ϑ_z for the center of trace. If the length of trace is commensurated with the size/dimension of Fresnel zone, then with the expansion ρ and ϑ_z in the series/row in terms of z and ϑ should be retained the members of the following order. As a result expression (1) will take the form

$$E(\vartheta) \cong A\cdot\frac{e^{2ikR}}{R^2}\varphi(\vartheta)\int_{-l/2}^{l/2} f_1(z)\left(1-\varkappa\frac{z}{R}\right)e^{ik\left(2z\cos\vartheta+\frac{z^2}{R}\sin^2\vartheta\right)}dz, \qquad (2)$$

where

$$\varkappa = 2\cos\vartheta - \frac{\frac{\partial\varphi}{\partial\vartheta}}{\varphi(\nu)}\sin\vartheta.$$

On the basis of expression (2), are obtained the formulas for

the efficient diameter of backscattering σ in the case of $f_1(z)$=const (uniform cylinder), $f_1(z)$=q+bz (electron concentration varies along the axis according to the linear law) and some other forms of the function $f_1(z)$. The obtained expressions for σ are equal with the analogous expressions, found under the assumption of Fraunhofer diffraction (σ_Φ). And as would expect, the greatest difference between σ and σ_Φ occurs with during the normal irradiation of trace. With the decrease of angle ϑ the difference between σ and σ_Φ is reduced. In the case of the cylinder uniform along the length whose radius is less or of the same order as wavelength λ, this difference proves to be unessential already when $\vartheta < \frac{\pi}{2} - 4 \div 5 \frac{l}{R}$. If the transverse size/dimension of trace is sufficiently great (in comparison with λ), then difference σ and σ_Φ can prove to be essential and in the inclined irradiation.

Analogous results are obtained also, if the electron concentration in the trace is so great that the trace can be represented by the ideally conducting cylinder. The solution of problem in this case is found in the approximation/approach of physical optics.

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Diffraction of electromagnetic waves on confined plasma with the presence of spatial dispersion.

V. B. Gil'denburg, I. G. Kondratyev.

The problem about scattering of plane electromagnetic wave on the uniform plasma objects with the sharp boundaries is solved taking into account the spatial dispersion, caused by thermal electron motion. Primary attention is paid to the research of the resonance effects, connected with the excitation in the plasma of natural transverse (electromagnetic) and longitudinal (plasma) oscillations.

In the presence of spatial dispersion (considered in the hydrodynamic approximation/approach) the field in the plasma is convenient to consider as the superposition of electromagnetic $(\mathbf{E}_e, \mathbf{H}_e \neq 0)$ and plasma $(\mathbf{E}_p \neq 0, \mathbf{H}_p = 0)$ the fields, which satisfy the conditions

$$\operatorname{div} \mathbf{E}_e = 0, \quad \operatorname{rot} \mathbf{E}_p = 0 \tag{1}$$

and described by the independent wave equations

$$\Delta \mathbf{E}_{e,p} + k_{e,p}^2 \mathbf{E}_{e,p} = 0. \qquad (2)$$

Here $k_e^2 = \frac{\omega^2}{c^2}\varepsilon$, $k_p^2 = \frac{\omega^2}{v_T^2}\varepsilon$, ω - the field frequency, c - the speed of light, v_T - average/mean thermal velocity, $\varepsilon = 1 - \frac{\omega_p^2}{\omega^2}$, ω_p plasma frequency. On the boundaries of object together with the continuity conditions of the tangential components of electrical and magnetic fields it is necessary to satisfy also the continuity condition of normal component of electric field (under the assumption of the mirror reflection of particles on the boundary of plasma). The identical form of wave equations for fields $\mathbf{E}_e$ and $\mathbf{E}_p$ makes it possible to claim that, at least, all those tasks, which are solved by the method of separation of variable/alternating in the case of cold plasma, without the special difficulties can be generalized, also, to that heated. As an example of this generalization is examined the task about the diffraction of plane wave on the bodies of the simplest geometric form: sphere and infinite cylinder.

For the sphere the solution is found out by field expansion inside, also, out of the plasma in terms of the vector spherical functions. The coefficients of expansion, which are the amplitudes of the multipole oscillations of different orders, are determined from the boundary conditions on the surface of sphere. If a radius of sphere a is small in comparison with the wavelength in the free space $\lambda=2\pi c/\omega$, then the frequency dependence of these coefficients carries

the pronounced resonance character: the amplitudes of the scattered waves of different multipole types sharply grow near the natural frequencies of the corresponding multipole oscillations of sphere. In the absence of dispersive processes in the plasma the cross section of scattering in the presence of the resonance reaches the value

$$Q_{\text{рез}} = \frac{\lambda^2}{2\pi}(2n+1), \tag{3}$$

where n - order of the resounding multipole (n=1 - dipole, n=2 - quadrupole, etc.). Resonance condition for the sphere of small sizes/dimensions (a<<λ) takes the form

$$(1-\varepsilon)\, j_n(k_p a) - \frac{k_p a}{n(n+1)}\, j_n'(k_p a)\,(\varepsilon n + n + 1) = 0 \tag{4}$$

($j_n(k_p a)$ - spherical Bessel functions) and when $V_T n \ll \omega a$ (that it is applicability condition for entire examination) determine two different series of resonance frequencies.

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One of them (corresponding to electromagnetic resonances) is approximately determined by the relationship/ratio

$$\varepsilon(\omega_n)\, n + n + 1 = 0, \tag{5}$$

but the second (corresponding strictly to plasma resonances) lies/rests at the region of low positive values ε, for which $k_p a \gg n$ and where with one and the same n equation (4) has many different roots. When $k_p a \gg n$ these roots are determined especially simply and they give for resonance frequencies ω_{nm} the following

expressions:

$$\omega_{nm}^2 = \omega_p^2 + \pi^2 \left(\frac{n+1}{2} + m\right)^2 \frac{V_T^2}{a^2}. \tag{6}$$

If the only mechanism energy loss for the natural oscillations of sphere are radiation losses, then plasma resonances (6), have a width of line, approximately/exemplarily $\left(\frac{\omega a}{c_T}\right)^4$ once less than electromagnetic (5).

Interesting special features/peculiarities possesses field in the sphere when $\varepsilon = 0$. The amplitudes of electromagnetic and plasma fields are converted in this case into infinity; however, total field as a result of their mutual compensation proves to be final.

Analogous results are obtained with the solution of the problem about the diffraction on the infinite plasma cylinder whose generatrix is perpendicular to electric field and wave vector of the incident plane wave.

Study of ellipsoid electromagnetic emitters in conducting medium.

Yu. Ya. Iossel', E. S. Kochanov, Ye. A. Svyadoshch.

Due to the strong fading of electromagnetic field in conducting medium for the series/row of practical tasks are used electrical type low-frequency emitters (elongated form) whose sizes/dimensions are considerably less than the wavelengths emitted by them.

Is examined the task about definition of both components of electromagnetic field at the distances, congruent with the sizes/dimensions of emitter.

I. Formulation of the problem. With the solution of problem the determining role plays not fading EMP, but the form of emitter and the location of electrodes on its surface.

Therefore are justified the following assumptions:

1) the equations of maxwell are substituted by the approximate system of equations

$$\mathrm{rot}\,\mathbf{H} = \mathbf{j},\ \mathrm{div}\,\mathbf{H} = 0,\ \mathbf{j} = \sigma\mathbf{E}, \qquad (1)$$

$$\mathrm{rot}\,\mathbf{E} = 0,\ \mathrm{div}\,\mathbf{E} = 0; \qquad (2)$$

2) the surface of emitter is substituted by the prolate spheroid and is introduced the spheroidal coordinate system.

Emitters are the asymmetrically excitable ellipsoids of revolution: metallic (ideally conducting) and dielectric.

II. Method of solution. In the approximation/approach in question the electric field is potential.

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Vector electric intensity is located employing the known procedure of the solution of the equation of Laplace in the spheroidal coordinate system as expansion in the series/row in terms of the associated functions of Legendre:

$$\mathbf{E} = -\operatorname{grad} \varphi_1 \tag{3}$$

$$\varphi = \sum_{n=1}^{\infty} \sum_{m=0}^{n} [A_n^m \cos m\alpha + B_n^m \sin m\alpha]\, Q_n^m(\xi)\, P_n^m(\eta), \tag{4}$$

where A_n^m, B_n^m - the constant coefficients, determined upon the satisfaction to boundary conditions on the surface of emitter.

The determination of the vector of magnetic intensity with the known methods (vector potential, the law of Biot and Savarr) leads to the fundamental difficulties, connected with the need of integrating the vector equation of Laplace, since in this case they are different

from zero all three components of vector potential.

For determining the components of magnetic field are proposed the following formulas, which satisfy (1):

$$H_\xi = \frac{\sigma(1-\eta^2)}{\sqrt{\xi^2-1}\sqrt{\xi^2-\eta^2}} \sum_{n=1}^{\infty} \sum_{m=0}^{n} m \frac{[A_n^m \sin m\alpha - B_n^m \cos m\alpha]}{n(n+1)} Q_n^m(\xi) P_n^{m\prime}(\eta); \quad (5)$$

$$H_\eta = \frac{\sigma(\xi^2-1)}{\sqrt{1-\eta^2}\sqrt{\xi^2-\eta^2}} \sum_{n=1}^{\infty} \sum_{m=0}^{n} m \frac{[A_n^m \sin m\alpha - B_n^m \cos m\alpha]}{n(n+1)} Q_n^m(\xi) P_n^m(\eta); \quad (6)$$

$$H_\alpha = \sigma\sqrt{\xi^2-1}\sqrt{1-\eta^2} \sum_{n=1}^{\infty} \sum_{m=0}^{n} [A_n^m \cos m\alpha + B_n^m \sin m\alpha] Q_n^m(\xi) P_n^{m\prime}(\eta) \quad (7)$$

with the same constant coefficients A_n^m and B_n^m, as in the expansion of potential.

The use of these expressions makes it possible to automatically write out the components of magnetic field according to the expression of potential.

III. Results of solution. Are given and are justified calculation formulas for determining electromagnetic field of the models being investigated. The results of the calculations of passage characteristics are compared with the results of measurements.

Error from the replacement of the complete system of equations of Maxwell by that approximated (1)-(2) is evaluated theoretically and it is made more precise according to the results of measurements.

Magnetic dipole in conducting medium.

O. G. Kozina.

Are investigated the fields of horizontal and vertical magnetic dipole in two-layered conducting medium. Primary attention is paid to the study of electromagnetic field in the near zone. Is carried out research of the applicability of the theorem about the reflection for different properties of interface.

Is investigated the possibility of restoring the properties of the second medium on the measured field in the first.

Are given the results of numerical calculations.

Radiation of electromagnetic waves by the electronic flux, which moves above the periodic structures.

O. A. Tret'yakov, S. S. Tret'yakova, V. P. Shestopalov.

Is examined the task about the radiation/emission of electromagnetic vibrations, caused by motion with constant velocity $v=\beta c$ of the flat/plane monochromatic electronic flux above the diffraction grating, comprised of the infinitely thin ideally conducting belts.

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Electronic flux moves perpendicularly to the generatrices of belts; the relationship/ratio between the wavelength λ, the period of lattice l and the width of belts d can be any. It is assumed that electromagnetic field as whole is moved above the lattice with the charge rate.

Since lattice is assumed to be not limited, and beam is the periodically charged/loaded plane, the unknown field is conveniently represented in the form of Fourier series. Unknown Fourier coefficients are found from the subordination of field to precise

boundary conditions; the obtained boundary-value electrodynamic problem of property then to Riemann-Hilbert's task. The solution of the latter takes the form of the infinite system of the linear algebraic equations:

$$
\begin{aligned}
&2cR_\sigma + \sum_{n=-\infty}^{\infty}{}' A_n \frac{|n|}{n} \chi_n V_\sigma^n = \varepsilon V_\sigma^0 \\
&2CR_m + \sum_{n=-\infty}^{\infty}{}' A_n \left(\frac{|n|}{n} \chi_n V_m^n - \delta_{mn}\right) = \varepsilon V_m^0;\ m = 0, \pm 1, \pm 2, \ldots,
\end{aligned} \qquad (1)
$$

where

$$
\begin{gathered}
R_m = \frac{1}{2} P_m(u),\ u = \cos\theta,\ \theta = \frac{\pi d}{l};\ R_\sigma = \sum_{m=-\infty}^{\infty} (-1)^m \frac{R_m}{1 + m\frac{\beta}{\varkappa}}; \\
V_\sigma^n = \sum_{m=-\infty}^{\infty} (-1)^m \frac{V_m^n}{1 + m\frac{\beta}{\varkappa}} \\
V_m^n = \frac{1}{2\pi^2 i} \int_{-\pi}^{\pi} R(e^{i\psi}) e^{-im\psi} \int_{\bar{\alpha}}^{\alpha} \frac{\zeta^n}{\zeta - \zeta_0} \sqrt{(\zeta - \alpha)(\zeta - \bar{\alpha})}\, d\zeta\, d\psi; \\
\overset{*}{R}(\zeta) = \begin{cases} [(\zeta - \alpha)(\zeta - \bar{\alpha})]^{-\frac{1}{2}}, & \bar{\alpha} < \zeta \leq \alpha \\ 0, & \alpha < \zeta < \bar{\alpha} \end{cases} \\
\zeta,\ \zeta_0 = e^{i\psi};\ \alpha = e^{i\theta},\ \bar{\alpha} = e^{-i\theta};\ \chi_n = 1 + iG_n; \\
G_n = \frac{|n|}{n} \frac{\left|1 + n\frac{\beta}{\varkappa}\right|}{1 + n\frac{\beta}{\varkappa}} \sqrt{\frac{\beta^2}{\left(1 + n\frac{\beta}{\varkappa}\right)^2} - 1};\ \delta_{mn} = \begin{cases} 1, & m = n \\ 0, & m \neq n. \end{cases}
\end{gathered} \qquad (2)
$$

$P_m^{(n)}$ - the polynomial of Legendre, C - unknown intermediate constant, A_n - the unknown Fourier coefficients field, factor ε determines the characteristics of beam. Series expansion parameter χ_n determines the convergence of system (1); it should be noted that χ_n ensures the more rapid convergence of system (1) in comparison with the analogous system, which describes wave diffraction on the

foil lattice.

Radiation/emission is possible only for Fouriers' those harmonics, for which is fulfilled the inequality

$$\beta^2 > \left(1 + n\frac{\beta}{\varkappa}\right)^2, \qquad (3)$$

where $\varkappa = l/\lambda$. Since $\beta<1$, then can be emitted only harmonics with the negative indices n, in this case occurs known connection/communication between the wavelength λ and the direction of its radiation/emission γ_n relative to the plane of the lattice

$$\lambda = \frac{l}{-n}\left(\frac{1}{\beta} - \cos\gamma_n\right). \qquad (4)$$

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Radiation/emission possesses directionality; if the condition for radiation/emission (3) is satisfied for the series/row of harmonics, then the directions of radiation/emission form discrete spectrum. Those harmonics, for which is satisfied the condition - $-n = \varkappa/\beta$, are emitted strictly perpendicular to the plane of lattice. Radiation/emission is symmetrical relative to the plane of lattice.

The obtained strict solution of the problem about the radiation/emission of waves by the electron beam, which moves with arbitrary constant speed, makes it possible to determine electromagnetic field to any degree of accuracy. Solution is obtained in the form, very convenient for the calculations on computer(s).

With the same prerequisites/premises is examined the task about the radiation/emission by the hollow cylindrical beam, which moves within the circular waveguide, period l and width rings d of which are arbitrary. In this case radiation conditions coincide with obtained conditions (3) for the flat/plane structure: electromagnetic field is found from the solution of the infinite system of linear algebraic equations, analogous (1). Are investigated the special features/peculiarities of the radiation/emission of electromagnetic vibrations in such structures.

Dispersive properties of transition layer.

N. V. Tsepelev.

Is examined the task about the propagation of oscillations in the medium with the transition layer, wave processes in which are described by system of equations

$$\Delta u_\nu = \frac{1}{v_\nu^2}\frac{\partial^2 u_\nu}{\partial t^2},\ \nu = 1, 2, 3,$$

where which $v_0 = \text{const}$, $v_1 = v_0 e^{\varepsilon z}$ and $v_2 = v_0 e^{\varepsilon H}$, and on the interfaces z=0 and z=H are satisfied the conditions

$$u_0 = u_1,\ \frac{\partial u_0}{\partial z} = \frac{\partial u_1}{\partial z}\ \text{при}\ z = 0,$$
$$u_1 = u_2,\ \frac{\partial u_1}{\partial z} = \frac{\partial u_2}{\partial z}\ \text{при}\ z = H.$$

Key: (1). with.

Perturbation source is placed at point with coordinates r=0, z=-h, moreover the field of displacement from it with -h<z<0 is given by the formula

$$\tilde{u}_0 = \int_0^\infty \left\{\frac{1}{2\pi i}\int_{\sigma-i\infty}^{\sigma+i\infty} F(k,\zeta)\, e^{k[iv_0\zeta-(h+z)\beta]}\, d\zeta\right\} J_0(kr)\, dk.$$

The solution of problem is constructed by common for ones the method of incomplete separation of variables by method and is given by the formula

$$u_\nu = \int_0^\infty \left\{\frac{1}{2\pi i}\int_{\sigma-i\infty}^{\sigma+i\infty} F(k,\zeta)\, U_\nu(k, z, \zeta)\, e^{k[iv_0\zeta-h\beta]}\, d\zeta\right\} J_0(kr)\, dk.$$

During the research of interference phenomena in the medium the study of the contour integrals, entering the solution, is reduced to the calculation of the sum of deductions at the singular points of function $U_{,}$, and to the integrals of some special ducts/contours.

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In the work is conducted the study of the singular points of function $U_{,}$, it is clarified their dependence on the parameters on the plane ζ they are done physical conclusions about the low-frequency processes, caused by transition layer.

Solution of spheroidal wave equation in an inhomogeneous medium.

E. A. Glushkovskiy, A. B. Izraylit, Ye. Ya. Rabinovich, B. M. Levin, Ye. F. Ter-Ovanesov.

Is examined the solution of spheroidal wave equation, in which the parameter of medium C=d/2k (d - interfocal distance, k - wave number) is the function of the coordinates

$$\frac{\partial}{\partial\xi}\left[(\xi^2-1)\frac{\partial u}{\partial\xi}\right]+\frac{\partial}{\partial\eta}\left[(1-\eta^2)\frac{\partial u}{\partial\eta}\right]-$$
$$-\left[\frac{1}{\xi^2-1}+\frac{1}{1-\eta^2}-C^2(\xi,\eta)\cdot(\xi^2-\eta^2)\right]\cdot u=0. \qquad (1)$$

To this equation it reduces the solution of the problems about the axisymmetric $\left(\frac{\partial}{\partial\varphi}=0\right)$ radiation/emission of the spheroid, sheathed from the dielectric whose electrical parameters (μ, ε) are the functions of coordinates.

If it is possible to represent $C^2(\xi,\eta)$ in the form

$$C^2(\xi,\eta)=C_0^2+C_\eta^2(\eta)+C_\xi^2(\xi) \qquad (2)$$

and to seek solution as expansion in terms of the angular spheroidal first-order functions $S(C_0,\eta)$, then applying to equation (1) the

integral transform of Greenberg, where as the kernel is undertaken function $S_{1n}(C_0, \eta)$, we will obtain

$$u=\sum_{n=1}^{\infty}\frac{\bar{u}_n}{N_{1n}}S_{1n}(C_0, \eta), \qquad (3)$$

where

$$N_{1n}=\int_{-1}^{1}S_{1n}^2(C_0, \eta)\,d\eta,$$

and $\bar{u}_n$ is the solution of system of equations

$$\frac{\partial}{\partial\xi}\left[(\xi^2-1)\frac{\partial\bar{u}_n}{\partial\xi}\right]-\left[\lambda_{1n}(C_0)-(C_0^2+C_w^2(\xi))\cdot\xi^2+\frac{1}{\xi^2-1}\right]\bar{u}_n=$$
$$=\sum_{t=1}^{\infty}(M_{nt}-\xi^2 Q_{nt}+C_w^2(\xi)\cdot P_{nt})\cdot\frac{\bar{u}_t}{N_{1t}}, \qquad (4)$$

where

$$M_{nt}=\int_{-1}^{1}c_v^2(\eta)\cdot\eta^2\cdot S'_{1n}(C_0, \eta)\cdot S_{1t}(C_0, \eta)\,d\eta$$

$$Q_{nt}=\int_{-1}^{1}c_v^2(\eta)\cdot S_{1n}(C_0, \eta)\cdot S_{1t}(C_0, \eta)\,d\eta$$

$$P_{nt}=\int_{-1}^{1}\eta^2 S_{1n}(C_0, \eta)\cdot S_{1t}(C_0, \eta)\,d\eta.$$

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System of equations (4) is reduced to the system of differential first-order equations

$$\left.\begin{aligned}&\frac{\partial\bar{u}_n}{\partial\xi}=V_n,\\&\frac{\partial V_n}{\partial\xi}=f(\xi)\cdot V_n+\varphi_n(\xi)\cdot\bar{u}_n+\sum_{t=1}^{\infty}\psi_{nt}(\xi)\,\bar{u}_t\end{aligned}\right\} \qquad (5)$$

System of equations (5) is solved by the methods, presented in the works of Persidskiy (DAN of the USSR, 1948) and peddler

(candidate dissertation, Rostov, 1955).

As the corollary is examined the solution of equation (1) for the case

$$C^{*}(\xi, \eta) = C_0^{*} + iC_1^{*},$$

where

$$C_1^{*} = \text{const}\,(\xi, \eta).$$

The given method applies to the solution of the nonhomogeneous equation, which corresponds to equation (1) with the right side, not equal to zero.

Radiation[redacted] of spheroidal radiator[redacted], covered with the magnetodielectric shell; numerical results.

A. B. Izraylit, T. I. Alekseyeva, Ye. F. Ter-Ovanesov.

The vibrator is a metallic spheroid (ellipsoid of revolution), excited by the axisymmetric circular slot, gashed in the center of radiator/resonator/element (relative to its length). Shell repeats the form of the radiator/resonator/element: its external surface is the surface of the spheroid, confocal with the metallic spheroid. The interior of shell, i.e., space between two confocal spheroids, is a homogeneous isotropic medium with the high values of dielectric and magnetic constants.

Problem is solved by the method of eigenfunctions. Are computed input admittances and radiation patterns of radiator/resonator/element depending on the parameters strictly of radiator/resonator/element and shell. Calculations are performed in electronic computer BESM-2M.

Diagrams of scattering from the surface of elliptical cylinder.

N. I. Dmitriyeva, L. N. Zakhar'yev, A. A. Lemanskiy, Z. I. Shteynfel'd.

Are communicated the results of the numerical calculation of the diagrams of scattering plane scalar wave from the metallic surface of elliptical cylinder. The calculation of the diagrams of scattering is carried out for dirichlet boundary conditions and Neumann on the surface of elliptical cylinder. The diagrams of scattering were calculated with the help of the known expressions in the form of series/rows according to the angular and radial eigenfunctions of the equation of Mathieu. The diagrams of scattering are designed for the series/row of the values of the parameters of elliptical surface and angle of incidence in the applied field. The parameters of elliptical surface were chosen in such a way that the calculation would make it possible to define, as transforms itself the diagram of scattering with a change in the form of elliptical surface from the circular cylinder to the thin band. The calculation of the diagrams of scattering makes it possible to trace, as depends stray field on the radius of curvature of the edge of elliptical cylinder, shading size/dimension of cylinder, angle of the diffraction of applied field.

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For the evaluation of the limits of the applicability of the approximation/approach of the physical theory of diffraction are equal the diagrams of scattering from the thin band, calculated by precise formulas and in the approximation/approach of the physical theory of diffraction.

Is stated the procedure of calculation of eigenvalues of the equation of Mathieu, Fourier coefficients angular eigenfunctions of elliptical cylinder, angular and radial Mathieu functions in electronic computer.

Solution with the help of the approximations of the task of diffracting the plane wave on ideally conducting large-diameter medium.

Yu. A. Yerukhimovich.

As the reference system of formulas are used field expressions in the distant zone, obtained in the quadratures by the method of vector potential, on the currents, induced on the surface of sphere by the incident plane wave. Into these dependences enter the values being subject to calculation

$$p_{1,2} = \int_0^{\pi} e^{-ix\cos\theta_1} J_1(\beta\sin\theta_1) \left\{ \begin{matrix} G(\xi) \\ \frac{iF(\xi)}{M} \end{matrix} \right\} d\theta_1,$$

where $G(\xi)$ and $iF(\xi)$ - Foch's universal functions, which have sufficiently complicated analytical representation,

$$x = 2ka\cos^2\frac{\theta}{2};\ \beta = ka\sin\theta;$$

$$\xi = -M\cos\theta_1;\ M = \left(\frac{ka}{2}\right)^{\frac{1}{3}};\ k = \frac{2\pi}{\lambda};$$

a - a radius of sphere, θ - the angle of observation point in the spherical coordinates R, θ, δ.

Interval $(0, \pi)$ of the values of variable/alternating θ_1 we divide/mark off into two intervals $0 \leqslant \theta, \leqslant \pi/2$ and $\pi/2 \leqslant 0, \leqslant \pi$.

In each of them it is possible to approximate $G(\xi)$ and $iF(\xi)$ by the dependences of form $(A+B\xi)e^{iC\xi}e^{-D\xi}$, where A, B, C, D do not depend on ξ.

The calculation of field components is performed with an accuracy down to the terms of order $(ka)^{-2/3}$.

Using such approximations, we obtain, that $P_{1,2}$ are expressed as the integrals of the form

$$S=\int_0^{\pi/2} e^{i\dot{\alpha}\cos\theta_1} J_1(\beta\sin\theta_1)\,d\theta_1,$$

where α - complex quantity, Re $I\alpha \leqslant 0$.

It is shown that

$$S=\frac{1}{3}\left[e^{i\dot{\alpha}}-e^{i\sqrt{\dot{\alpha}^2+\beta^2}}+i2u_1(\dot{w},\beta)\right],$$

$$\dot{w}=\sqrt{\dot{\alpha}^2+\beta^2}-\dot{\alpha}=\dot{\alpha}_1-\dot{\alpha}.$$

$U_1(\dot{w}, \beta)$ - cylindrical function from two variable/alternating (Lommel's function).

Further for calculating the fields should be produced only the operations of differentiation.

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Using different asymptotic representations of Lommel's

functions, we come to the following results.

In the region angle $0 \leqslant \theta \leqslant \theta_{rp} = 2\arccos\frac{1}{\mu^2}$ field components are represented in the form

$$E_{\theta,\varphi} \sim E_1 + E^{exp}_{\theta,\varphi},$$

where $E_1 = \left(1 - \frac{i}{\alpha_1}\right)e^{-i\alpha_1} + i\frac{1}{\alpha}I_0(\beta)$ - the "refined" geometric optic/optics, and $E^{exp}_{\theta,\varphi}$ consist of several exponentially decreasing terms, two of which are predominant (index of exponential curve in all terms is proportional to value M).

It should be noted that in the case in question there are no members, obtained during the solution by Huygens-Kirchhoff process, connected with the cylindrical functions, which indicates the inaccuracy of the correction term to the geometric optic/optics, given by Huygens-Kirchhoff's method from the illuminated region.

In the region of angles $\theta_{rp} \leqslant \theta = 180°$ correct result gives solution by Huygens-Kirchhoff's method, while the exponentially damped terms are relatively small.

The obtained dependences are significantly simpler than known up to now asymptotic expressions. They are expressed as the elementary functions (in the illuminated region) and through Lommel's tabulated functions (in the shadow zone).

The obtained approximations it is possible to use with the solution of the problem of diffraction on other smooth bodies of more complex form.

Radiation from the open end of flat and axisymmetric horns with the chamfered edges (scalar and vector task).

B. Ye. Kinber, A. D. Gondr.

In present report is stated:

1) the refinement of the method of solving two-dimensional problem about the horn, which is smoothly closed with the waveguide, whence attacks assigned type primary wave;

2) the generalization of method to the case of axisymmetric of the scalar and vector tasks when the methods of geometric optic/optics not at all can be used, since the internal caustic curve or the centerline of rays/beams can defend from the wall of horn in the portion of wavelength.

Excitation of sphere in the presence of dielectric layer.

V. N. Volovskiy, N. P. Mironcheva.

Are obtained field expressions during the excitation of the conducting sphere with the concentric layers of isotropic dielectric by arbitrary sources. Solution is registered in the form of series/rows according to the eigenfunctions. As an example are examined the cases of exciting the sphere by annular slot with the zero and fundamental harmonics of electric field. Are given numerical results for these tasks. During the symmetrical excitation of sphere the slot was taken final width. The parameters of layer $\varepsilon_1\mu_1$ and external space $\varepsilon_2\mu_2$ were considered arbitrary.

With the narrow slot the series/rows descend badly/poorly; therefore was carried out an improvement in the convergence.

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Primary attention is given to research of current distribution according to the surface of sphere, external conductivity of slot, and also field in the layer and above it. Is given the dependence of current distribution on the thickness of dielectric, losses and

position of slot. It shows that with the specific thickness of the layer the current distribution carries the character of standing wave. With an increase in the losses the traveling-wave ratio of current increases, and amplitude drops with the approximation/approach to a pole.

The dependence of the external conductivity of slot on the frequency with the sufficient thickness of the layer greatly calls to mind the curves of input admittance of thin antennas. Here also occurs the phenomenon of resonance. Resonance phenomena are especially sharply pronounced in the case of narrow slot and perfect dielectric. For the layer with the losses resonance curves become duller.

Besides this, is investigated the case, when on the boundary layer - external space it is possible to assign Leontovich boundary conditions. Is here given the dependence of conductivity on the dielectric constant of external space and thickness of the layer. During the asymmetric excitation are examined the directed properties of spherical antenna.

Is examined a change of the radiation patterns in the dependence on the position of slot, thickness of the layer and its parameters.

While conducting of numerical calculations great difficulties were met during the calculation of spherical Bessel functions.

Is carried out a comparative evaluation of the series/row of methods according to the calculation of these functions.

The propagation of low-frequency waves in flat and cylindrical layers, which are located in contact with the elastic medium.

P. V. Krauklis, L. A. Molotkov.

Is examined the propagation of waves in plane layer, which lies on the elastic half-space, and in the cylindrical layer, surrounded outside by elastic medium. Layers and their ambient media are assumed to be uniform ones and ideally elastic ones. On the interfaces are satisfied the conditions of the rigid or sliding contact.

The solution of the equations of Lame is constructed by the method of incomplete separation of variables and is represented in the form of the integrals of Fourier and Mellin. Research of the obtained solutions consists of the isolation/liberation from entire field of the displacement of "normal waves" and of the explanation of their properties. In contrast to the normal waves, usually examined/considered in the literature (for example, Love's waves), the chosen waves can have further fading as a result of the radiation/emission of energy from the layer.

This task is reduced to the determination of the special features/peculiarities of the integrands of Mellin's internal

integrals. For this purpose are investigated the roots of dispersion equations. On the basis of these research it is possible to isolate "normal waves" and to indicate the dependence of their phase speeds and attenuation factors on the frequency.

Primary attention in the work is paid to the study of the processes of propagation in the range of frequencies ν, determined by the inequality

$$0 \leqslant \nu \leqslant \frac{v_s}{h}, \qquad (1)$$

in which v_s - speed of transverse waves in the layer, and h - thickness of the layer. Special interest in the low-frequency waves is connected, mainly, with the fact that in the seismic experiments basic part of the spectrum is placed in interval (1). Furthermore, in this frequency region the wave picture in a decisive manner depends on contact conditions.

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Essential is also the fact that in interval (1) the wave picture proves to be simplest and a number of normal waves is small.

Let us pause at the characteristic of the low-frequency waves, observed in the layers and their ambient media. In the medium with the flat/plane of decked boats is propagated Rayleigh wave. The phase

speed of this wave is equal to wave velocity of Rayleigh which is observed in the elastic half-space, if we exclude from the system layer. In the cylindrical layer to Rayleigh wave is analogous the group of the normal waves which are observed in the elastic medium with the cylindrical groove.

If we are bounded to the examination of solid contact, then the waves indicated under condition (1) will prove to be only. However, in the case of sliding contacts, besides these waves, in the media in question is observed one additional wave. v_0 of the propagation velocity of the latter along the layer when $\nu = 0$ is determined by the equality

$$v_0 = 2\sqrt{\frac{\mu(\lambda+\mu)}{\rho(\lambda+2\mu)}}, \qquad (2)$$

in which λ, μ - permanent Lames, and ρ - density.

With an increase in the frequency phase speed monotonically decreases, in this case the dispersion in the case of cylindrical layer proves to be more noticeable than for plane layer.

The wave in question proves to be surface or volumetric depending on the relationship/ratio between value v_0 and transversing speed v_s' of the environment. When $v_0 < v_s'$ does not occur the leak/leakage of energy from the layer and the wave, for example in plane layer, attenuates with a distance of r along the layer as $r^{-1/2}$.

But if is satisfied condition $v_s' < v_0$, then the wave in question tests/experiences exponential decay, connected with the radiation/emission of energy into the medium ambient layer. Wave forming in this case in the medium is propagated at a rate of v_0 and therefore it can be named low-frequency leading wave. The latter also tests/experiences exponential decay which in the case of high frequencies proves to be more intense. Therefore out of gap/interval (1) head indicated barely is observed. When $v > \frac{v_s}{h}$ is observed the high-frequency leading wave which proves to be another nature.

The conducted investigations makes it possible to explain seismic experiments on the humid species/rocks and to indicate the character of the contact between the media. Furthermore, the results of research of the propagation of waves in the cylindrical layer explain wave development with seismic logging.

Lamb's problem for the elastic heterogeneous half-space.

A. G. Alenitsyn.

1. Is examined task about determination of field of displacement U(x, y, z, t)= (U_x, U_y, U_z), satisfying with z>0 equations of elasticity

$$LU \equiv (\lambda+\mu)\nabla(\nabla, U) + \mu\nabla U + (\nabla U)\nabla\lambda + (\nabla\mu, \nabla U) + \\ + (\nabla\mu, \nabla)U - \rho U_{tt} = 0, \quad (1)$$

to initial conditions

$$U|_{t=0} = U_t|_{t=0} = 0, \quad (2)$$

and to boundary conditions with z=0:

$$t_{zx} = t_{zy} = 0, \quad t_{zz} = e(t)\delta(x). \quad (3)$$

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It is assumed that Lame's coefficients λ and μ and density ρ - the arbitrary smooth functions of coordinate z, and that task is flat/plane, i.e., $U = U(x, z, t) = (u_x, 0, u_z)$.

2. Exact solution of task (1)-(3) takes form

$$U(x, z, t) = \frac{1}{\pi}\int_0^\infty dk \frac{1}{2\pi i}\int_M \begin{pmatrix} G_x(z, k, s)\sin kx \\ G_z(z, k, s)\cos kx \end{pmatrix} e^{kts}\frac{ds}{s}, \quad (4)$$

where $(G_x G_z) = G(z, k, s)$, M - duct/contour of Mellin, G satisfies system of ordinary differential equations of form

$$G'' + kAG' + k^2BG + kCG + DG' = 0 \quad (5)$$

Boundary conditions are satisfied, if G is the following combination of two linearly independent solutions $G^{(p)}$ and $G^{(s)}$ system (5):

$$G = (-D_s G^{(p)} + D_p G^{(s)})\, \Delta^{-1} \nu^{-1},\ \nu = \lambda + 2\mu,$$

where

$$D_p(k, s) = \left(\frac{d}{dz} G_x^{(p)} - k G_z^{(p)}\right)\Big|_{z=0},\quad \Delta = D_p E_s - E_p D_s;$$

$$E_p(k, s) = \left(\frac{d}{dz} G_z^{(p)} + k \frac{\lambda}{\nu} G_x^{(p)}\right)\Big|_{z=0}$$

expressions for D_s and E_s are analogous. (Solutions $G^{(p)}$ and $G^{(s)}$ they are chosen so that would be satisfied the initial conditions (2).).

3. Using asymptotic behavior G with $k \to \infty$, it is possible to investigate wave field in vicinity of fronts of different waves. Asymptotic behavior G it is simplest to obtain, after registering (5) in the form of the system

$$Z' = kHZ + kZ \qquad (6)$$

of four equations for vector $Z = (G_x, G_z, k^{-1}G_x', k^{-1}G_z')$. In region Re $s \geq \sigma, \geq 0$, Re $k \to +\infty$ is applicable the classical asymptotic behavior of Tamarkin. But in the regions, which contain the turning points, determined in this case from the equations $m_p^2 = 0$ or $m_s^2 = 0$, where $m_p^2 = 1 + n_p^2 s^2$, $m_s^2 = 1 + n_s^2 s^2$, $n_p^2 = \frac{\rho}{\nu}$, $n_s^2 = \frac{\rho}{\mu}$, it is necessary to use the more complicated formulas whose part is obtained in the work for the first time (uniform asymptotic behavior in the vicinity of the turning

points of the systems of form (6)).

4. Use/application of classical asymptotic behavior and steepest descent method gives formulas known from ray method for special features/peculiarities of field at usual fronts of longitudinal and transverse waves, and also at front of Rayleigh wave.

5. If n_p or n_s grow, in half-space there are geometrical shadows. The use/application of an asymptotic behavior in the vicinity of the turning point and method of the nonanalytic part of the field leads to the formulas for the dominant term of the special feature/peculiarity of wave field at the appropriate front of slide, completely analogous such formulas in the case of Lamb's task for the wave equation.

6. If n_p grows, leading wave is excited by shadow longitudinal wave and has shadow character. The special feature/peculiarity, common for a leading wave, is substituted in this case to the special feature/peculiarity of the type of the front of the slide: occurs the continuity of field with all derivatives.

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Diffraction of flexural wave on the circular obstruction in the plate.

Yu. K. Konenkov.

Is examined plane flexural wave $u_i = e^{ikR\sin\varphi}$ falling to the obstruction, in the plate which has the form of the circle of radius R. Stray field is sought in the form of the series/row

$$u_r = \sum_{n=-\infty}^{n=+\infty} [a_n H_n(kr) + \beta_n H_n(ikr)] e^{in\varphi}.$$

Coefficients a_n and β_n are found from boundary conditions. Let the boundary conditions be formulated in the following form: some linear differential operators $L_{r\varphi}$ and $M_{r\varphi}$, that function on the sum of the incident and reflected field, they become zero on the duct/contour along the boundary with the obstruction. Let us introduce the designations:

$$L_{r\varphi}H_n(kr)\,e^{in\varphi} = L_n(kr)\,e^{in\varphi},\; L_{r\varphi}e^{ikr\sin\varphi} = L(kr,\varphi),$$
$$M_{r\varphi}H_n(kr)\,e^{in\varphi} = M_n(kr)\,e^{in\varphi},\; M_{r\varphi}e^{ikr\sin\varphi} = M(kr,\varphi).$$

Then boundary conditions lead to the series/rows:

$$\sum_{n=-\infty}^{n=+\infty} [a_n L_n(kR) + \beta_n L_n(ikR)]\,e^{in\varphi} = -L(kR,\varphi).$$
$$\sum_{n=-\infty}^{n=+\infty} [a_n M_n(kR) + \beta_n M_n(ikR)]\,e^{in\varphi} = -M(kR,\varphi)$$

Whence, according to Fourier theorem, it follows:

$$\alpha_n L_n(kR) + \beta_n L_n(ikR) = -\frac{1}{2\pi}\int_{-\pi}^{+\pi} L(kR, \varphi)\, e^{-in\varphi}\, d\varphi,$$

$$\alpha_n M_n(kR) + \beta_n M_n(ikR) = -\frac{1}{2\pi}\int_{-\pi}^{+\pi} M(kR, \varphi)\, e^{-in\varphi}\, d\varphi.$$

Solving this system, we find:

$$\alpha_n = \frac{L_n(ikR)\int_{-\pi}^{+\pi} M(kR, \varphi)\, e^{-in\varphi}\, d\varphi - M_n(ikR)\int_{-\pi}^{+\pi} L(kR, \varphi)\, e^{-in\varphi}\, d\varphi}{2\pi\, [L_n(kR)\, M_n(ikR) - L_n(ikR)\, M_n(kR)]},$$

$$\beta_n = \frac{M_n(kR)\int_{-\pi}^{+\pi} L(kR, \varphi)\, e^{-in\varphi}\, d\varphi - L_n(kR)\int_{-\pi}^{+\pi} M(kR, \varphi)\, e^{-in\varphi}\, d\varphi}{2\pi\, [L_n(kR)\, M_n(ikR) - L_n(ikR)\, M_n(kR)]}.$$

As in all cases when the solution of the problem about the diffraction is found in the form of series/row, solution is convenient with kR<<1.

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In this case is found the solution of the following problems:

1) scattering on the eyelet:

$$u_r = \sqrt{\frac{2}{\pi kr}}\, e^{i\left(kr - \frac{\pi}{4}\right)} \left\{ \frac{\pi i \sigma}{4(1-\sigma)}(kR)^2 + \frac{\pi i (1+\sigma)}{8(1-\sigma)}(kR)^4 \ln kR - \right.$$
$$- \left[\frac{(1,46372 + \sigma\, 3,46372)\,\pi i}{32(1-\sigma)} + \frac{\pi^2\sigma^2}{16(1-\sigma)^2}\right](kR)^4 +$$
$$\left. + \frac{\pi i (3+\sigma)}{32(1-\sigma)}(kR)^4 \cos\left(\varphi - \frac{\pi}{2}\right) + \ldots \right\},$$

σ - Poisson ratio;

2) scattering in the rigidly attached section of radius R:

$$u_r = \sqrt{\frac{2}{\pi kr}}\, e^{i\left(kr - \frac{\pi}{4}\right)} \left\{ -1 + \frac{\pi i}{2} \frac{\cos\left(\varphi - \frac{\pi}{2}\right)}{\ln kR - \left(0{,}11593 + \frac{\pi i}{4}\right)} + \ldots \right\};$$

3) scattering on the opening/aperture with the supported edge:

$$u_r = \sqrt{\frac{2}{\pi kr}}\, e^{i\left(kr - \frac{\pi}{4}\right)} \left\{ -1 + \frac{\pi i}{2} \frac{\cos\left(\varphi - \frac{\pi}{2}\right)}{\ln kR - \left(0{,}11593 + \frac{\pi i}{4}\right) + \frac{1}{1 - \sigma}} + \ldots \right\}.$$

4) Scattering on the boundary of two plates with the different parameters:

$$u_r = \sqrt{\frac{2}{\pi kr}}\, e^{i\left(kr + \frac{\pi}{4}\right)} \{J(\nu, \varepsilon, \sigma, \sigma')(kR)^2 + \ldots\},$$

$$J(\nu, \varepsilon, \sigma, \sigma') = \frac{\frac{\pi}{2}\left\{\frac{\nu^4}{2}[\varepsilon(1-\sigma) + (1+\sigma')] + \varepsilon[\varepsilon\sigma - 1 - \sigma']\right\}}{2\varepsilon\{(1+\sigma') + \varepsilon(1-\sigma)\}},$$

ε - the relation of flexural rigidities; ν - refractive index, σ' - Poisson ratio plate - obstruction, 5) scattering from the section of plate, depending on in the circle/circumference:

$$u_r = \sqrt{\frac{2}{\pi kr}}\, e^{i\left(kr - \frac{\pi}{4}\right)} \left\{ -1 + \frac{\pi i}{2} \frac{\cos\left(\varphi - \frac{\pi}{2}\right)}{\ln kR + \mathrm{const}} + \ldots \right\}; \quad |\mathrm{const}| \sim 1.$$

Dispersive properties of Love's waves in an elastic heterogeneous sphere.

Z. A. Yanson.

Is conducted research of Love's waves (interfering waves), generated by rotary type source in the elastic uniform spherical layer $R_0 \leqslant r \leqslant R_1$, which covers heterogeneous elastic sphere $0 \leqslant r \leqslant R_0$. Wave velocity SH in the sphere exceeds the appropriate speed in the layer and monotonically it grows with decrease of r. The media in question are characterized by densities ρ_1, $\rho_0(r)$ and by elastic constants, respectively equal to λ_1, μ_1 and $\lambda_0(r)$, $\mu_0(r)$.

Task is reduced to the solution of the equations of Lame under zero initial data and boundary conditions, which escape/ensue from the assigned surface stress of $r=R_1$ (source is arranged/located in the pole of environment) and conditions of solid contact on the interface of two media. The solution of problem is constructed by the method of the incomplete separation of variables, and for the wave field in the layer we obtain representation in the form of series/row according to the associated functions of Legendre $P^{(1)}_{p-\frac{1}{2}}(\cos\theta)$, coefficients of which take the form of Mellin's contour integrals. The latter are computed from the deductions in zero of characteristic

(dispersion) equation of task. Then sum from the parameter by Watson's method is converted into the contour integral, and after the series/row of conversions for the wave field we obtain expression in the form of the infinite sum of Love's harmonics v_{kl}.

Each such harmonic is treated as interfering wave of the k order, completed l of revolutions on the layer.

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Research of Love's waves is conducted at the high frequencies with the help of the asymptotic behavior of cylindrical functions, and also by means of the use of an asymptotic behavior of the solution of the differential equation, generated by the radial part of the equation of Lame with change r in the interval $(0, R_*)$.

Dispersion equation at different frequencies is approximated by the transcendental equations of different form.

Is conducted the qualitative and quantitative study of dispersive curves and their comparison with the appropriate curves for: 1) the case of uniform sphere even 2) plane layer, which lies on the uniform half-space.

Results of the calculations of the fields of the once reflected waves near the initial point.

N. S. Smirnova.

Research of recent years, calculations dedicated to questions of the fields of waves, observed in the region exit points of leading wave, made it possible to a certain degree to explain, were such the special features/peculiarities of wave fields in this region.

Were proposed the new methods of the calculations which made it possible to obtain the head values of fields near exit point of leading wave to the surface, were carried out the proofs of the methods indicated, and are also worked out diagrams and methods of calculations in the high speed machines.

The use/application of the calculation procedures indicated made it possible to explain the basic character of the behavior of the fields of the waves reflected in the special region depending on elastic constant media and on the form of the function of effect.

Calculation was produced based on the example of once reflected waves PP, which extend in uniform-isotropic elastic medium which was

characterized by the parameters

$$\gamma = \frac{v_s^0}{v_p^0},\ \Delta_0 = \frac{v_s^1}{v_p^1},\ \delta = \frac{v_s^0}{v_s^1},\ \sigma = \frac{\rho_0}{\rho_1}.$$

(1) и $\frac{2h}{\lambda}$, (2) где v_s^0, v_s^1, v_p^0, v_p^1, ρ_0 и ρ_1 —

Key: (1). and. (2). where.

with respect to the speed of transverse and longitudinal waves and density, h - thickness of the layer, and $\lambda = v_p^0 \cdot T$, where 2T - time of action of source.

Was calculated the vertical component of the wave PP reflected to exit point r_0 and sum wave (reflected PP and head PPP) after exit point r_0.

As a result of calculations were obtained the theoretical seismograms of the waves reflected near the maximum ray/beam, which differed from the theoretical seismograms, calculated according to the old calculation methods. On the basis of these seismograms were constructed the graphs of a change in the reflection amplitude near the initial point in the dependence on a change in the parameters δ and 2h/λ.

It turned out that its maximum values the reflection amplitude takes not at exit point r_0, but at certain point $r_M > r_0$.

With an increase in the parameter 2h/T maximums are displaced to the side of smaller values of r, their value increase and they become sharper. With the decrease of this parameter maximum is displaced to the side of high values and becomes more slanting.

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Diffraction of plane hydroacoustic wave on a system of cracks in an elastic plate.

D. P. Kouzov.

Are examined diffraction phenomena in the following model: ideal compressible liquid, which fills half-space, is covered from above with elastic plate; in the plate there are two infinitely thin cracks, situated on the parallel lines. The falling/incident disturbance/perturbation is assigned in the form of the plane simple harmonic wave, which moves from to the depth of liquid in the direction, orthogonal to the direction of cracks (two-dimensional task). It is clarified, that the diffraction in the system in question is accompanied by the considerable resonance phenomena (depending on the distance between the cracks, the angle of incidence and frequency).

The mathematical formulation of the problem consists of the

following.

Is sought the solution of the equation of Helmholtz

$$\Delta u - k^2 u = 0$$

in the lower half-plane ($-\infty<x<+\infty$, $0<y<+\infty$), continuous up to axis Ox. On axis Ox is assumed to be that carried out the boundary condition

$$Lu \equiv \frac{\partial^5 u(x,0)}{\partial x^4 \partial y} - k^2\delta_0 \frac{\partial u(x,0)}{\partial y} + \nu_0 k^2 u(x,0) =$$

$$= \sum_{s=1}^{4} [A_s^+ \delta^{(s-1)}(x-a) + A_s^- \delta^{(s-1)}(x+a)].$$

In this case constants $A_s^\pm$ are such, that occur boundary-contact relationships/ratios

$$\lim_{x\to\pm a\pm 0} \frac{\partial^3 u(x,0)}{\partial x^2 \partial y} = 0; \quad \lim_{x\to\pm a\pm 0} \frac{\partial^4 u(x,0)}{\partial x^3 \partial y} = 0.$$

Difference of $u-u_0$ (where $u_0=\exp[-ik(x\cos\varphi_0+y\sin\varphi_0)]$ is considered satisfying the principle of maximum absorption. Here δ_0, ν_0 – some functions of the mechanical characteristics of plate and liquid; $\delta(x)$ - delta-function; 2a - distance between the cracks. All lengths in the task are considered dimensionless; dimensionlessness is introduced by division into the thickness of plate.

Is constructed a precise mathematical solution of stated problem. Is conducted asymptotic research of solution with $k<<1$ and $F>>1$, where $F=4\pi\frac{a}{l_0}$ (l_0 - length of flexural wave in the plate).

From the solution are selected ground waves $u_{\pm}$ (u_+ . it moves in the direction of the increase of coordinate x, u_- - to the reverse side) and cylindrical wave u_0.

Let us designate the relation of two corresponding components of wave field for the taken apart task and tasks with one crack by letter R with corresponding index

$$R_a = \frac{u_a}{u_a^0}$$

(superscript ° it marks the elements/cells of wave field for the case of one crack) let us name this relation the "function of effect".

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The function of effect for the straight/direct ground wave takes the following form:

$$R_+ = e^{-i\left(\frac{F}{2} + \frac{\pi}{10}\right)} \left\{ \frac{e^{iF} - e^{-i\frac{\pi}{5}}}{e^{iF}\cos\frac{\pi}{10} - e^{-i\frac{3\pi}{10}}} \cos(ka\cos\varphi_0) - \right.$$
$$\left. - \frac{e^{iF} + e^{-i\frac{\pi}{5}}}{e^{iF}\cos\frac{\pi}{10} - e^{-i\frac{3\pi}{10}}}\, i\sin(ka\cos\varphi_0) \right\}.$$

Let us consider for the certainty the case when phase displacement for the incident wave between the left and right cracks is absent or is multiple 2π ($ka\cos\varphi_0 = s\pi$, s=0, +-1, +-2,...). Then

$$R_{+}=e^{-i\left(\frac{F}{2}+\frac{\pi}{2}\right)}\frac{e^{iF}-e^{-i\frac{\pi}{5}}}{e^{iF}\cos\frac{\pi}{10}-e^{-i\frac{3\pi}{10}}}$$

and when $\frac{2a}{l_0}=n-\frac{1}{10}$ occurs the complete growing of straight/direct ground wave. When $\frac{2a}{l_0}\approx n-\frac{3}{20}$ straight/direct ground wave grows in the amplitude approximately 7 times.

For cylindrical wave the function of effect depends on viewing angle φ:

$$R_0=-2i\left\{\frac{e^{iF}\cos\frac{\pi}{10}-e^{-i\frac{\pi}{10}}}{e^{iF}\cos\frac{\pi}{10}-e^{-\frac{3\pi i}{10}}}\cos(ka\cos\varphi_0)\cos(ka\cos\varphi)-\right.$$
$$\left.-\frac{e^{iF}\cos\frac{\pi}{10}-e^{-i\frac{\pi}{10}}}{e^{iF}\cos\frac{\pi}{10}+e^{-i\frac{3\pi}{10}}}\sin(ka\cos\varphi_0)\sin(ka\cos\varphi)\right\}.$$

Therefore here occurs not only resonant amplification or weakening of wave process as a whole, but also a change in the character of radiation pattern. In particular, can occur splitting/fission two basic of the lobes/lugs of diagram into certain number of of narrower. It should also be noted that not at what values of parameters k, a, φ_0 the cylindrical disturbance/perturbation vanishes completely.

Spatial problem about the action of unsteady pressure wave on the plate of arbitrary planform, which moves in the flow of gas.

L. A. Galina, V. A. Kovaleva.

In the report is examined the task about the action of acoustic pressure wave on the plate of arbitrary planform which moves with supersonic speed. In this case are investigated the diffraction processes, which occur under a similar influence of the wave whose profile/airfoil is assumed to be arbitrary. Furthermore is investigated the case of acoustic effect of the type of the noise, when pressure profile in the incident wave is random function.

Is obtained effective solution in the quadratures for the plate of rectangular planform. Is somewhat more complex the case of plate, which in the plan/layout is triangle. In the general case the task is reduced to the solution of some integral equations.

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Some problems of short waves theory.

G. P. Shindyapin.

1. Questions about regular and irregular reflection of weak shock waves from rigid wall and interpenetration of weak shock waves belong to type of tasks which cannot be investigated with the help of only acoustic theory and is caused need for account of effect of basic flow parameters from value of overpressure. The general/common/total principles of short waves theory, which considers in the first approximation, this dependence, were developed by O. S. Ryzhov and S. A. Khristianovich.

Equations of the dynamics of the compressible gas for the case of plane-parallel flows give in this case the system of the short waves:

$$\frac{\partial\mu}{\partial\tau} + (\mu - \delta)\frac{\partial\mu}{\partial\delta} + \frac{1}{2}\frac{\partial v}{\partial y} + \frac{1}{2}\mu = 0,$$

$$\frac{\partial v}{\partial\delta} - \frac{\partial\mu}{\partial y} = 0,\ M = p/np_0, \qquad (1.1)$$

where the dimensionless functions μ, v, δ, v are connected with the projections of speed on radius-vector u and perpendicular to it v and

with the components of the cylindrical system r, ϑ by the equalities

$$u = a_0 M_0 \mu = a_0 \dot{M},$$
$$v = a_0 M_0 \sqrt{\frac{n+1}{2} M_0}\, \nu,$$
$$r = a_0 t \left(1 + \frac{n+1}{2} M_0 \delta\right),$$
$$\vartheta = \sqrt{\frac{n+1}{2} M_0}\, y, \quad \tau = \ln t. \qquad (1.2)$$

The differential equation of shock wave front, being propagated for the gas with overpressure p_1 and particle speed of q_1, takes the form

$$\frac{d\delta}{dy} = \pm \sqrt{2\left[\delta\left(1 + \frac{1}{M_0}\frac{\partial M_0}{\partial \tau}\right) + \frac{\partial \delta}{\partial \tau}\right] - (\mu + \mu_1)}\,. \qquad (1.3)$$

Moreover at the shock wave front Hugoniots's condition for normal component of speed is satisfied automatically in view of equality $M=p/np_0$ in entire flow. The condition of retaining/preserving/maintaining the tangential component to front leads to the equality of the type

$$u\psi - v = u_1(\psi + \vartheta + \alpha), \qquad (1.4)$$

where ψ — angle between the direction of radius-vector and the normal to the wave front, α - angle between the direction of particle speed before the wave front and the axis ϑ=0.

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2. Regular reflection of weak shock waves from rigid wall, as is known, takes place when front of plane shock wave OK with

overpressure p_1 extends along rigid wall with small fracture α, which coincides with angle of incidence and satisfies condition $\alpha > \alpha^* = 2\sqrt{\frac{n+1}{2}M_1}$. The front of the wave reflected consists of the segment of line OB, which composes with the perpendicular to the wall angle of reflection β, with overpressure p_0 and the small arc BC, where occurs an intense pressure drop from p_0 to p_1. So that between the wave reflected and the wall we have a zone ABCD of the short wave for which boundary conditions (1.4) at the front of the wave BC

$$(\mu-\mu_1)\sqrt{2\delta-(\mu-\mu_1)}-\nu=\mu_1(\alpha_0+y),$$
$$\alpha_0=\alpha/\sqrt{\frac{n+1}{2}Mo} \qquad (2.1)$$

reflected at the front of the maximum wave of lowering pressure AB

$$\mu y-\nu=0, \qquad (2.2)$$

on the wall DA

$$\nu=0, \qquad (2.3)$$

with the approach on BC to the point C let us require so that the front reflected would pass into the sonic circle/circumference, i.e.

$$\mu=\mu_1 \text{ with } \delta=\mu_1. \qquad (2.4)$$

Taking as M_0 value of M at point O and using flow conditions in the zone of short wave, we find representation β, M_0 and the coordinates of points $B(\delta_B, y_B)$, $O(\delta_0, 0)$ through α, M_1.

The solution of the system of short waves, which makes it

possible to satisfy all these conditions, we find in the form

$$\mu = a\delta + a\left(a - \frac{1}{2}\right)y^2 + a_1,$$
$$\nu = \frac{a(2a+1)\left(a - \frac{1}{2}\right)}{3}y^3 - [2a\left(a - \frac{1}{2}\right)\delta + (2a+1)a_1]'y. \quad (2.5)$$

where a_1 and a are determined accordingly conditions at points B and C.

The equation of the front of wave (1.3) reflected in this case succeeds in integrating and obtaining for the coordinates of front the locked analytical expressions. Solution, thus, makes it possible to trace a change in the picture of reflection with a change of the initial data in the range of regular reflection.

3. Irregular reflection of shock waves from rigid wall when angle of incidence $\alpha<\alpha^*$, is characterized by presence of three-shock configuration, which consists of falling/incident front AK with overpressure p_1, wave Maxa OA, along which overpressure falls from p_0 in basis/base of wave in wall to p_2 at point A, and front of wave AB reflected, where overpressure falls from p_2 to p_1, at certain point B. I.e., in the region OABD behind the front of the wave of Mach between the front reflected and the wall we will have a zone of the short wave, boundary of condition for which (1.4) on AK:

$$M = P_1/np_0,$$

AB:

$$\mu_1 \psi^0 + \nu_1 = 0, \quad (3.1)$$

$$M = p/np_0, \mu\psi^0 - \nu = \mu_1 (\alpha_0 + y + \psi^0), \quad (3.2)$$

OA:

$$M = p/np_0, \mu\psi^0 + \nu = 0, \psi^0 = \psi \Big/ \sqrt{\frac{n+1}{2} M_0}. \quad (3.3)$$

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On wall DO we have

$$\nu = 0, \quad (3.4)$$

and with the approach on AB to point B

$$\mu = \mu_1 \text{ with } \delta = \mu_1. \quad (3.5)$$

Taking as M_0 value of M in the basis/base of Mach's wave (wave approaches the wall at the right angle), we will obtain from (1.3) the coordinate of point 0(1/2, 0) and for the flow near triple point $A(\delta_A, \chi_0)$ condition system according to (3.1), (3.2), (3.3) and (1.3). Supplementing to these conditions the geometric condition AG=OG, where point G - point of intersection of perpendicular at point A to front AK with the wall, we determine the coordinates of point $A(\delta_A, \chi_0)$, value α_0, β_0 and μ_2 through μ_1 and we can trace a change in the maximum relative overpressure in wall p_0/p_1 in entire range of irregular reflection.

The solution of the system of short waves, which makes it possible to satisfy boundary conditions in the region OABD, takes the form

$$\begin{gathered}\mu = {}^1/_2 q\,(1-q)\,y^2 + a\,(q - {}^1/_3)^{-5/4}\left[q^3 + (b-1)\,q + \frac{1-2b}{5}\right] + d,\\ \delta = qy^2 - a\,(q - {}^1/_3)^{-5/4}\left(q - \frac{1-2b}{5}\right) + d,\\ \nu = {}^1/_3 q^3 y^3 - a\,(q - {}^1/_3)^{-5/4}\left[q^3 + (1-2b)\,q^2 - (1-2b)\,q + \frac{1-2b}{5}\right] y - dy.\end{gathered} \tag{3.6}$$

Writing/recording for it conditions at points A and O, we will obtain the system of five equations for determining the parameters a, b, d, q_1, q_0, which by reducing to one equation relatively q_1 makes it possible to numerically calculate dependence q_1 and, consequently, also the remaining parameters on the initial data M_1, α in entire range of irregular reflection.

4. Task about interpenetration of weak shock waves in the case of regular interaction leads to picture, which consists of falling/incident fronts OK_1, OK_2 with overpressures p_1, p_2 which intersect at small angle α (angle between normals to fronts at point of intersection) and fronts OB_1, OB_2 reflected, where overpressure falls from value of p_0 at point O to p_1, p_2 at points B_1, B_2 respectively. For zone OB_1B_2 of short wave boundary conditions (1.4) on OB_1:

$$(\mu - \mu_1)\,\psi_1^0 + \nu = \mu_1\,(\alpha^0 - \theta^0), \tag{4.1}$$

$$OB_2\colon\ (\mu - \mu_2)\,\psi_2^0 - \nu = \mu\theta^0, \tag{4.2}$$

where θ - component of the coordinate system with the axis θ=0 of the

perpendicular OK_2. At points B_1, B_2 we have conditions of the type $\mu=\mu_1$ with

$$\delta_1\left(1+\frac{1}{M_0}\frac{\partial M_0}{\partial\tau}\right)+\frac{\partial\delta_1}{\partial\tau}=\mu_1. \tag{4.3}$$

Satisfaction of conditions (4.1), (4.2) near the point of interaction 0 leads to the determination of the parameters α_1, α_2, β_1, β_2, v_0, M_0 and the law of the motion of point itself. The research by that obtained in this case for dependence for M_0 on the initial data M_1, M_2, α permits to derive critical relationship/ratio for these data characteristic regular interaction. At the same time in the case of the waves of equal intensity all obtained relationships/ratios pass in the relationships/ratios of the task of regular reflection.

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The solutions of the equations of short waves, which make it possible to satisfy conditions at points O, B_1, B_2, are solutions of type (3.7). Satisfaction of these conditions leads to the system nine equations relative to a, b, d, q_1, q_2, q_0, y_1, y_2, y_0 whose reducing to two equations relative to q_0, y_0 makes it possible to numerically solve the task for any initial data, which lie in the range of regular interaction.

Theory of the reflection of wind waves from the vertical wall.

Yu. M. Krylov.

Is placed and is solved the problem about finding of the field of the statistical characteristics of wind waves with their reflection from the vertical wall. Task is examined within the framework of the linear spectral wind wave theory.

The agitated surface of sea before the wall is recorded in the form of the sum of a large quantity of elementary incoming and reflected waves with different amplitudes, frequencies, directions of propagation and random phases. The increase of the surface of sea, obliged to the addition of incoming and reflected waves, is written/recorded in this form

$$z_{ij} = a(\mu_i, \theta_j)\sqrt{\Delta\mu\Delta\theta}\{\sin[\mu_i t + \varepsilon_{ij} + k_i(x\cos\theta_j + y\sin\theta_j) + \sin[\mu_i t + \varepsilon_{ij} - k_i(x\cos\theta_j + y\sin\theta_j)]\},$$

where μ_i — frequency of elementary wave, k_i — wave number, θ_j — the striking angle, x, y - the coordinate of point on the surface of sea, t - time, $\Delta\mu$, $\Delta\theta$ - increase in frequency and direction of propagation. Phase ε_{ij} of each elementary wave is assumed the random variable, evenly distributed in the range from 0 to 2π. Quantity $a(\mu_j, \theta_j)$ is considered as the integrated function μ_i and θ_j. Is

introduced the function

$$e(\mu_i, \theta_j) = \frac{1}{2} g\rho a^2 (\mu_i, \theta_j),$$

called the energy spectrum of incident waves. The agitated surface of sea before the wall is constructed in the form of sum $z = \sum_{ij} z_{ij}$. In view of the chance of phases ε_{ij} value z is the random function of coordinates and time.

Is obtained the following general/common/total expression for the root-mean-square divergence σ(x) the increase of floating surface from the rest position at a distance of x from the wall:

$$\frac{\sigma(x)}{\sigma_0} = 2^{\frac{1}{2}} \left[1 + \frac{\int\limits_{\mu_1}^{\mu_2} \int\limits_{\theta_1}^{\theta_2} e(\mu, \theta) \cos(2kx \cos\theta) \, d\mu d\theta}{\int\limits_{\mu_1}^{\mu_2} \int\limits_{\theta_1}^{\theta_2} e(\mu, \theta) \, d\mu d\theta} \right]^{\frac{1}{2}}.$$

Here σ_0 - corresponding root-mean-square divergence in the absence of wall; (μ_1, μ_2) and (θ_1, θ_2) - the ranges of frequencies and directions where spectrum e(μ_1, θ) is different from zero.

is carried out the calculation of the field of the mean square deviations of increase for the case of a deep water, on the basis of the concrete/specific/actual form of spectral function e(μ, θ), found by the author.

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Is obtained the following formula for the calculation $\sigma(x)$:

$$\frac{\sigma(x)}{\sigma_0} = 2^{\frac{1}{2}} \left[1 + \frac{S(2\pi x/\bar{\lambda}_0^*)}{S(0)}\right]^{\frac{1}{2}};$$

$$S(2\pi x/\bar{\lambda}_0^*) = \frac{\pi}{2} \int_0^\infty f_0(\omega) \left[2I_0\,(4\pi x\omega^2/\bar{\lambda}_0^*) - \frac{I_1\,(4\pi x\omega^2/\bar{\lambda}_0^*)}{2\pi x\omega^2/\bar{\lambda}_0^*}\right] d\omega,$$

$$f_0(\omega) = \omega^{-7} \exp\left[-\frac{0,785}{\omega^4}\right], \quad \bar{\lambda}_0^* = \frac{g\bar{\tau}^2}{2\pi},$$

$\bar{\tau}_0$ - average observed period of the wind waves incoming to the wall; I_0, I_1 - Bessel function of zero and first order respectively.

As a result of the carried out research it is discovered, that the field of wave heights, which appears about the vertical wall with the reflection from it of wind waves, qualitatively and quantitatively differs from appropriate interference pattern, created by simple harmonic wave.

Waves on the surface of viscous fluid.

L. V. Cherkesov, V. V. Pastushenko.

Is examined the following spatial problem. To the floating surface of the viscous incompressible fluid of infinite depth is applied the periodic moving system of the pressures of form $p=p_0 a(y) \exp[i(kx-\omega t)]$. It is necessary to find the form of waves forming in this case.

Accepting as the reference system of equations the linearized Navier-Stokes equations, the velocity components and pressure let us represent in the form

$$u = -\varphi_x - \psi_y,\ v = -\varphi_y + \psi_x + f_z,\ w = -\varphi_z - f_y,\ p = \rho(\varphi_t - gz)$$

if φ, ψ, f satisfies the equations

$$\Delta\varphi = 0,\ \psi_t = \nu\Delta\psi,\ f_t = \nu\Delta f \qquad (1)$$

The boundary conditions, which characterize wave motion, will be registered as the projections of the stresses/voltages, which function on the surface of liquid. Expressing them through the functions φ, ψ, f we will obtain with $z=0$

$$\varphi_{tt} + g(\varphi_z + f_y) + 2\nu(f_{yzt} + \varphi_{zzt}) = -\frac{i\omega}{\rho} P_0 a(y) \exp[i(kx - \omega t)]$$
$$2\varphi_{xz} + \psi_{yz} + f_{xy} = 0,\ 2\varphi_{yz} - \psi_{xz} - f_{zz} + f_{yy} = 0. \qquad (2)$$

In this case the type of floating surface is determined by the

formula

$$\zeta=\frac{1}{g}[\varphi_t+2\nu(\varphi_{zz}+f_{zy})]_{z=0}-\frac{P_0}{\rho g}a(y)\exp[i(kx-\omega t)].$$

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As a result the task is led to the determination of functions φ, ψ, f, which satisfy equations (1) and boundary conditions (2). Furthermore, it is here necessary to supplement condition at infinity φ, ψ, $f\to 0$ with $z\to-\infty$. Solving this problem, we find expression for the increase of the liquid:

$$\zeta=\frac{P_0}{\rho g}\left[\frac{i}{\sqrt{2\pi}}J-a(y)\right]\exp[i(kx-\omega t)],$$

$$J=\int_{-\infty}^{\infty}\frac{\bar{a}(m)\Delta_1(m)}{\Delta(m)}e^{im\sigma y}\,dm,$$

where $\sigma=\omega^2g^{-1}$, $\Delta_1(m)$, $\Delta(m)$ - some functions m. The obtained exact solution of task is investigated for function a(y) of form $a(y)=\begin{cases}1, & |y|<b\\ 0, & |y|>b\end{cases}$ with the help of the introduction of low parameter $\varepsilon=\omega^3\nu g^{-2}$. As a result, with an accuracy to ε to the first degree inclusively and for the high values of ky, is obtained this asymptotic representation for ζ:

$$\zeta=[A\sin(kx+y\sqrt{\sigma^2-k^2}-\omega t)+\varepsilon B\cos(kx+y\sqrt{\sigma^2-k^2}-\omega t)]e^{-\gamma y}+ \\ +O(e^{-ky}). \qquad (3)$$

Formula (3) in detail is analyzed. If we in (3) complete passage to the limit when $\varepsilon\to 0$, the obtained expression within the limit coincides with the appropriate expression for the ideal fluid.

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Is solved analogous problem for the liquid of final depth. Is obtained integral representation for the increase of floating surface. Due to the complexity of expressions could not carry out analysis as was done above. This can be done, if to examine long waves. The equations of long waves for the viscous fluid can be obtained from the system of equations of Navier-Stokes as the approximation/approach of the lowest order in the process of the construction of disturbances/perturbations. This process consists of the formal expansion of all values in terms of degrees of the ratio of the depth of liquid to the wavelength. As a result are obtained the equations of the form

$$u_t = -g\zeta_x - \frac{1}{\rho} P_{0x} + \nu u_{zz} + X,$$

$$v_t = -\frac{1}{\rho} P_{0y} + \nu v_{zz} + Y,$$

$$\zeta_t = -\int_{-h}^{0} (u_x + v_y)\, dz.$$

In this case pressure P is determined by the formula

$$P = \rho g(\zeta - z),$$

where P_0 - pressure on the floating surface, X, Y - horizontal mass forces.

On the basis of these equations of long waves, is solved the following problem. Let the viscous fluid of depth h revolve with the

constant angular velocity σ around the vertical axis together with the coordinate system. To the floating surface are applied the pressures of form $P = P_0\bar{a}(y)\exp[i(kx - \omega t)]$. It is necessary to determine the type of free surface of liquid and to investigate the generatrices of wave. Problem is solved with the help of the Fourier transform. Is conducted detailed analysis of the obtained solution is located the condition, under which is possible the formation of long waves.

Effect of waves on the immersed circular cylinder at the arbitrary [redacted] course angle.

A. Sh. Afremov.

Is examined flow of the ideal ponderable liquid about the motionless immersed circular cylinder of infinite elongation with the horizontal axis. To the cylinder attack regular two-dimensional waves with an amplitude of a, the frequency σ and a length of $\lambda = \frac{2\pi}{\nu}$. The direction of the run of waves composes angle χ with the axis of cylinder. Problem is solved in the usual ones of the assumption of small waves theory. The velocity potential of liquid Φ must satisfy known free-surface conditions, condition of nonpassage through the surface of cylinder S, conditions for radiation/emission and decrease up to zero with an infinite increase in the depth.

The potential of incident waves φ takes the form:

$$\varphi = \frac{ag}{\sigma} e^{i\nu x \cos\chi + i\nu z \sin\chi + \nu y + i\sigma t}. \qquad (1)$$

Here and subsequently has in mind only the real part of the complex expression.

In formula (1) it is marked: g - the acceleration of gravity, t - time, x, y, z - coordinate axes. X axis coincides with the axis of

cylinder, y axis is directed upward, the origin of coordinates is undertaken on the floating surface.

Potential Φ is sought in the form:

$$\Phi = (F_1 + F_2)\, e^{i\nu x \cos\chi + i\sigma t} + \varphi,$$

where functions $F_1(y, z)$ and $F_2(y, z)$ satisfy the equation

$$\Delta F_{1,2} - k^2 F_{1,2} = 0, \qquad (2)$$
$$k = \nu \cos\chi,$$

and to the boundary conditions

$$\frac{\partial}{\partial n}(F_1 + F_2) = -e^{-ikx - i\sigma t} \frac{\partial}{\partial n}\varphi \,\Big|_{\text{на } S}, \qquad (3)$$

$$\nu(F_1 + F_2) - \frac{\partial}{\partial y}(F_1 + F_2) = 0\,\Big|_{y=0}. \qquad (4)$$

Function F_1 satisfies equation (2) everywhere out of s, while function F_2 with y<0.

Solution is determined by the method of successive approximations.

In the first approximation, is assumed $F_2^{(1)} = 0$ and function $F_1^{(1)}$ is determined according to condition (3). In the polar coordinate system (r, θ)whose beginning is undertaken on the axis of cylinder, $F_1^{(1)}$ has the form:

$$F_1^{(1)} = -\frac{ag}{\sigma} e^{-\nu H} \sum_{n=-\infty}^{\infty} \frac{I'_n(\nu r_0 \cos\chi)}{K'_n(\nu r_0 \cos\chi)} e^{in\theta} K_n(\nu r \cos\chi)\, \mathrm{tg}^n\left(45^\circ + \frac{\chi}{2}\right). \qquad (5)$$

Here

K_n — MacDonald's function, I_n — the Bessel function of the first order from a number of alleged argument, H - submersion depth of the axis of cylinder, r_0 - radius of cylinder.

In the second approximation/approach function $F_2^{(2)}$ is determined from condition (4).

We have:

$$F_2^{(2)} = -\frac{ag}{\sigma}\int_{-\infty}^{\infty} b(\mu)\,\frac{(\sqrt{\mu^2+k^2}+\nu)^2}{\mu^2-\nu^2\sin^2\chi_0}\,e^{i\mu x+y\sqrt{\mu^2+k^2}}. \tag{6}$$

where

$$b(\mu) = \frac{e^{-\nu H - H\sqrt{\mu^2+k^2}}}{2\sqrt{\mu^2+k^2}}\sum_{n=-\infty}^{\infty}\frac{I_n'(kr_0)}{K_n'(kr_0)}\,\frac{k^n}{(\sqrt{\mu^2+k^2}-\mu)^n}\,\mathrm{tg}^n\left(45^\circ+\frac{\chi}{2}\right).$$

Singular point $\mu = -\nu\sin\chi$ is bypassed from below, and $\mu = \nu\sin\chi$ —

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functions $F_1^{(2)}$ and $F_2^{(2)}$ have the following asymptotic expressions:

$$F_{1\,z\to\pm\infty}^{(1)} \sim 0$$

$$F_{2\,z\to+\infty}^{(2)} = \frac{2\pi i}{\sin\chi}\frac{ag}{\sigma}e^{-2\nu H - i\nu z\sin\chi + y\nu}\sum_{n=-\infty}^{\infty}\frac{I'_n(kr_0)}{K'_n(kr_0)}$$

$$F_{2\,z\to-\infty}^{(2)} = \frac{2\pi i}{\sin\chi}\frac{ag}{\sigma}e^{-2\nu H + i\nu z\sin\chi + y\nu}\sum_{n=-\infty}^{\infty}\frac{I'_n(kr_0)}{K'_n(kr_0)}\operatorname{tg}^{2n}\left(45^\circ + \frac{\chi}{2}\right). \quad (7)$$

As the second approximation/approach for F_1 is accepted function $F_1^{(2)}$, determined from condition (3) taking into account $F_2^{(2)}$.

As can be seen from formulas (7), before the cylinder appears the two-dimensional agitation reflected at angle to axis $-\chi$, and after the cylinder the waves caused by its presence are propagated in the same direction, as encountering.

Relationships/ratios (3-4) are approximately valid only when $\chi \neq n\pi$ (n — integer), since when $\chi \to n\pi$ $F_2 \to \infty$, and the method of successive approximations placed into the calculation proves to be inapplicable.

The flow forces, which function on the cylinder, are counted on Lagrange's linearized integral.

For lateral force R_z is an expression:

$$R_z = \int \overline{R}_z(x)\,dx,$$

where $\bar{R}_x$ — load, which falls in the section of cylinder with abscissa x, moreover:

$$\bar{R}_x = -e^{-\nu H}\alpha_0\gamma\pi r_0^2\left[\sin\chi\cdot(m_1+m_2)-m_4\left(1+\frac{m_2}{m_1}\right)\right]\cos(\sigma t+ + \nu\chi\cos\chi)+e^{-\nu H}\alpha_0\gamma\pi r_0^2 m_4\sin(\sigma t+\nu x\cos\chi),$$

where

$$m_1=\frac{2I_1(kr_0)}{kr_0},\quad m_2=-\frac{2I_1'(kr_0)}{K_1'(kr_0)}\frac{K_1(kr_0)}{kr_0},$$

$$m_3=m_1f_1,\ m_4=m_1f_2,$$

$$f_1=\text{v. p.}\int_{-\infty}^{\infty}\frac{ue^{-2\nu H\sqrt{u^2+\cos^2\chi}}}{2\sqrt{u^2+\cos^2\chi}}\sum_{n=-\infty}^{\infty}\frac{I_n'(kr_0)}{K_n'(kr_0)}\times$$

$$\times\operatorname{tg}^n\left(45^\circ+\frac{\chi}{2}\right)\cdot\left(\frac{\sqrt{u^2+\cos^2\chi}}{\cos\chi}\right)^n\frac{(\sqrt{u^2+\cos^2\chi}+1)^2}{u^2-\sin^2\chi}\,du,$$

$$f_2=\pi e^{-2\nu H}\sum_{n=-\infty}^{\infty}\frac{I_n'(kr_0)}{K_n'(kr_0)}\left[\operatorname{tg}^{2n}\left(45^\circ+\frac{\chi}{2}\right)-1\right],$$

α_0 - amplitude of the angle of the wave slope of incident waves, γ - specific gravity/weight of water.

For functions f_1 and f_2 are carried out numerical calculations and are given graphs.

Analogous formulas are obtained for R_y.

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III. Use [redacted] of theory of diffraction and adjacent questions (in particular, some tasks, connected with the statistics, nonlinearity, and also with the use of geometric-optical methods).

Method of parabolic equation in a theory of the propagation of waves in the randomly inhomogeneous medium.

L. A. Chernov.

The account of shadings with the wave diffraction on uneven surfaces.

F. G. Base, I. M. Fouque.

In the approximation/approach of Kirchhoff is solved the problem about the diffraction of plane simple harmonic wave $\varphi_0(\mathbf{R}) = e^{ikR}$ on the uneven pad S of finite dimensions. If observation point R_0 is located in fraunhofer region and surface ideally reflecting, then for the stray field we have formula:

$$\varphi(\mathbf{R}_0) = \frac{e^{ikR_0}}{4\pi R_0}\int_S (\mathbf{qn})\, e^{i\mathbf{qr}}\eta(\mathbf{k}, \varkappa; \mathbf{k})\, d\mathbf{r}. \qquad (1)$$

Here n - normal to the reflecting surface, $\mathbf{q} \equiv \mathbf{k} - \varkappa$, $\varkappa \equiv k\frac{\mathbf{R}_0}{R_0}$.

Formula (1) differs from earlier used by the presence under the integral of function $\eta(\mathbf{k}, \varkappa; \mathbf{r})$, diffusing surface $z = \zeta(\mathbf{r})$: $\eta(\mathbf{k}, \varkappa; \mathbf{r}) = 1$, considering shadings/blanketings if point r on the surface is illuminated with the incident wave and is visible from observation point, and it becomes zero in all remaining cases.

For the one-dimensional inequalities the averaging of formula (1) on all realizations of random function $z=\zeta(x)$ leads to the following expression for the coefficient of reflection $f(\alpha)$ (coherent scattering):

$$f(\alpha)=\int_{-\infty}^{\infty} d\zeta W(\zeta)\, e^{iq_z\zeta} \int_{-\mathrm{tg}\,\alpha}^{\mathrm{tg}\,\alpha} d\dot{\zeta} W'(\dot{\zeta})\, P_2(\alpha, \alpha/\zeta, \dot{\zeta}). \qquad (2)$$

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Here α - angle of slip, $W(\zeta)$ and $W'(\dot{\zeta})$ - the density of distribution of heights/altitudes ζ and slope tangents $\dot{\zeta}=\frac{\partial\zeta}{\partial x}$ of surface, and $P_2(\alpha, \beta \mid \zeta, \dot{\zeta})$ - conditional probability that the point of surface with the height/altitude ζ and the derivative $\dot{\zeta}$ is not shaded by other points of the same surface with respect to source and observation point (β - angle of elevation observation point). Compact formulas are obtained only in the following limiting cases:

a) $\mathrm{tg}\alpha >> \sigma_2$ ($\sigma^2_2 \equiv \overline{\dot{\zeta}^2}$) - weak shadowing

$$f(\alpha)=\Phi(\alpha)\left[1-\frac{\psi(\alpha)}{\mathrm{tg}\,\alpha}\right]\int_{-\infty}^{\infty} d\zeta W(\zeta)\, e^{iq_z\zeta};$$

$$\Phi(\alpha)\equiv\int_{-\mathrm{tg}\,\alpha}^{\mathrm{tg}\,\alpha} W'(\dot{\zeta})\, d\dot{\zeta};\ \psi(\alpha)\equiv\int_{\mathrm{tg}\,\alpha}^{\infty}(\dot{\zeta}-\mathrm{tg}\,\alpha)\, W'(\dot{\zeta})\, d\dot{\zeta}; \qquad (3)$$

b) $\mathrm{tg}\ \alpha << \sigma_2$ - strong shadowing

$$f(\alpha)=\Phi(\alpha)\int_{-\infty}^{\infty} d\zeta W(\zeta)\exp\left\{iq_z\zeta-2\frac{\psi(\alpha)}{\mathrm{tg}\,\alpha}\int_{\zeta}^{\infty} W(\zeta')\, d\zeta'\right. . \qquad (4)$$

It is interesting to note that the latter/last formula in the opposite limiting case of weak shadowings ($\mathrm{tg}\ \alpha >> \sigma_2$) passes in (3) and can, thus, be considered as interpolation.

The account of shadings/blanketings for the intensity of stray field leads to the appearance of a factor less than unity, that depends on the angle of slip α of the incident wave and angle of elevation β observation point:

$$I(\alpha, \beta) \equiv \overline{\varphi(\mathbf{R}_0)\varphi^*(\mathbf{R}_0)} = I_0(\alpha, \beta)\,\Phi(\alpha, \beta)\frac{1 - \exp\left\{-\frac{\psi(\alpha)}{\operatorname{tg}\alpha} - \frac{\psi(\beta)}{\operatorname{tg}\beta}\right\}}{\frac{\psi(\alpha)}{\operatorname{tg}\alpha} + \frac{\psi(\beta)}{\operatorname{tg}\beta}};$$

$$\Phi(\alpha, \beta) \equiv \int_{-\operatorname{tg}\alpha}^{\operatorname{tg}\beta} d\zeta\, W'(\zeta)\left(1 - \frac{q_x}{q_z}\zeta\right)^2.$$

$I_0(\alpha, \beta)$ - the intensity of stray field without taking into account shadings/blanketings.

Diffraction of electromagnetic waves on the bodies, placed into the weakly inhomogeneous medium.

I. G. Kondratyev, M. A. Miller.

Are considered some general/common/total questions of the diffraction of electromagnetic waves on the bodies, arranged/located in the inhomogeneous medium. It is assumed that the significant dimensions of body L are negligible in comparison with the sizes/dimensions of region L_E, within limits of which the amplitude of applied field noticeably is changed, i.e.

$$L_E \gg L \gtrsim \lambda \qquad (1)$$

(where $\lambda = \frac{2\pi}{k} = \frac{\lambda_0}{\sqrt{\varepsilon\mu}}$ — wavelength in the medium). Then strictly the diffraction aspect of the task, connected with the satisfaction of boundary conditions on the body surface in the field of the incident wave, does not differ from the appropriate task for uniform space. However, ray tubes in the surrounding inhomogeneous medium can test/experience essential deformations, as a result of which diffraction field far from the body, and also general/common/total characteristics of scattering body are changed. In particular, in heterogeneous media is possible the ideal focusing of rays/beams, which brings in the approximation/approach of geometric optic/optics to the unlimited increase of the intensity of radiation/emission and,

therefore, to the unlimited increase in the apparent diameter of scattering. Of course in actuality this effect seals itself as a result of the heterogeneous structure of ray/beam near the caustic curve or the focus, and also due to the further refraction divergence.

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All these special features/peculiarities are in detail traced based on the example of the plane-layered medium $\varepsilon(z)$, $\mu(z)$, within which are arranged/located the emitter and the reflecting unit. Concrete/specific/actual calculations were performed for the media with the linear law of a change in dielectric constant $\varepsilon = \mathrm{const} \quad az$. As a result are established/installed the zones, within which the apparent diameter of scattering (among other things the apparent diameter of backscattering) differs significantly from the actual diameter, determined for the body in the uniform space. As reflectors were examined: a) quasi-static objects with the assigned tensor of polarizability, b) ideally reflectors of simple form (cylinder, sphere) in the approximation/approach of physical optics.

Unfortunately, only in some special cases to get rid of limitation (1). They include the system in which the emitter and reflecting object in the piecewise-uniform medium, for example are

located through the different sides from plane-parallel plate $\varepsilon = \text{const}$.

Special interest produces the effect of the "inflation" of field near region $\varepsilon = 0$ (or respectively $\mu = 0$). The limiting value of amplitude is actually determined by absorption $(\varepsilon^{*} = \varepsilon_g - i\varepsilon_м)$ or spatial dispersion, and the sizes/dimensions of the region of nonuniform field prove to be order $L_E \sim \frac{\zeta_м}{\alpha}$, so that condition (1) is satisfied only for the relatively small objects. However, because of the resonance increase of the amplitude of stray field on them can occur more efficient than even at the turning point of ray/beam.

At the end of the report are considered the possibilities of generalizing the results for the anisotropic media.

Compression of radio-study pulse in a dispersive medium with the random heterogeneities.

P. V. Bliokh.

Is examined the propagation of wave packet in the dispersive medium:

$$u\,(t,\,z) = A\,(t,\,z)\,e^{i(\omega_0 t - k(\omega_0)z)}.$$

The amplitude function A(t, z), which characterizes the form of wave packet, is assumed to be that slowly varying in comparison with the exponential factor.

Introducing the spectral density of the initial disturbance

$$G\,(\omega) = \int_{-\infty}^{+\infty} u\,(t,\,0)\,e^{-i\omega t}\,dt,\ \text{представим}^{(1)}\ A\,(t,\,z)\ \text{в виде}^{(2)}$$

$$A\,(t,\,z) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} G\,(\omega)\,e^{i[(\omega-\omega_0)t-(k(\omega)-k(\omega_0))z]}\,d\omega.$$

Key: (1). let us represent. (2). in the form.

After decomposing $k(\omega)$ in the series/row according to degrees $(\omega-\omega_0)$ and being limited to quadratic terms, it is possible to ascertain that the amplitude function satisfies the equation:

$$\frac{\partial A}{\partial z} = i\,\frac{k_0''}{2}\,\frac{\partial^2 A}{\partial \tau^2}. \qquad (1)$$

Here $k_0'' = \left.\frac{d^2k}{d\omega^2}\right|_{\omega=\omega_0}$, $\tau = t - z\left.\frac{dk}{d\omega}\right|_{\omega=\omega_0}$.

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Equation (1) coincides with the equation of the transverse diffusion of amplitude for the fronts of approximately plane waves with the diffraction.

The role of transverse coordinate plays variable/alternating τ, and to "length of wave" corresponds parameter k_0''.

This analogy makes it possible to immediately consider characteristic scale of the deformation of the impulse/momentum/pulse, which extends through the dispersive medium. The deliquescence of impulse/momentum/pulse is determined by the width of "Fresnel zone":

$$+\Delta\tau|_{z=\mathrm{const}} = \Delta t = \sqrt{zk_0''}.$$

If the initial pulse duration near point z=0 $T_0 \gg \sqrt{zk_0''}$ or $z \ll \frac{T_0^2}{k_0''}$.— work passes to the region of Fresnel diffraction (compare with condition $z \ll \frac{a^2}{\lambda}$, a — the size/dimension of slot in the shield). In this case, as is known, is possible the focusing of wave with the help of the lens, which deforms plane front of wave into the cylindrical with the center at the point of focusing z_Φ. The size/dimension of focal spot is equal to $\sim \frac{\lambda}{a} z_\Phi$. Analogously can be

realized the temporary/time focusing of wave packet, with which the impulse/momentum/pulse is reduced, reaching at point z_Φ the minimum duration

$$T_{min} \sim \frac{k_0''}{T_0} z_\Phi. \qquad (2)$$

Respectively pulse power grows $T/T_{min} \sim \frac{T_0^2}{k_0'' z_\Phi}$ once.

During the propagation of wave packet in the dispersive medium with the random heterogeneities to the process of compressing the impulse/momentum/pulse prevents the wave dissipation on the heterogeneities.

It is shown that there is certain critical length of route $z_{кр}$, beginning with which the temporary/time focusing of impulse/momentum/pulse becomes impossible.

As an example of dispersive medium with the random heterogeneities is examined the ionosphere. Are determined the conditions of compressing the radio pulse, passing through entire thickness of the ionosphere.

The spatial dispersion of inhomogeneous medium.

Yu. A. Rykhov, V. V. Tamoykin, V. I. Tatarekiy.

There is examined a question about the propagation of waves in the medium with the fluctuations of dielectric constant. For describing the middle field in the medium it is convenient to introduce concept efficient dielectric constant. $\varepsilon_{ij}^{эф}(\omega, k)$, which in the isotropic medium with fluctuations is tensor. The tensor character (dependence on k) of dielectric constant is caused by the spatial dispersion, caused by the heterogeneities of medium.

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Is investigated the case of isotropic plasma with the fluctuations of electron density. Is obtained the expression for $\varepsilon^{эф}(\omega\, k)$ with ω the close ones to

$$\omega_0 = \sqrt{\frac{4\pi e^2 \bar{N}}{m}},$$

which makes it possible to describe field in the plasma at frequencies, at which the plasma is medium with the strong fluctuations of dielectric constant.

Application of asymptotic methods to the analysis of the measurement of radiation patterns in the near zone.

V. B. Tseytlin, B. Ye. Kinber.

1. Earlier was obtained expression (1)

$$C \sim \int_S ([\mathbf{E}_{\rm I}\mathbf{H}_{\rm II}] - [\mathbf{E}_{\rm II}\mathbf{H}_{\rm I}], \mathbf{n})\, ds, \qquad (1)$$

which connects wave amplitude C in circuit of receiving antenna with radiation field of transmitting antenna I $E_{\rm I}H_{\rm I}$ and field $E_{\rm II}, H_{\rm II}$ with work of antenna II on transmission (without taking into account repeated interactions). Expression (1) is used for: 1) the error analysis of the measurement of radiation patterns and KND, the investigated and auxiliary antennas caused by proximity and by finite dimensions of the latter; 2) the determination of radiation pattern from the measurements in the near zone. In each case are examined two versions of the measurement: a) $\frac{\lambda R}{D^2_{\rm I,II}} > 1$; b) $\frac{\lambda R}{D^2_{\rm II}} < 1$; where R - distance between the antennas, $D_{\rm I}, D_{\rm II}$ - the sizes/dimensions of antennas.

2. For analysis of task a) for fields $E_{\rm I}H_{\rm I}, E_{\rm II}H_{\rm II}$ are used asymptotic expansions

$$\begin{Bmatrix}\mathbf{E}\\ \mathbf{H}\end{Bmatrix} = \frac{e^{ik\rho}}{\rho} \sum_{n=0}^{\infty} \begin{Bmatrix}\mathbf{F}_n(\varphi, \vartheta)\\ \mathbf{N}_n(\varphi, \vartheta)\end{Bmatrix} \frac{1}{(k\rho)^n}, \qquad (2)$$

where functions $\mathbf{F}_n, \mathbf{N}_n$ are connected with recurrent

relationships/ratios of index n, and F_e, N_e - radiation patterns on electrical and magnetic field. Substitution (2) in (1) for both antennas and asymptotic integration for the method of steady state they lead to the expression

$$C \sim \frac{4\pi i e^{ikR}}{kR}\left[1-\frac{i}{2kR}(\nabla')^2-\frac{1}{8(kR)^2}(\nabla')^4+O\left(\frac{1}{kR}\right)^3\right](\mathbf{F}_I\mathbf{F}_{II}), \qquad (3)$$

where

$$\nabla' = \left[\boldsymbol{\vartheta}_I\frac{\partial}{\partial\vartheta_I}+\boldsymbol{\varphi}_I\frac{\partial}{\partial\varphi_I}\right]-\left[\boldsymbol{\vartheta}_{II}\frac{\partial}{\partial\vartheta_{II}}+\boldsymbol{\varphi}_{II}\frac{\partial}{\partial\varphi_{II}}\right], \qquad (4)$$

$\boldsymbol{\vartheta}_I, \boldsymbol{\varphi}_I, \boldsymbol{\vartheta}_{II}, \boldsymbol{\varphi}_{II}$ — the unit vectors of spherical coordinates, in which are registered the fields of antenna I, II. Their polar axes are assumed parallel to the oriented spherical coordinates operator ∇' it takes the more complicated form. The values of all functions, entering in (3), are taken for the direction of the line of communications.

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3. Formula (3) makes it possible to calculate measuring errors of modulus/module and phase of radiation pattern and KND of antenna. For similar broadside antenna arrays with the central symmetry of field in the plane of aperture from (3) it follows that

$$\frac{P_{пр}}{P_{пер}} = \left(\frac{\lambda}{4\pi R}\right)^2 G_I G_{II}\left\{(\mathbf{F}_I\mathbf{F}_{II})^2-\left(\frac{kD^2}{2R}\right)^2\left[(\mathbf{F}_I\mathbf{F}_{II})\hat{\nabla}^4(\mathbf{F}_I\mathbf{F}_{II})-(\hat{\nabla}^2(\mathbf{F}_I\mathbf{F}_{II}))^2\right]\right\}, \qquad (5)$$

where

$$\hat{\nabla} = kD_I(\hat{\nabla}'_I-\varkappa\hat{\nabla}'_{II}),\quad \varkappa=\frac{D_{II}}{D_I}<1,$$
$$\hat{\nabla}'_{I,II} = \boldsymbol{\vartheta}_{I,II}\frac{\partial}{\partial(kD_{I,II}\vartheta_{I,II})}+\boldsymbol{\varphi}_{I,II}\frac{\partial}{\partial(kD_{I,II}\varphi_{I,II})}, \qquad (6)$$

i.e. asymptotic expansion contains the even degrees of parameter kD²2R. If polarizations of the tested and auxiliary antennas coincide, then from (5) follows formula for distance R_μ, on which the measuring error of KND is equal to μ:

$$R_\mu = \frac{D^2}{\lambda}\sqrt{\frac{\tau}{\mu}}, \qquad (7)$$

where τ is determined through the derived diagrams of auxiliary antenna and the relation of the sizes/dimensions of antennas.

In particular, for two equal square irregularly excited antennas

$$R_\mu = \frac{0,88}{\sqrt{\mu}}\frac{D^2}{\lambda}. \qquad (7a)$$

But if the size/dimension of auxiliary antenna is much lower than the size/dimension of subject, then

$$R_\mu = \frac{0,33}{\sqrt{\mu}}\frac{D^2}{\lambda}. \qquad (7б)$$

proceeding the same their formula (3), it is possible to show that the measuring error of lateral radiation in the case of antenna with the rounded duct/contour decreases with an increase in the number of minor lobe. For example, for the antenna with the circular aperture

$$R_\mu = \frac{1,4D^2}{\lambda\sqrt{\mu}\left(m+\frac{3}{4}\right)}, \qquad (8)$$

where m - number of minor lobe.

4. For antennas with complex diagrams (4) it contains also odd degrees of parameter $kD^2/2R$, which increases error of measurement. Is obtained formula for the measuring error of KND of the horn antennae of the form

$$\mu = \mu\left[\frac{kD^2_{ij}}{R},\ A_q\left(\frac{kD^2_{ij}}{8r_{ij}}\right),\ B_q\left(\frac{kD^2_{ij}}{8r_{ij}}\right)\right], \qquad (9)$$

$$(q = 1, 2, 3;\ i = 1, 2;\ j = \mathrm{I}, \mathrm{II}),$$

where r_{ij} — length of horns, D_{ij} — sizes/dimensions of the apertures of horns.

Functions A_q and B_q are designed and for them given the graphs. Calculation according to (9) gives a good agreement with the experiment when $\frac{R\lambda}{D^2} > 1{,}5$.

5. Solution of task b) is important for error analysis of measurements of diagrams and diameters scattering objects, located in section of plane wave, formed/shaped with auxiliary antenna. For field $E_I H_I$ as before is used expansion (2). For field $E_{II} H_{II}$ is used the expansion of the form

$$\begin{Bmatrix} \mathbf{E}_{II} \\ \mathbf{H}_{II} \end{Bmatrix} = e^{-ikx} \sum_{n=0}^{\infty} \begin{Bmatrix} \mathbf{E}_n \\ \mathbf{H}_n \end{Bmatrix} \frac{1}{(kr_0)^n}\Bigg|_{x=0}, \qquad (10)$$

where r_0 - significant dimension of antenna II, radius of the section of plane wave. Are obtained the formulas, analogous (3), in which the error has two components - on the point of steady state (due to the

gradient of amplitude along the wave front) and contour integral (due to the edge/boundary waves). Is made an evaluation of errors and are examined some methods of their decrease.

6. While conducting of measurements for exception/elimination of errors it is necessary to convert operator in (3) and analogous operator in measurement by method, described in p. 5. Inversion (3) gives

$$\frac{4\pi i e^{ikR}}{kR} F = \Big\{1 + \frac{i}{2kR}\Big[a_1 + \frac{\partial^2}{\partial\theta^2} + \frac{\partial^2}{\partial\varphi^2}\Big] + \frac{1}{8(kR)^2}\Big[a_2 + a_3\frac{\partial^2}{\partial\varphi^2} + a_4\frac{\partial^2}{\partial\theta^2} + a_5\frac{\partial^2}{\partial\varphi\partial\theta} - \frac{\partial^4}{\partial\varphi^4} - \frac{\partial^4}{\partial\theta^4} - 2\frac{\partial^4}{\partial\varphi^2\partial\theta^2}\Big]\Big\} C, \quad (11)$$

where a_i — function of the derived radiation patterns of auxiliary antenna. In particular, in the measurement of KND of major lobe from (11) it follows that

$$\mu = \frac{\frac{\partial^2\psi}{\partial\varphi^2} + \frac{\partial^2\psi}{\partial\theta^2}}{kR} + o\left(\frac{1}{kR}\right)^2, \text{ где}^{(1)}\ C = C_0 e^{i\psi}. \quad (12)$$

Key: (1). where.

From (12) follows the validity of the method of the measurement of KND by the defocusing of irradiator and the estimation for its error.

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The theory of standing finite waves on the floating surface and on the interface of heavy liquid of two layers of final depth and different density.

Ya. I. Sekerzh-Zen'ko.

Is examined the heavy ideal incompressible fluid, which consists of two layers of different density and arranged/located one above the other. Both layers are assumed final depth. Liquid is bounded above by floating surface, and from below by horizontal flat bottom.

In Lagrange's variable/alternating is given the complete formulation of the problem about the plane standing waves on the floating surface and on the interface of the liquid in question. Is indicated the method of the construction of the complete solution of problem in the form of series/rows according to the degrees of certain low parameter and besides without secular components/terms/addends. To the end/lead are designed first two and

partially the third approximation/approach. Are obtained the approximate equations of floating surface and interface and the approximate equation, which connects frequency with amplitude and wavelength. Are noted the basic properties of the finite waves in question, which differ them from the results of the solution of problem in the linear setting.

In the literature known to us we did not meet the works, dedicated to the task in question. The case of the free final oscillations of the interface of two unlimited heavy liquids of different densities is for the first time examined in our article "On the theory of the free final oscillations of the interface of two unlimited heavy liquids of different densities" (DAN of the USSR, 136, No 1, 1961. It is reported on the I All-Union symposium about the wave diffraction in g. To Odessa in 1960; see also our article in the labor/works of the Marine Hydrophysical Institute of the Academy of Sciences of the USSR, Vol. 23, 1961). The case of the liquid whose upper layer has finite depth, and lower is infinitely deep, is examined in our work, reported on the II All-Union symposium on the wave diffraction in Gor'kiy in 1962.

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Some tasks of wave dynamics in elastoplastic media.

I. G. Filippov.

In the present work is proposed the simplified formulation of the dynamic problems in the elastoplastic medium of Prandtl-Reuss which we will consider uniform, isotropic. Furthermore, the deformation of medium we will consider small and the volume strain of that reversed.

The general/common/total relationships/ratios between the components of the deviators of the stress tensor of the strain velocity tensor, as is known, take the following form:

$$\begin{gathered} 2G\,d/dt\,(\varepsilon_{xx} - \varepsilon) = d/dt\,(\sigma_{xx} + p) + m\,(\sigma_{xx} + p) \\ G\,d/dt\,(\varepsilon_{xy}) = d/dt\,(\sigma_{xy}) + m\sigma_{xy}, \end{gathered} \tag{1}$$

where $\varepsilon_{xx}, \varepsilon_{xy}, \ldots$ - amounts of deformations, ε - average/mean volume strain, σ_{xx}, σ_{xy} - the value of stresses/voltages, p - mean pressure m - the parameter of plasticity, depending on the unknown functions.

Expressions for $\sigma_{yy}, \sigma_{zz}, \sigma_{xz}, \sigma_{yz}$ are obtained from (1) by the cyclic permutation.

The condition of plasticity we accept in the form of Mises

$$(\sigma_{xx}-\sigma_{yy})^2+(\sigma_{yy}-\sigma_{zz})^2+(\sigma_{zz}-\sigma_{xx})^2+6\,(\sigma_{xy}^2+\sigma_{xz}^2+\sigma_{yz}^2)=2k^2. \qquad (2)$$

Furthermore, we consider that

$$p=\frac{2(\nu+1)G}{\nu-2}\varepsilon, \qquad (3)$$

where $1/\nu$ - Poisson number.

$$m=\frac{dW}{dt}. \qquad (4)$$

In the general case

where W - value of plastic work, moreover in view of positiveness m relationships/ratios (1) are valid with satisfaction of condition dW/dt>0 and condition (2). In the opposite cases of m=0 relationships/ratios (1) are degenerated and relationship/ratio for the elastic deformations.

In order to convert relationships/ratios (1) relative to the components of strain tensor, let us act as follows.

Let us introduce the relative deformations

$$\zeta_1=\varepsilon_{xx}-\varepsilon_{yy},\ \zeta_2=\varepsilon_{yy}-\varepsilon_{zz},\ \zeta_3=\varepsilon_{zz}-\varepsilon_{xx},\ \zeta_4=\varepsilon_{xy},\ \zeta_5=\varepsilon_{yz},\ \zeta_6=\varepsilon_{xz} \qquad (5)$$

and the auxiliary functions

$$\zeta_i^0(t)=A_i f(t),\ A_3=-(A_1+A_2),\ i=1,\ldots,6, \qquad (6)$$

where A_i - some number- functionals, and f(t) - characterizes a

change in the relative deformations in the course of time.

Let us decompose the physical space Q(x,y,z) into the final sum of subspaces Q_j, moreover $Q = \sum_j Q_j$.

Assuming relative deformations by the piecewise-continuous functions of coordinates x,y,z, in each subspace Q_j let us replace ζ_i with their average/mean values kA by functions from t.

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There is this function f(t) and there are such numbers A_i in each Q_j, that

$$\left| \iiiint\limits_{Q_j \times [0 \leqslant t < T_0]} [\zeta_i(x, y, z, t) - \zeta_i^0(t)]^2 \, dx dy dz dt \right|^{1/2} = \min_j \qquad (7)$$

in the sense of the root-mean-square approximation/approach where T_0 - duration of the process of loading (dW/dt>0).

Using relations (1), (2), (5), (6) in sense (7), for m(t) we will obtain the ordinary differential equation

$$\frac{dm}{dt} + m^2 - m \frac{d^2 f}{dt^2} \Big/ \frac{df}{dt} = \alpha_0^2 f'^2, \quad m = \alpha_0 f' \operatorname{th}[\alpha_0 f(t)], \qquad (8)$$

where

$$\alpha_0^2 = \frac{G}{k^2}(2A_1^2 + 2A_2^2 + 2A_3^2 + 3A_4^2 + 3A_5^2 + 3A_6^2). \qquad (9)$$

Note. It is possible to consider the elastoplastic medium taking into account the phenomena of strengthening. For example, counting in

(2)

$$k^2 = H^{-1}\left(G\int H\sigma_{ij}de^p_{ij} + k_0^2\right), \tag{10}$$

where H and k^2_0 - parameters of medium, H depending on the components of the deviator of strain tensor, and using condition (7), for m we will obtain the expression

$$m = H'_t(A_i, t)/H(A_i, t). \tag{11}$$

Parameter H makes sense of weight function relative to plastic work, and the process of loading is characterized by inequality $H'_t > 0$.

Considering the parameter of plasticity that approximately having form (9) or (11) and converting relationships/ratios (1) relatively $\sigma_{xx}, \sigma_{xy}, \ldots,$ by the introduction of the "generalized potentials" Φ and Ψ_i according to the formulas

$$u = \frac{\partial\Phi}{\partial x} + \frac{\partial\Psi_1}{\partial y} - \frac{\partial\Psi_2}{\partial z},\ v = \frac{\partial\Phi}{\partial y} - \frac{\partial\Psi_1}{\partial x},\ w = \frac{\partial\Phi}{\partial z} - \frac{\partial\Psi_2}{\partial x}, \tag{12}$$

equations of motion in the absence of external forces it is possible to reduce to the form

$$\begin{gathered}\frac{\partial^2\Phi}{\partial x^2} + \frac{\partial^2\Phi}{\partial y^2} - \frac{1}{a^2}\frac{\partial^2\Phi}{\partial t^2} = \frac{4b^2}{3a^2}e^{-\int_0^t m dt'}\int_0^t m e^{\int_0^{t'} m dt''}\Delta\Phi dt' \\ \frac{\partial^2\Psi_i}{\partial x^2} + \frac{\partial^2\Psi_i}{\partial y^2} - \frac{1}{b^2}\frac{\partial^2\Psi_i}{\partial t^2} = \frac{b^2}{a^2}e^{-\int_0^t m dt'}\int_0^t m e^{\int_0^{t'} m dt''}\Delta\Psi_i dt',\end{gathered} \tag{13}$$

where

$$a = \sqrt{(\lambda + 2G)/\rho},\ b = \sqrt{G/\rho},\ \lambda = 2G/(\nu - 2),\ e = 1, 2.$$

With m=0 equation (13) they pass in the equations for the

elastic media.

Approximate equations (13) simpler than the reference system of equations make it possible to solve the problems of the propagation of waves and task of diffraction in the elastoplastic medium of Prandtl-Reuss. Unknown constants $A_i(j)$ are determined from (7) after the solution of one or the other problem.

By the given method are solved general problem of diffracting the elastoplastic wave on the arbitrary duct/contour C and task of diffraction in the semi-infinite section/cut, and is also solved the problem of the propagation of wave in the elastoplastic medium which with m=0 gives the solution of the problem about the propagation of periodic disturbances/perturbations in the elastic medium.

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Incidence of plane electromagnetic wave on the layer of nonlinear substance.

L. A. Ostrovskiy, Ye. I. Yakubovich.

Recently increasing interest acquire such types of the boundary-value problems of electrodynamics for solving which usual geometric-optical and diffraction methods are unsuitable. Some of such types form the tasks of nonlinear electrodynamics, in particular, nonlinear optic/optics. In the report is examined the propagation of electromagnetic waves in the heterogeneous layer, which contains nonlinear medium. If nonlinear are the dispersive properties of medium (moreover, losses can be negative, i.e., medium can contain the excited molecules), then a change in the wave amplitude is determined by two factors - heterogeneity and nonlinear dissipation. Complication of the problem in comparison with the nonlinear is caused another need for the account of interaction in the layer of waves with different wave numbers. Therefore the apparatus for spectral field expansion to the plane waves does not lead to simplification in the task.

In the report are derived the equations, which describe a change in amplitude and phase of monochromatic field in the heterogeneous nonlinear medium (nonlinearity is caused by the substance, which consists of the two-level molecules). On the basis of the general solution of these equations for the one-dimensional case is examined the task about the incidence/drop on the layer of plane wave. It is located field distribution in the layer, and also coefficients of reflection V and transmission R. These values depend both on the frequency ω and on amplitude A of the incident wave. As in the linear case, layer has specific resonance frequencies, but values their others.

Special is the case of negative damping (effective layer). For this layer with the small amplitude of the field of the incident wave is possible the ambiguity in the determination of harmonic solutions; however, not all these solutions are stable in view of the possibility of generation.

Are examined the special features/peculiarities of reflection from the thin (order of wavelength λ) and thick (thickness of the layer is great in comparison with λ) layers.

Investigated also a continuous incidence in the wave on the heterogeneous layer. With certain polarization of the incident wave

this task with the help of the separation of variables, can be reduced to the one-dimensional.

The method used in the report, apparently, can be generalized to the series/row of other non-one-dimensional quasi-optical problems, which relate to the nonlinear media.

The propagation of quasi-harmonic waves in a heterogeneous dispersive medium.

L. A. Ostrovskiy.

The problem about the propagation of quasi-harmonic (modulated) waves in the inhomogeneous medium is solved most frequently in the approximation/approach of geometric optic/optics and is reduced to the determination of value and direction of the radial velocity at each point, and also change of the amplitude of field along the ray/beam. However, for the dispersive media this approximation/approach hardly ever proves to be sufficient in view of the fact that at the distances, congruent with the characteristic scale of heterogeneity, can substantially be manifested the deformation of the envelopes of wave due to the dispersion. This deformation is of independent interest.

In the report is proposed the method of integrating the wave equation for some classes of similar tasks, which makes it possible to determine change in time and space of "slow" values - the frequency of wave $\omega(r,t)$ and amplitude $A(r,t)$.

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These values satisfy nonlinear (in spite of the linearity of initial equations) equations in the partial derivatives which can be integrated. The obtained solutions are applied for the study of the passage of the quasi-harmonic wave through the heterogeneous layer. It is shown, in particular, that under specific conditions the dispersion leads to the grouping of wave energy in the very narrow intervals; the frequency of wave in the same intervals sharply varies; then are formed regions with the "three-frequency" solution. *In the general case of change* ω and A they are connected and are determined by "competition" between the dispersion and heterogeneity of medium.

The simplest results are obtained for the one-dimensional tasks; generalization to the three-dimensional case is achieved with the help of the examination of the "ray tubes" whose diameter varies at each point in the course of time. It should be noted that similar methods of solution of hyperbolic equations prove to be efficient and in the more complicated cases (for example, waves in the linear unsteady ones or in the nonlinear media).

Reflection of atmospheric waves from solid obstructions.

G. S. Golitsyn.

In the isothermal atmosphere heterogeneous in density there are two transmission modes, caused by the combined action of the elasticity of air and by the stratification of the atmosphere under the effect of gravitational force. These waves are described by the following dispersion equation:

$$\omega^4 - \omega^2 (k^2 + \alpha^2) c^2 + N^2 c^2 k^2 \cos^2 \varphi = 0, \qquad (1)$$

where ω - frequency, k- wave number, $\alpha = \gamma g/2c^2$, $\gamma = c_p/c_v$, g - the acceleration of gravity, c - the speed of sound, $N = \sqrt{\gamma - 1}\, g/c$, φ - the angle between the direction of wave vector and the horizontal.

From (1) it follows that

$$\omega^2 = \frac{1}{2} c^2 [(k^2 + \alpha^2) \pm \sqrt{(k^2 + \alpha^2)^2 - N^2 c^{-2} k^2 \cos^2 \varphi}]. \qquad (2)$$

Frequencies with sign (+), for which $\omega > \alpha c$, with $k >> \alpha = 1/2H$, where H - height of homogeneous atmosphere, they correspond to the usual acoustic waves (the acoustic branch). Frequencies with the sign (-) correspond to the internal gravity waves for which $\omega \leq N \cos \varphi < \alpha c$. Both transmission modes possess frequency and spatial dispersion. The reflection of these waves from the solid flat/plane boundary

possesses a number of interesting special features/peculiarities.

With the reflection must the holding frequency and the projection of wave vector on the plane of obstruction. Furthermore, on the interface must be equal to zero normal to the surface of the section of the component of speed. In the waves of the type in question the" terminus" of the velocity vector describes certain ellipse in the vertical plane, passing through the wave vector. From the considerations of symmetry it is obvious that the oscillations in the wave reflected will occur in the same plane. For simplicity we will count this plane of the perpendicular plane of obstruction.

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Requirements of the retention/preservation/maintaining frequency and tangential component of wave vector give the following conditions:

$$k_1^2(\omega^2 - N^2\cos^2\varphi_1) = k^2(\omega^2 - N^2\cos^2\varphi), \quad (3)$$

$$k_1\sin\theta_1 = k\sin\theta, \quad (4)$$

where θ, θ_1 - angles of incidence and reflection, and index $_1$ relates to wave (4)reflected. If ψ - angle between the horizontal of obstruction, then all three angles are connected with the relationship/ratio

$$\psi + \varphi + \theta = \frac{\pi}{2} = \psi + \varphi_1 - \theta_1. \quad (5)$$

Conditions (3)-(5) are sufficient for determining of unknown values

k_1, θ_1, φ_1 from the assigned values of k, θ, φ.

With the reflection from horizontal plane $(\psi=0)$ $k=k_1$, $\varphi=\varphi_1$, $\theta=\theta_1$, i.e., are observed the normal conditions for reflection. If $\psi\neq 0$ (slope of mountain), then reflection occurs with a change in the wavelength and $\theta_1\neq\theta$. To the process of solving system (3)-(5) it is possible to give geometric interpretation. Equation (3) is in polar coordinates (k_1,φ_1) the equation of ellipse for the acoustic branch and hyperbolas for the gravity waves. Equations (3), (4) are symmetrical relative to k and k_1, θ and θ_1, φ and φ_1. The terminus of the wave vector of the wave reflected and the origin of the wave vector of the incident wave must be located on the intersection of these curves of the second order with perpendiculars to the reflecting plane, conducted at distance k sin θ from the origin of coordinates (terminus of the wave vector of the incident wave). Condition (5) makes it possible to select from the points of intersection that which corresponds to the wave, reflected back in the atmosphere.

The requirement of a change in the phase on π with the reflection ensures equality to zero normal component of speed on the interface.

Propagation of acoustic-gravitational waves in an unhomogeneous atmosphere.

L. A. Dikiy.

Any strong localized disturbance/perturbation excites in the atmosphere the whole set of waves with the different periods. Usually these waves observe with the help of the microbarographs, which fix the pulsations of pressure in the range from the fractional minutes to 10 or somewhat more than minutes. The signal velocity proves to be order 300 m/s. During the theoretical study of such waves it is clarified, that they cannot be described second order equation for the time as in the "pure/clean" acoustics, but only by the system of equations of higher order, which joins acoustic and gravitational oscillations. In the present work is constructed the theoretical model of such waves upon consideration of the real dependence of temperature on an altitude (with two temperature minimums, at heights/altitudes ~17 and 86 km). With the help of the electronic computer are obtained the dispersive curves, which show the dependence of natural frequencies on the wave numbers or on the phase speeds. In this case entire/all set of curves falls into two beams. The waves, which correspond to one of them, can be somewhat

conditionally treated as acoustic ones (although the archimedian forces play in them known role). Waves, which correspond to other, respectively gravitational. The periods of acoustic waves are less, approximately, 5 min. Phase speeds - are more than 300 m/s. For the gravitational ones, on the contrary, the periods more than 5 min., speeds less than 300 m/s.

Special attention is given to the energy distribution of oscillations according to the height/altitude. Are separately calculated four forms of the energy: kinetic energy of the horizontal component of speed, energy of vertical component, potential energy, connected with changes in the pressure (elastic energy) and with changes in the entropy (thermobaric, or gravitational). The given graphs are shown, as with the decrease of wavelength wave energy is crowded in the waveguide layers. For the acoustic waves this of the zone of the minimums of temperature.

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The phase speeds of short acoustic waves are close to 300 m/s, dispersion almost is absent. Energy of short gravity waves is crowded in the zone of large relative stability, ~110 km. The phase speeds of these waves are close to zero, dispersion is great. For the comparison of the separate forms of energy it is possible to use the

following relationships/ratios. e_r, e_B, e_y, e_T - density of horizontal, vertical, elastic and thermobaric energy. Then

$$e_y = e_T \cdot h/\gamma H,$$
$$e_T = e_B \cdot \sigma_*^2/\sigma^2,$$

where H(z) - so-called, statically equivalent height of homogeneous atmosphere H=RT/g, γ - the relation of heat capacities, h - dynamically equivalent height/altitude, equal to c_Φ^2/g (c_Φ - phase wave velocity). Further, σ - frequency, σ_* - this so-called frequency of Brunt-Vaisala $\sigma_*^2 = \beta/\gamma H$, where β - steady-state stability factor $\beta=(\gamma-1)\ g+\gamma gH'$. Thus, with an increase in the phase velocity (or h) increases the portion of elastic energy in comparison with the horizontal. With an increase in the period increases the portion of thermobaric energy in comparison with the vertical. For the comparison of the vertical and horizontal energies with each other is proposed the formula

$$\frac{\bar{e}_B}{\bar{e}_r} = -\frac{c_\Phi}{\sigma} \cdot \frac{d\sigma}{dc_\Phi},$$

where $\bar{e}_B$, $\bar{e}_r$ - this e_B, e_r, averaged on the air column. Here $d\sigma/dc_\Phi$ is computed along the dispersive curve. Thus, the relation of horizontal and vertical energy can be calculated according to the inclination/slope of dispersive curve to the coordinate axes. Latter/last formula allows/assumes this interpretation

$$c_{rp} = c_\Phi \cdot \frac{\bar{e}_r}{\bar{e}_r + \bar{e}_B},$$

where c_{rp} - the group velocity of waves. Group velocities calculated according to this formula nowhere exceed 310 m/s, although the phase

speeds are not limited. The group velocities of gravity waves much less than the phase speeds which and themselves are small; therefore the group velocities of short gravity waves are close to zero. Gravity waves more slowly diverge than acoustic, consequently more slowly they attenuate at the particular point of space. Besides very low speeds, the observation of these waves impedes also the fact that their energy in essence is thermobaric, but is not elastic, which only and they can note microbarographs.

Work gives the graphs, which illustrate the waveguide character of the propagation of short waves.

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Ray description of wave beams.

L. S. Dolin.

Work examines some properties of the spectrum of the correlation function of the randomly heterogeneous monochromatic monochromatic field, which lead under specific conditions to the purely energy description of diffraction effects.

It is known that the correlation function

$$\Gamma(\mathbf{r}_1, \mathbf{r}_2) = \overline{E(\mathbf{r}_1) E^*(\mathbf{r}_2)}$$

of the scalar wave field E(r) satisfies system of equations

$$(\Delta_1 + k_0^2)\,\Gamma(\mathbf{r}_1, \mathbf{r}_2) = 0, \qquad (1)$$

$$(\Delta_2 + k_0^2)\,\Gamma(\mathbf{r}_1, \mathbf{r}_2) = 0. \qquad (2)$$

If we in equations (1), (2) switch over to new variable/alternating $\rho = r_1 - r_2$, and $\mathbf{r} = \frac{\mathbf{r}_1 + \mathbf{r}_2}{2}$ and then sum these equations and subtract one from another, then we will have:

$$(\nabla_\rho \nabla_r)\,\Gamma(\rho, \mathbf{r}) = 0, \qquad (3)$$

$$\left(\frac{1}{4}\Delta_r + \Delta_\rho + k_0^2\right)\Gamma(\rho, \mathbf{r}) = 0. \qquad (4)$$

Expanding function $\Gamma(\rho, \mathbf{r})$ into the three-dimensional integral of Fourier

$$\Gamma(\rho, \mathbf{r}) = \iiint\limits_{-\infty}^{\infty} F(\mathbf{k}, \mathbf{r})\, e^{-i\mathbf{k}\rho} d^3\mathbf{k}, \qquad (5)$$

for its spectrum F we will obtain the following system of equations:

$$(\mathbf{k}\nabla_{\mathbf{r}})\, F(\mathbf{k}, \mathbf{r}) = 0, \qquad (6)$$

$$\left(\frac{1}{4}\Delta_{\mathbf{r}} + \mathbf{k}_0^2 - \mathbf{k}^2\right) F(\mathbf{k}, \mathbf{r}) = 0. \qquad (7)$$

The first of these equations shows that the spectrum of correlation function is retained along the rays/beams, parallel to vector $\mathbf{k}$: $\frac{d}{ds} F(\mathbf{k}, \mathbf{r}_0 + \mathbf{k}s) = 0$; the second - determines the class of the possible distributions F(k, r) in the plane, normal to the direction of this vector.

It is not difficult to see that if on the boundary of certain region is known the true distribution F, then the continuation of the spectrum inside the region can be realized with the help of the equation of transfer (6). Of course in the general case this does not simplify the solution of the boundary-value problem for $\Gamma(\rho, r)$, since for finding the spectrum on the boundary is necessary knowing the correlation function in certain of its vicinity, up to the distances from the boundary of the order of the greatest of the sizes/dimensions of the region of correlation field.

However, in some special cases, for example for the fields of the type of wave beam, boundary-value problem actually is reduced to this continuation, and then the use of equation (6) proves to be very

convenient.

In fact, let $\Gamma(\rho,r)$ be the slow to scale of length function r with the characteristic scale of heterogeneity L. It is not difficult to show that then when $|\rho| \ll \frac{L^2}{\lambda}$ the correlation function approximately is represented in the form

$$\Gamma(\rho, r) = \int_{4\pi} I(n, r) e^{-ik_0 n\rho} d\omega_n, \qquad (8)$$

moreover, the function I confronting under integral - angular spectrum of correlation function - it as before satisfies the equation of the transfer:

$$(n\nabla_r) I(n, r) = 0. \qquad (9)$$

it is possible to consider that this function defines certain field of radiant energy with the same as in field E(r), by the energy density distribution. Relationship/ratio (8) establishes connection/communication between the ray intensity of this field and the correlation properties of field E(r).

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For the fields of the type of wave beam the angular spectrum I is different from zero in the small solid angle about certain direction (for the certainty we will consider it as that coinciding with the direction of z axis of the Cartesian coordinate system) and can be expressed directly through the distribution of correlation in

the cross section of the beam:

$$I(\mathbf{n}, \mathbf{r}_s) = \left(\frac{k_0}{2\pi}\right)^2 \iint\limits_{-\infty}^{\infty} \Gamma(\rho_s, \mathbf{r}_s)\, e^{ik_0\mathbf{n}\rho_s}\, d^2\rho_s$$

($\mathbf{r}_s$ - radius-vector in plane z=const).

Thus, the calculation of the field of correlation according to its specified distribution in certain beam section (for example, in plane z=0) is reduced to the solution of the equation of transfer (9) under boundary condition $[I(\mathbf{n}, \mathbf{r})]_{z=0} = \tilde{I}(\mathbf{n}, \mathbf{r}_s)$, where $\tilde{I}$ - Fourier-transform $[\Gamma(\rho, \mathbf{r})]_{z=0}$, i.e. it consists of finding the planar-angular distribution of radiant energy in region z>0 according to the known distribution in plane z=0. Energy density is located in this case by the simple addition of ray intensity, which arrive into observation point from different points of plane z=0.

Analogously is solved the problem about the passage of the wave beam through the flat/plane shield with the assigned transformation ratio (which, in particular, can be random). It is reduced to the task about scattering of the pencil of rays on the rough surface, the indicatrix of scattering by which is represented in the form of the spectrum of the correlation function of the gear ratio/transmission factor of converter on the field.

Of course the ray method in question is applicable, in

particular, and to the regular fields. On the obtained spectrum of the I diffracted field in this case there can be restored the field itself E(r).

In the conclusion the equation of transfer (9) is generalized to the fields in the randomly heterogeneous media. Equation obtained in this case coincides with the equation of the transfer of radiant energy in the turbid medium, the role of the indicatrix of scattering playing the spectrum of the correlation function of the fluctuations of refractive index. In contrast to the usual equation of transfer this equation, just as (9), describes the propagation of radiation/emission taking into account diffraction effects.

Goubou wave dissipation on heterogeneities of beam guide.

R. B. Vaganov.

1. Ray waveguide, or, briefly, beam guide, is system of phase correctors (lenses or mirrors), being guided beam plane waves. Their own waves of ray waveguide compose the complete system of orthogonal functions; therefore any field distribution in plane z=const can be mathematically decomposed according to the functions of modes. This expansion makes physical sense, if transverse wave numbers γ in γ-distribution are limited $\gamma_{max} \ll k$.

If the field of the wave (for example, to the x- component of magnetic field) incident to the inadequate corrector we designate through $U_{\nu,n}(z_1)$, the field of the wave transmitted through the corrector is represented in the form of the series/row

$$U_{\nu,n}(z_1)\cdot T = \sum_{\nu',n'} C_{\nu,n,\nu',n'}\cdot U_{\nu',n'}(z_1).$$

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Here T - coefficient of transmission, calculated in the approximation/approach of geometric optic/optics, ν, n and ν', n' - azimuth and radial indices of incident and outgoing waves,

respectively $C_{\nu,n,\nu',n'}$ - the excitation coefficients of the waves of high orders. For the approximate computation are used the eigenfunctions of system from the correctors, unconfined of the radius:

$$C_{n,\nu,n'\nu'} \cong \frac{(-1)^{n-n'} n'!}{\pi (n'+\nu')!} \int_0^{\infty} \int_0^{2\pi} (\rho/\rho_{z_2})^{\nu'} (\rho/\rho_{z_1})^{\nu} L_n^{\nu}(\rho/\rho_{z_1})^2 L_{n'}^{\nu'}(\rho/\rho_{z_2})^2 e^{-\frac{\rho^2}{2}\left(\frac{1}{\rho_{z_1}^2}+\frac{1}{\rho_{z_2}^2}\right)} \times$$
$$\times e^{-j(\psi_1-\psi_2)} \cos \nu\varphi \cos \nu'\varphi \frac{\rho d\rho}{\rho_{z_1}\rho_{z_2}} d\varphi,$$

where $L_n^{\nu}(x)$ - generalized Lyagerr polynomials,

$$\rho_z^2 = \rho_0^2\left[1+\left(\frac{z}{k\rho_0^2}\right)^2\right], \psi = k\bar{z} - (2n+\nu+1) \operatorname{arc\,tg} \frac{\bar{z}}{k\rho_0^2} + \frac{\bar{z}}{2k\rho_0^2}(\rho/\rho_z)^2.$$

It is shown that the error, connected with this replacement, exponentially decreases with an increase in parameter $c=ka^2/L$, where k - wave number, a - a radius of corrector, L - the distance between the correctors. If $|T|=1$, then distortion carries purely phase character and action of the inadequate corrector it is reduced to the dissipation of energy of ray wave into the waves of adjacent numbers. For determining the character of the propagation of waves in the real system the transformation ratios are calculated on the assumption that the system is loss-free. Then to waves are assigned the losses, introduced by real correctors. These losses include radiation losses, also, in the case of the dielectric lenses of loss in the dielectric.

2. With transverse displacement of lens between forming on shifted lens modes and basic wave appear three-dimensional/space

beats. Beats form the beam, which in ideal system obeys the law of paraxial optic/optics. By the shifts/shears of lenses it is possible to return it to the optical axis, moreover its structure completely is restored. This phenomenon is analogous with the compensation for parasitic waves in the multiwave waveguide systems. If displacement is small in comparison with the sizes/dimensions of spot on the lens, then to optical laws is subordinated beam in the real system when the sum of the radiation and dielectric losses of fundamental wave and wave $TEM_{1,}$ are equal.

3. In beam guide with random uncorrelated displacement compensation does not occur. Are accumulated waves of the high orders whose energy in turn, is scattered on the waves of the following orders. Are designed coupling coefficients in the power in the chain/network of the waves which appear from the fundamental wave on the displacement of lenses and changes in their optical power. Are calculated the dielectric losses of all waves of chain/network. These data, and also known radiation losses make it possible to determine losses to the unit of the length of beam guide with the real lenses, if is known the standard deviation of lenses from the ideal geometry. Calculation is performed on the assumption that the chain/network is broken due to the large radiation losses on each lens, and is established certain repeating distribution according to the power between the modes.

4. Is carried out experimental check of basic conclusions of the works. It is shown that the coupling coefficients calculated according to the obtained formulas between the modes coincide well with the experiment even during the displacement of the order of a radius of spot on the lens. Under the condition

$$\frac{n\,\mathrm{tg}\,\delta}{n-1} \simeq \eta_{10},$$

where n - refractive index of dielectric, η_{10} - the radiation losses of wave TEM_{10}, losses to the conversion into the waves of high orders virtually completely are compensated by artificial shift/shear.

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Geometric optics of open resonators.

V. P. Bykov.

In the report is developed the geometric method of the study of the open resonators. It is shown that during certain modification this method makes it possible to obtain the natural frequencies of oscillation and the position of the caustic surfaces, which correspond to these oscillations.

Is studied the propagation of rays/beams in the mirror general ellipsoid and it is shown that the caustic surfaces of the families of rays/beams in it are the surfaces of the second order, confocal to mirror ellipsoid. Is given the classification of the natural oscillations of mirror ellipsoid.

In all are four vibration modes. Two of them (developing in the barrel-shaped resonators), actually, relate to the oscillations of so-called "of that whispering of galleries", by studied Rayleigh. Remaining two are of greatest interest for studying the oscillations in the gas quantum generators.

Along some lines, which lie on the caustic surfaces, the propagation of the wave front occurs at speed, equal to the speed of light in the vacuum. Such lines are the intersections of caustic surfaces and the geodetic, concerning intersections. For the steady-state oscillations it is necessary that the wave front, which extends on one of the paths, would lag behind the front, which extends on another path, to the integer of wavelengths. This requirement makes it possible to write three quantum conditions, that are determining natural frequencies and position of the caustic curves of natural oscillations. In this case it is necessary to consider "phase jump on the caustic".

In the ellipsoidal coordinate system quantum conditions take the following form:

$$k\int_{0}^{\lambda_1^0}\sqrt{\frac{(\lambda_1^0-\lambda_1)(\lambda_3^0-\lambda_1)}{(a_1-\lambda_1)(a_2-\lambda_1)(a_3-\lambda_1)}}\,d\lambda_1=\left(2l+\frac{1}{2}\right)\pi,$$

where k - wave number, λ_1 - first ellipsoidal coordinate λ^0_1 and λ^0_3 - parameters, which are determining the position of caustic surfaces, l - integer.

For three unknowns k, λ^0_1 and λ^0_3, are three quantum conditions.

The quantum conditions of different vibration modes are distinguished by numerical factors in the right side, integration limits and by parameters, which are determining the position of caustic surfaces.

Hyperelliptic integrals on the left side of the quantum condition explicitly are not taken. However, in the specific cases, using symmetry of task and these or other physical simplifications, these integrals are reduced to the simpler and are taken explicitly.

Since the value of electromagnetic field exponentially drops after the caustic surface, then, having a little stepped back from this surface, it is possible to break the mirror surface of ellipsoid, as a result of which appears the open resonator.

The sense of transition to the open resonator consists in the fact that in this case are suppressed all vibration modes, except those chosen. This fact is very important, since the density of oscillations in the locked ellipsoid is very great at wavelengths much smaller sizes/dimensions of resonator.

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Work examines the following specific cases: 1) the oscillation between two spherical mirrors, 2) the oscillation between two

spherical mirrors in the presence between them of two inclined plane-parallel plates (Brewster's so-called windows), 3) oscillation in the barrel-shaped resonators. In the first and third cases are obtained the simple formulas, which are determining frequencies and position of caustic curves. The second case is interesting in that small disturbance/perturbation, introduced by plane-parallel plates, causes a qualitative change in the vibration mode.

The geometric method, developed in the present work possesses large simplicity and clarity. It supplements well wave approach, permitting for more bending to use its results.

A statistical analysis of the process of the propagation of beam in a weakly deformed circular metal tube.

V. A. Zyatitskiy, B. Z. Katsenelenbaum.

1. Coefficient of reflection of ray/beam, which falls to metallic surface at glancing angle and polarized so that in parallel to metal, it is very close to unity. On this basis/base it was proposed to transmit light/world up to the large distances within the circular metal tube: polarized beam repeatedly is reflected from the metal and covers considerable distances with the low losses.

In the real duct of wall it is slightly deformed: normal to the surface of metal at different points forms different angles with the surface of ideal cylinder. These angles further are considered as two random functions of the number of reflection. Into this diagram enter both small and large random deformations of surface, random curvatures with the large radius of curvature, etc. Entire calculation is produced for one ray/beam within the limits of the applicability of geometric optic/optics.

2. Comparatively simply is produced analysis of deformations

during which meridional (passing through axis) ray/beam remains meridional; example to this deformation is change in radius of duct with retention/preservation/maintaining of straightness of axis. It causes an increase in average slip angle: equal to $2\mu^2/\alpha$, where α - slip angle, μ^2 - the mean square of the inclination/slope of standard/normal. This effect leads to the increase in the fading, connected with the fact that with the increase/growth α is reduced the reflection coefficient and, furthermore, is reduced the distance between two mirror points. Fading at large distances increases in this case as the cube of distance.

3. Divergence of section of duct from circle causes departure/attendance of ray/beam from meridian plane. With each reflection of helical (nonmeridian) ray/beam occurs the energy exchange between two polarizations. This process is described by the iterative equations:

$$\begin{aligned} p_{n+1} &= r_p\,(p_n \cos^2\gamma_n + q_n \sin^2\gamma_n) \\ q_{n+1} &= r_q\,(p_n \sin^2\gamma_n + q_n \cos^2\gamma_n) \end{aligned} \qquad (*)$$

Here p_n, q_n - energy of basic and parasitic polarization after the n reflection, r_p and r_q - coefficients of reflection ($r_q < r_p$) and γ_n - angle between the planes of incidence in the n-th and (n+1)th reflections. Angle γ_n random variable; $\gamma_n = 2\sum_{i=1}^{n} \nu_i$, where ν_i - angle between the direction of normal to the metal and the radius-vector at the point of the i reflection. Equations (*) do not consider a phase difference of rays/beams, being combined into one ray/beam after

reflection; analysis shows that this energy addition is lawful.

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With not very large n and $\gamma_n \ll 1$ system (*) has with $p_0=1$, $q_0=0$ a solution:

$$\overline{\ln p_n} = -n \ln \frac{1}{r_p} - n^2 \cdot 2\nu_0^2 + n^3 A \nu_0^4, \qquad (**)$$

where ν_0^2 is an average/mean value of random variable ν_i^2 and A(A>0) depends on relation $\frac{r_p}{r_q}$. First term describes ohmic fading, the second - energy loss to the transition into another polarization. Helical nature produces with each reflection the energy loss of order γ_i^2 (even if further it follows the undeformed duct), and $\overline{\gamma_i^2}$ linearly increases with n ($\bar{\nu}_i = 0; \overline{\nu_i^2} = \nu_0^2; \overline{\nu_i \nu_j} = 0, i \neq j; \overline{\gamma_n} = 0; \overline{\gamma_n^2} = 4\nu_0^2 n$).

With this is connected squareness on n of second term. Third component/term/addend is caused by reverse process - the transition of energy from q- of polarization in p- polarization.

4. With large n it is possible to give only estimates of the magnitude p_n and q_n. For the complete statistical analysis of process (*) was produced its imitation on computer(s). As ν_i were introduced the random numbers with different values ν_0^2 and two (uniform and gaussian) laws of distribution. Calculation was produced for the series/row of values r_p and r_q for initially polarized

p_0=1, q_0=0) and the nonpolarized (p_0=1/2, q_0=1/2) light/world. Were found averages $\overline{p_n}$ and $\overline{q_n}$ (averaging for each n on hundred realizations) and dispersion p_n and q_n. Is established/installed the law of fading $\overline{p_n}$ and $\overline{q_n}$ at the values of the parameters, for which (**) is already not applicable. It turned out that the efficient statistical analysis of this complicated process, which contains "accumulation of noise" ($\overline{\gamma_n^2} \sim n$), requires relatively short machine time.

Diffractometer as instrument, which uses phenomenon of diffraction for multichannel spectral or correlation analysis of random processes.

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Optical equipment makes it possible to carry out both the consecutive and parallel analysis of spectra and correlation functions of transparent structures. Diffractometer is one of such instruments. Observing the diffraction pattern of Fraunhofer from these structures or their combinations it is possible to obtain the spectra or correlation functions simultaneously for a large number of processes, registered, for example, in the different rows of photographic film in the form of variable/alternating transparency. A maximum number of simultaneously working channels in this case depends on the quality of optic/optics and film. Instrument virtually can treat very large quantity of information for a comparatively small time, possessing in this essential advantage in comparison with the radio engineering devices/equipment and even with TsVM [ЦВМ - digital computer].

With the help of the instrument it is possible to also realize distribution of signals and detection of weak signals against the background of noise. Resolution and dynamic range, determined for the sine waves, depend on the size/dimension of the "window" of optical system and quality of display system.

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Instrument can be used and as the optimum matched filter for the detection of the signal of special form. In this case is used the diffraction pattern of Fresnel.

Very essential for the work of instrument proves to be the quality of film, on which is produced the recording of the analyzed signal.

The distortions of amplitude and the phases of light wave, which appear after passage through "the uniform" lit film (the "noises of film") limit both resolution and dynamic range. With the noises of film it is possible to fight, using an immersion.

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